Gluon thermodynamics at intermediate coupling

Jens O. Andersen [1]

Norwegian University of Science and Technology
Department of Physics

Montreal, Canada
June 30 2010

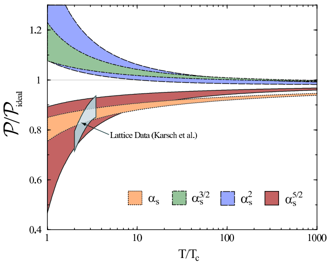
[1] In collaboration with Eric Braaten, Emmanuel Petitgirard, Lars Kyllingstad, Lars Leganger, Mike Strickland, and Nan Su. PRL **83**, 2139 (1999), PRD61 **014017** (2000), PRD **61**, 074016 (2000), PRD **63**, 105008 (2001), PRD **64**, 105012 (2001), PRD **66**, 085016 (2002), PRD **70**, 045001 (2004), PRD **71**, 025011 (2005), PRD **78**, 076008,(2008), JHEP **0908**, 066 (2009), PRD **80**, 085015 (2009), PRL **104** 122003 (2010).

Heavy-ion collisions

- Understanding of the quark-gluon plasma essential for heavy-ion collisions
    - At RHIC temperatures are at 400 MeV $\sim 2T_c$
    - At LHC temperatures will even higher: $T \sim 4 - 5T_c$
- Strongly coupled plasma at RHIC
- Is a quasiparticle approach of weakly interacting particles appropriate at LHC [2]?
- What about lattice data?

[2] Blaizot, Iancu and Rebhan 2001.

Weak-coupling expansion



Perturbative free energy vs temperature for QCD with $N_F = 2$ and $N_c = 3$. Lattice results from Karsch et. al. 03.

- The weak-coupling expansion of the free energy of QCD has been calculated to order $\alpha_s^3 \log \alpha_s$ [a].
- Temperatures expected at RHIC energies are $T \sim 0.3$ GeV corresponds to $\alpha_s(2\pi T) \sim 1/3$ or $g_s \sim 2$.
- Successive terms contributing to $\mathcal{F}$ form a decreasing series only if $\alpha_s \simeq 1/20$ or $T \sim 10^5$ GeV.

[a] Arnold and Zhai, 94/95, Kastening and Zhai 95, Braaten and Nieto 96, Kajantie, Laine, Rummukainen, and Schröder 02.

Screened perturbation theory

- Massless ϕ^4-theory :

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{g^2}{24}\phi^4$$

- Add and subtract a (thermal) mass terms [3]

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}m^2\phi^2 + \frac{1}{2}m_1^2\phi^2 - \frac{g^2}{24}\phi^4$$

- $m_1 = m$ and we recover the original theory.
- Treat $\frac{1}{2}m_1^2\phi^2$ and $\frac{g^2}{24}\phi^4$ as interactions on equal footing.

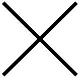
[3] F. Karsch, A. Patkos, and P. Petreczky, PLB401, 69 (1997).

Screened perturbation theory

$$\begin{aligned}
\mathcal{L} &= \mathcal{L}_{\rm free} + \mathcal{L}_{\rm int} \; ; \\
\mathcal{L}_{\rm free} &= \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - \frac{1}{2}m^2\phi^2 \\
\mathcal{L}_{\rm int} &= \frac{1}{2}m_1^2\phi^2 - \frac{g^2}{24}\phi^4
\end{aligned}$$

Expanding around an ideal gas of massive particles.

Quartic interaction:



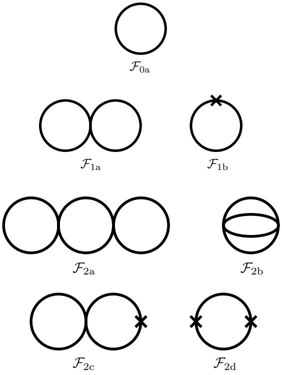
$= -\frac{g^2}{24}$

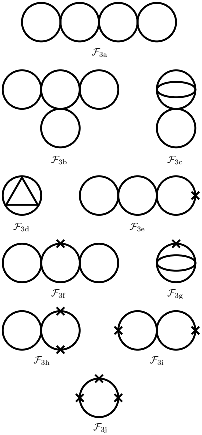
Mass insertion:

$= \frac{1}{2}m_1^2$

The diagrams

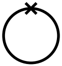
The free energy $\mathcal{F}$ is the sum of these diagrams:





