Resummation: screened perturbation theory versus weak coupling

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Outline

1. Introduction
2. Effective field theory
3. Screened perturbation theory
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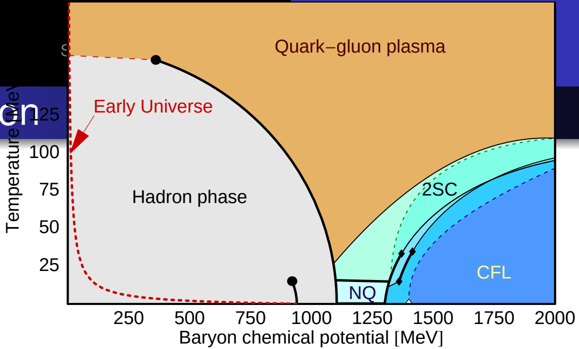
Introduction

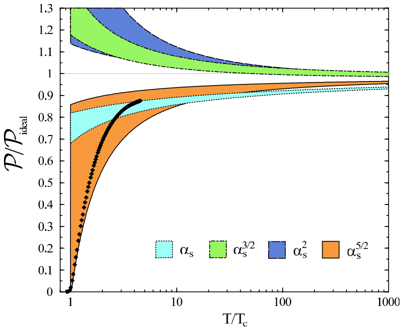


- Phase diagram of QCD [2]

[2] Taken from Shovkovy's homepage

Introduction

- Weak-coupling expansion of the pressure through order $\alpha_s^{5/2}$ [3].



[3] Zhai and Kastening, PRD **52**, 7232 (1995): Braaten and Nieto PRD **53**, 3421(1996), Boyd et al (lattice)

Introduction

- Pressure:
  - Can be calculated to order $\alpha^{5/2}$ either using resummed perturbation theory or effective field theory
  - Poor convergence properties
  - Breaks down at order α^3 due to infrared divergences (magnetic mass problem)
  - Three scales in the problem> T, gT, and g^2T
  - Can use effective field theory methods to calculate effective theory for scale g^2T
  - Put it on the lattice
  - g^6 contribution from scale T can be calculated from perturbation theory.

Dimensional reduction

- Fields (anti)periodic in Euclidean time $\beta = 1/T$
- Free propagators

$$\Delta = \frac{1}{p_0^2 + p^2} ,$$

- Nonzero Matsubara modes have a mass of order T and decouple.
- Construct an effective field theory in three dimensions for static mode
- Effective field theory valid for soft scale gT

Recipe

- Recipe:
  - Write down the most general three-dimensional Lagrangian consistent with the symmetries
  - Determine the coefficients by matching
  - Power counting
  - Use the effective Lagrangian in actual calculations

$$\mathcal{L}_{\rm eff} = \frac{1}{2}(\nabla\phi)^2 + \frac{1}{2}m^2\phi^2 + \frac{g_3^2}{24}\phi^4 + ...$$

- Also include coefficient of unit operaror f!
- Coefficients of $\mathcal{L}_{\rm eff}$ are power series in g^2

$$
\begin{aligned}
\mathcal{F}_{\rm hard} &= -\frac{\pi^2 T^4}{90}\left\{1-\frac{5}{4}\alpha\right. \\
&+\frac{15}{4}\left[L+\frac{1}{3}\gamma_E+\frac{31}{45}+\frac{4}{3}\frac{\zeta'(-1)}{\zeta(-1)}-\frac{2}{3}\frac{\zeta'(-3)}{\zeta(-3)}\right]\alpha^2 \\
&+\frac{15}{16}\left[\frac{\pi^2}{\epsilon}-12L^2-\left(\frac{1084}{45}+8\gamma_E-8\pi^2+32\frac{\zeta'(-1)}{\zeta(-1)}\right.\right. \\
&-16\left.\frac{\zeta'(-3)}{\zeta(-3)}\right)L-\frac{134}{9}-\frac{25}{3}\gamma_E^2-\frac{1}{27}\zeta(3) \\
&+\frac{31}{15}\gamma_E-\frac{\pi^2}{2}+4\gamma_E\pi^2-\frac{206}{9}\frac{\zeta'(-1)}{\zeta(-1)}-\frac{16}{3}\gamma_1+8\gamma_E\frac{\zeta'(-3)}{\zeta(-3)} \\
&+\frac{4}{3}\gamma_E\frac{\zeta'(-1)}{\zeta(-1)}-8\left(\frac{\zeta'(-1)}{\zeta(-1)}\right)^2-\frac{20}{3}\frac{\zeta''(-1)}{\zeta(-1)} \\
&\left.\left.-\frac{2}{3}C'_{\rm ball}+2C^a_{\rm triangle}+\pi^2 C^b_{\rm triangle}\right]\alpha^3+\mathcal{O}(\epsilon)\right\}\,.
\end{aligned}
$$

