Submission Date: 16. May 2023

Original Author: Andrew Evans, Imperal College, Contact Author: N. P. Høj, HOJ Consulting

Contact Email: nph@hoj.ch

Title: FN-criterion lines, criticism by Evans and Verlander

The results of quantitative risk analyses will be in terms of the e.g. the number of fatalities, n and the associated probability (or frequency), f. In case of systems or accidents with a range i=1..x of possible consequences n_i and associated frequencies f_i the result of the risk analysis will be

$$R = \sum_{i=1..x} f_i \cdot n_i \qquad (1)$$

An FN-curve, $F(n_j) = \sum_{i=j..x} f_i$, may be used to illustrate the results in a double-logarithmic diagram. The acceptability of the risk may be evaluated in the next step of the risk assessment.

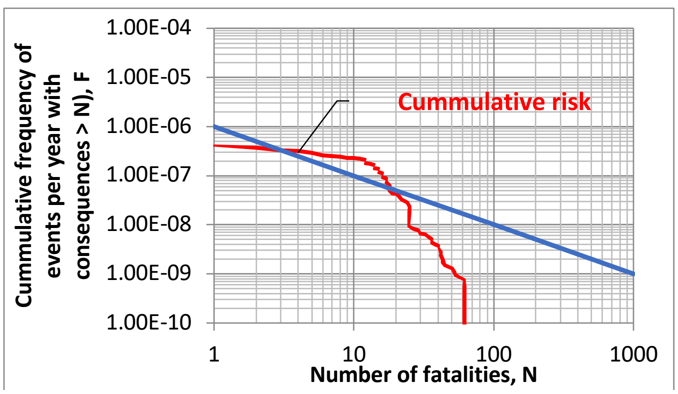

Figure 1. Example of an FN-curve and an FN-criterion line with slope = - 1.

Risk acceptance is often formulated as a criterion line in the double logarithmic representation. The criterion line can be described by its intersection with the n=1 axis and the slope.

Criticism against using FN-criterion lines has been published by Evans and Verlander[1]. In the notation of Evans and Verlander the FN-curve is described by the equation F = F(n) and the criterion function by F = C(n). The tolerability of the system is represented by comparing these two functions, which in the logarithmic representation of the FN-curve leads to the criterion:

$$log[F(n)] - log[C(n)] \leq 0 \; for \; all \; n = 1,2,\ldots \Longrightarrow$$

$$\max_n\{log[F(n)] - log[C(n)]\} \leq 0 \Leftrightarrow$$

$$\max_n\left\{log\left[\frac{F(n)}{C(n)}\right]\right\} \leq 0 \Leftrightarrow \max_n\left\{\frac{F(n)}{C(n)}\right\} = 1 \qquad (2)$$

A family of parallel criterion lines are considered, F = v·C(n), where v is a positive number in the range [0; ∞]. There will always be one member of the family of criterion line for which the FN-curve is exactly tolerable.

$$\max_n\{log[F(n)] - log[vC(n)]\} \leq 0 \Leftrightarrow \max_n\left\{\frac{F(n)}{C(n)}\right\} = v \qquad (3)$$

v is the intolerability measure of the system. Larger v indicates more intolerable systems.

[1] What is wrong with criterion FN-lines for judging the tolerability of risk? A. Evans & N Verlander, Risk Analysis, Vol. 17, No. 2, 1997.

If the criterion function is a straight line with a slope of α and an intersection with N=1 at L:

$$log[C(n)] = \log(L) - \alpha \log(n) \Rightarrow C(n) = \frac{L}{n^{\alpha}} \quad (4)$$

Therefor the intolerability measure for straight-line criterion function is:

$$v = \max_{n}\left\{\frac{F(n)n^{\alpha}}{L}\right\} \quad (5)$$

If a decision is made between i competing systems, the system with the least intolerability should be chosen. This can for a straight-line criterion function be formulated as:

$$\min_{i}\{v_i\} = \min_{i} \max_{n}\left\{\frac{F_i(n)n^{\alpha}}{L}\right\} \quad (6)$$

Based on tests of FN-curves which are just acceptable to a FN-criterion line with slope -1 conclusions can be drawn that FN-criterion lines may be less suited for evaluating and comparing risk, and it conflicts with the basic goals of risk evaluation.