Calculating $\mathcal{F}$

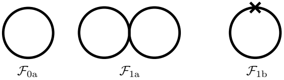
- Expand in powers of g^2 (loop expansion).
- Expand each diagram in powers of $m/T \sim g$:



$$
\begin{aligned}
&= -\frac{1}{2}m_1^2 T \sum_{p_0} \int_p \frac{1}{P^2+m^2} \\
&= -\frac{1}{2}m_1^2 T \left[\int_p \frac{1}{p^2+m^2} + \sum_{p_0\neq 0} \int_p \frac{1}{P^2+m^2} \right] \\
&= -\frac{1}{2}m_1^2 T \left[\int_p \frac{1}{p^2+m^2} + \sum_{p_0\neq 0} \int_p \frac{1}{P^2} \left(\frac{m^2}{P^2} + \frac{m^4}{P^4} + \cdots \right) \right]
\end{aligned}
$$

- Truncate expansion at g^7.

Calculating $\mathcal{F}$



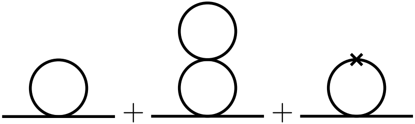
$$
\begin{aligned}
\mathcal{F}_0 &= -\frac{\pi^2 T^4}{90}\left[1 - 15\hat{m}^2 + 60\hat{m}^3 + ...\right] \\
\mathcal{F}_1 &= -\frac{\pi^2 T^4}{90}\alpha\left[\frac{5}{4} - 15\hat{m} + ...\right] - \frac{\pi^2 T^4}{90} 15\hat{m}_1^2\left[1 - 6\hat{m} + ...\right] , \\
\alpha &= \frac{g^2}{4\pi} \\
\hat{m} &= \frac{m}{2\pi T} .
\end{aligned}
$$

Calculating $\mathcal{F}$

- Ultraviolet divergences removed by renormalization of the vacuum, m^2 and g^2.
- UV-divergences and counterterms are temperature dependent(!).
- Temperature-dependent divergences are systematically substracted out.
- Final result obtained by setting $m_1 = m$. Need a prescription for m as a function of g and T.

Tadpole mass

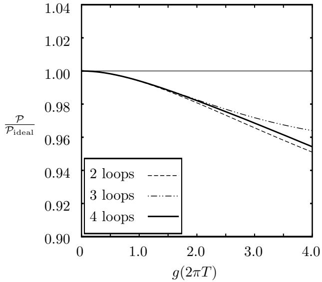
We choose m to be the tadpole mass,

$m^2 =$  $+ \cdots = g^2 \left.\frac{d\mathcal{F}}{dm^2}\right|_{m_1^2=m^2}$

- Self-consistent gap equation for m. Well-defined to all loop orders.
- Selective resummation of diagrams from all loop orders in the original (massless) theory.

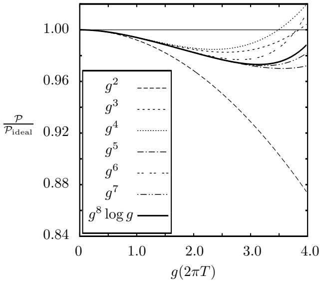
Comparison

Screened perturbation theory:



4

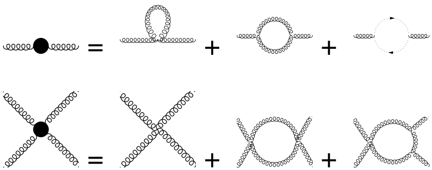
Weak-coupling expansion:



[4] JOA and L. Kyllingstad, PRD **78**, 076008 (2008), R.R. Parwani and H. Singh, PRD **511**, 4518 (1995), E. Braaten and A. Nieto, PRD **51**, 6990 (1995), Gynther et al JHEP **04** 094 (2007), JOA, L. Kyllingstad, and L. Leganger JHEP **08**, 066 (2009).

Hard-thermal-loop perturbation theory

- Extension of SPT to gauge theories.
- Cannot simply add and subtract a mass term since this would violate gauge invariance.
- Must use effective progators and vertices that are encoded in the HTL correction term



Hard-thermal-loop perturbation theory

- HTL perturbation theory is a reorganization of the perturbative series for gauge theories which is similar in spirit to SPT.

