Summary TFY4305 2014

Here is a short summary of the most important topics covered in 2014.

Chapter 2

Flow on the line:

$$\dot{x} \quad = \quad f(x) \; , \tag{1}$$

Fixed points x^* are given by

$$f(x^*) \quad = \quad 0 \; . \tag{2}$$

Linear stability of fixed point x^* given by sign of

$$f'(x^*) \; . \tag{3}$$

Potentials $V(x)$

$$f(x) \quad = \quad -\frac{dV}{dx} \; . \tag{4}$$

Stable fixed points correspond to local minima of $V(x)$ etc.

Chapter 3

Saddle-node bifurcation:

$$\dot{x} \quad = \quad r + x^2 \; , \tag{5}$$

where r is a parameter. Creation of fixed points. Normal forms and stability diagram.

Transcritical bifurcation

$$\dot{x} \quad = \quad rx - x^2 \; . \tag{6}$$

Exchange of stability of fixed points.

Pitchfork bifurcation:

$$\dot{x} \;=\; rx - x^3 \;. \tag{7}$$

Subcritical and supercritical pitchfork bifurcations. Hysteresis.

Imperfect bifurcations

$$\dot{x} \;=\; h + rx - x^3 \;, \tag{8}$$

where h is imperfection parameter. Stability diagram in h, r-space.

Chapter 4

Flows on the circle:

$$\dot{\theta} \;=\; f(\theta) \;, \tag{9}$$

where $f(\theta)$ must be a *periodic* function.

Nonuniform oscillator

$$\dot{\theta} \;=\; \omega - a \sin\theta \;, \tag{10}$$

where $a \geq 0$ and $\omega > 0$. Oscillation period

$$T \;=\; \frac{2\pi}{\sqrt{\omega^2 - a^2}} \;. \tag{11}$$

Saddle-node bifurcation at $\theta = \pi/2$ and bottleneck near $\theta = \pi/2$. Ghost or saddle-node remnant. Overdamped pendulum.

Chapter 5

Linear systems in two dimensions. Phase portraits. Stable and unstable manifold. Attracting, globally attracting, and Liapunov stable, and unstable fixed points. Classification of fixed points (saddles, nodes, centers, centers and spirals) and borderline cases (stars centers, non-isolated fixed points) and eigenvalues/eigenvectors.

Chapter 6

Phase portraits, existence and uniqueness of trajectories (curves never cross). Fixed points and closed curves. Nullclines. Fixed points and linearization. Effects of nonlinear terms and qualitatively correct prediction of linear theory for saddles, nodes, and spirals.

Lotka-Volterra model of competition of two species. Basin of attraction for fixed points. Conservative systems and absence of attracting fixed points. Homoclinic orbits. Nonlinear centers. Reversible systems and nonlinear centers. The pendulum and its phase portrait.

Index theory and index of a closed curve and of a point. Properties of the index of a curve. Index theory to rule out closes orbits.

Chapter 7

Limit cycles in the plane. Simple examples. Ruling out closed orbits in gradient systems and using Liapunov functions. Dulac's criterion. Poincare-Bendixon theorem to prove existence of closed curves (recall all conditions!). Construction of trapping regions. Examples and applications.

Chapter 8

Saddle-node bifurcations, transcritical bifurcations, and pitchfork bifurcations in two dimensions. Behavior of eigenvalues. Hopf bifurcations (subcritical and supercritical) and behavior of eigenvalues. Degenerate bifurcations. Chemical reactions. Global bifurcation of cycles: Saddle-node bifurcation and infinite-period bifurcation. Existence of closed orbits and Poincare maps (Damped pendulum with external torque).

Chapter 9

Nothing

Chapter 10

Fix points for discrete maps $x_{n+1} = f(x_n)$:

$$x^* \;=\; f(x^*) \; . \tag{12}$$

Linear stability is given by $f'(x^*)$. Use *cobweb* as a graphical technique to determine stability of fixed point. Superstability.

Logistic map $x_{n+1} = rx(1-x)$ and period doubling at specific values of r when $f'(x^*) = -1$

(flip bifurcations). Stability of p-cycles. Period-doubling route to chaos. Periodic windows and chaos. Orbit diagrams. Bifurcation diagrams. Universality and renormalization for unimodal maps.

Chapter 11

Countable and uncountable sets. Cantor set as example of a fractal. Properties of Cantor-like sets (uncountability, zero measure). Similarity and box dimension of fractals. von Koch curve. Period doubling route to chaos and fractal structure of attractor.

Chapter 12

Baker's map and examples 12.1.1-12.1.2. Lozi map: fixed points and their stability.