It is stated that "*There are two objections to the minimax or FN-criterion. First, by concentrating on just one extreme feature of a statistical distribution, the minimax criterion ignores other features which are relevant to a decision. In this way, the minimax criterion can lead to decisions that appear unreasonable. Second, and more seriously, minimax criteria are, in the language of decision theory, incoherent. That is, they give inconsistent preferences, or, when applied in the present context they give inconsistent judgments about tolerability of risk*."

The inconsistency of criterion FN-lines is proved by comparing two engineering systems 1 and 2: Consistency requires that if both systems are judged tolerable separately, the combined system should also be judged tolerable, and if both systems are judged intolerable separately, the combined system should also be judged intolerable. Furthermore, if both separate systems are judged to be *just* tolerable, the combined system must be judged to be *just* tolerable.

It follows that the tolerability threshold, in term of expected fatalities or the utility of these, U, for the combined system must be $U_1 + U_2$ (the sum of the thresholds for the individual systems). It can be proved that if the individual thresholds are satisfied the combined is always satisfied.

Similarly, the FN-criterion function, C(n) for the combined system must be $C_1(n) + C_2(n)$. If both engineering systems are just tolerable $F_1(n) = C_1(n)$ and $F_2(n) = C_2(n)$. It follows that consistency then requires (for every accident size, n) that

$$C(n) = F(n) = F_1(n) + F_2(n) = C_1(n) + C_2(n) \quad (7)$$

The general consistency condition requires in the case of the FN-criterion, in terms of the intolerability measure, v, that the following condition is fulfilled:

If $v_1 = 1$ and $v_2 = 1$, then v must be 1, where

$$v_i = \max_{n}\left\{\frac{F_i(n)}{C_i(n)}\right\} \; for\ i = 1,2 \text{ and } v = \max_{n}\left\{\frac{F_1(n)+F_2(n)}{C_1(n)+C_2(n)}\right\} \quad (8)$$

It cannot be proved mathematically that this is always satisfied. On the contrary, a counter example can be provided, to demonstrate that the condition (8) is not generally satisfied: Say, both systems are described by single-step FN-curves: System 1 has a frequency of 0.1 accidents/year and a consequence of 10 fatalities, system 2 has a frequency of 0.01 accidents per year and a consequence of 100. Both of these systems are just tolerable and satisfy the criterion line of C(n) = 1/n.

$$v_1 = \max_{n}\left\{\frac{F_1(n)}{C_1(n)}\right\} = 0.1 \cdot 10 = 1 \text{ and } v_2 = \max_{n}\left\{\frac{F_2(n)}{C_2(n)}\right\} = 0.01 \cdot 100 = 1 \quad (9)$$

For the combined system

$$C(n) = C_1(n) + C_2(n) = \frac{1}{n} + \frac{1}{n} = \frac{2}{n} \tag{10}$$

Therefore,

$$v = \max_n \left\{\frac{F_1(n)+F_2(n)}{C_1(n)+C_2(n)}\right\} = \max_n \left\{\frac{[F_1(n)+F_2(n)]\cdot n}{2}\right\} = \frac{0.11\cdot 10}{2} = 0.55 \neq 1 \tag{11}$$

Hereby it is demonstrated that the condition in (8) is not satisfied, which is sufficient to show that the condition is not satisfied in general, and that the FN-criterion lines are inconsistent in general.

The paper by Evans and Verlander concludes that "*FN-criterion lines is equivalent to a minimax rule for taking decisions under uncertainty. Such rules can give inconsistent decisions when a given decision problem is presented in different ways. They can also give decisions that appear unreasonable, because they do not make use of all relevant information. Statistical decision theory suggests an alternative and preferable rule of minimizing the expected disutility, that is average harm, from accidents*".