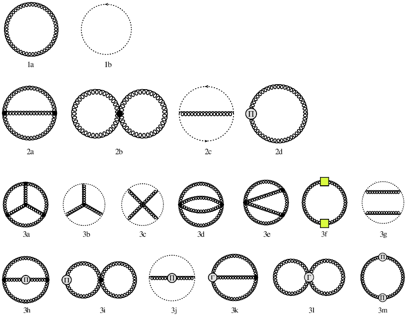
$$\mathcal{L}_{\rm HTLpt} = \left(\mathcal{L}_{\rm QCD} + \mathcal{L}_{\rm HTL}\right)\Big|_{g\rightarrow\sqrt{\delta}g}\;,$$

$$\mathcal{L}_{\rm HTL} = -\frac{1}{2}(1-\delta)m_D^2{\rm Tr}\left(G_{\mu\alpha}\left\langle\frac{y^\alpha y^\beta}{(y\cdot D)^2}\right\rangle_y G^\mu{}_\beta\right)$$

$$+(1-\delta)\, im_f^2\bar{\psi}\gamma^\mu\left\langle\frac{y^\mu}{y\cdot D}\right\rangle_y\psi\;.$$

- HTLpt is defined by expanding in powers of δ.

Feynman diagrams



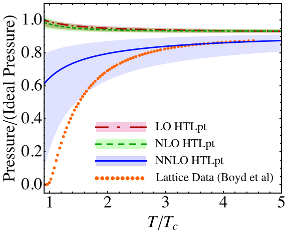
- Double expansion in g^2 and m_D^2

Pure-glue QCD

$$
\begin{aligned}
\Omega_{\rm NNLO} \quad = \quad & \mathcal{F}_{\rm ideal}\left\{1-\frac{15}{4}\hat{m}_D^3+\frac{N_c\alpha_s}{3\pi}\left[-\frac{15}{4}+\frac{45}{2}\hat{m}_D-\frac{135}{2}\hat{m}_D^2-\frac{495}{4}\left(\log\frac{\hat{\mu}}{2}+\frac{5}{22}+\gamma_E\right)\hat{m}_D^3\right]\right.\\
&+\left(\frac{N_c\alpha_s}{3\pi}\right)^2\left[\frac{45}{4\hat{m}_D}-\frac{165}{8}\left(\log\frac{\hat{\mu}}{2}-\frac{72}{11}\log\hat{m}-\frac{84}{55}-\frac{6}{11}\gamma_E-\frac{74}{11}\frac{\zeta'(-1)}{\zeta(-1)}+\frac{19}{11}\frac{\zeta'(-3)}{\zeta(-3)}\right)\right.\\
&\left.\left.+\frac{1485}{4}\left(\log\frac{\hat{\mu}}{2}-\frac{79}{44}+\gamma_E+\log 2-\frac{\pi^2}{11}\right)\hat{m}_D\right]\right\}.
\end{aligned}
$$

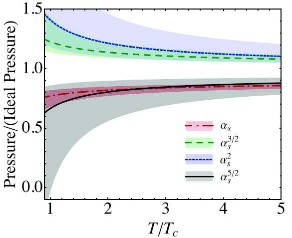
- Variational mass has complex solution.
- Weak-coupling expansion of Debye mass involves magnetic mass and is IR-divergent.
- Use m_E^2 from dimensional reduction. Gauge invariant and well defined to all orders.

Comparision -lattice



HTLpt [a]

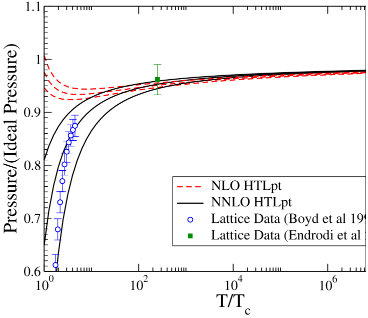
[a] JOA, N. Su, and M. Strickland PRL **104**, 122003 (2010).



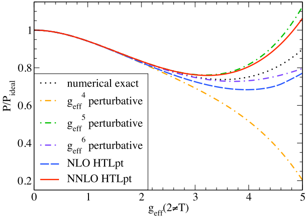
Weak-coupling expansion [a]

[a] Arnold and Zhai, 94/95, Kastening and Zhai 95, Braaten and Nieto 96, Kajantie, Laine, Rummukainen, and Schröder 02.

Comparision - lattice II



Comparision - Large-N_f



Exact: G. Moore, A. Ipp, A. Rebhan, JHEP **0301** 037 (2003); Perturbative:A. Gynther, A. Kurkela, and A. Vuorinen, PRD **80** 096002 (2009); HTLpt: JOA, N. Su, and M. Strickland PRD **80** 085015 (2009). (2010).

Summary and Outlook

- Poor convergence of perturbation theory is a generic problem in scalar and gauge theories at finite temperature
- SPT and HTLpt can be used to improve the convergence of perturbative calculations. Good agreement with lattice for $T \simeq 3T_c$.
- NNLO calculations of QCD with fermions underway.
- HTL perturbation theory can be used to calculate dynamic quantities systematically in a gauge-invariant manner. Relevant to LHC.