$$g_3^2 = g^2 T \left[1 - \frac{3g^2}{(4\pi)^2}(L + \gamma_E) - \frac{3g^2}{(4\pi)^2} \left(L^2 + 2\gamma_E L + \frac{\pi^2}{8} - 2\gamma_1 \right) \epsilon \right]$$

$$
\begin{aligned}
\tilde{m}^2 &= \frac{1}{24}g^2T^2\Bigg\{1+\frac{g^2}{(4\pi)^2}\left[L+2-\gamma_E+2\frac{\zeta'(-1)}{\zeta(-1)}\right] \\
&-\frac{6g^4}{(4\pi)^4}\Bigg[\frac{5}{2}L^2+\frac{19}{18}L+\frac{2851}{864}-\frac{95}{48}\gamma_E^2 \\
&+3\gamma_E L-\frac{119}{144}\gamma_E-\frac{1}{144}\zeta(3)-7\gamma_1 \\
&+\frac{\zeta'(-1)}{\zeta(-1)}\left(2L+\frac{113}{72}+\frac{17}{12}\gamma_E\right)-\frac{1}{4}\frac{\zeta''(-1)}{\zeta(-1)}+\frac{25}{32}\pi^2 \\
&-2\gamma_E\log(2\pi)+2\log^2(2\pi)-\frac{1}{24}C'_{\rm ball}+\frac{1}{4}C_l\Bigg]+\mathcal{O}(\epsilon)\Bigg\}\,, \\
\Delta m^2 &= \frac{g^4T^2}{24(4\pi)^2\epsilon}\left[1-\frac{6g^2}{(4\pi)^2}(L+\gamma_E)-\frac{6g^2}{(4\pi)^2}\left(L^2+2\gamma_E L+\frac{\pi^2}{8}-\right.\right. \\
&= \frac{g_3^4}{24(4\pi)^2\epsilon}\,.
\end{aligned}
$$

- Calculations in the effective theory

$$\mathcal{L}_{\rm eff}{}^{0} = \frac{1}{2}(\nabla\phi)^2 + \frac{1}{2}m^2\phi^2$$

$$\mathcal{L}_{\rm eff}{}^{\rm int} = \frac{g_3^2}{24}\phi^4$$

$$
\begin{aligned}
\mathcal{F}^{(s)} &= -\frac{m^3 T}{12\pi} + \frac{g_3^2 m^2 T}{8(4\pi)^2} + \frac{g_3^4 m T}{96(4\pi)^3}\left[8L_m + 9 - 8\log 2\right] \\
&+\frac{g_3^6 T}{768(4\pi)^4}\Big[-4 + 16\log 2 - 16L_m - 42\zeta(3) \\
&+\pi^2(1 + 2\log 2 + 4L_m)\Big] \\
&-\frac{g_3^8 T}{288m(4\pi)^5}\Bigg[L_m^2 + \frac{1}{4}L_m - 2L_m\log 2 - \frac{15}{64} - \frac{3}{8}\pi^2 \\
&+\frac{9}{8}\pi^2\log 2 + \frac{23}{4}\log 2 + 6\log^2 2 \\
&-6\log 3 - \frac{81}{16}\zeta(3) + 5\mathrm{Li}_2(\tfrac{1}{4}) + 9\mathrm{C}_{4\mathrm{j}}\Bigg]\;,
\end{aligned}
$$

- Hard contribution to the pressure

- Soft contribution to the pressure

- Total pressure to order g^7

- Can seletively resum higher orders by not expanding the parameters m^2 and g_3^2

Screened perturbation theory

- Screened perturbation theory [4]
  - Weak-coupling expansion is an expansion about an ideal gas of massless particles

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)^2 + \frac{g^2}{24}\phi^4$$

  - Perhaps better to expand about an ideal gas of massive particles

$$\mathcal{L}^0 = \frac{1}{2}(\partial_\mu\phi)^2 + \frac{1}{2}m^2\phi^2$$

$$\mathcal{L}^{\rm int} = -\frac{1}{2}m_1^2\phi^2 + \frac{g^2}{24}\phi^4$$

  - Variational calculations

[4] Karsch, Patkos, and Petreczky, PLB **401** 69 (1997).

Screened perturbation theory

- Recipe
  - Treat m^2 $\mathcal{O}(1)$ term
  - Treat $\frac{1}{2}m_1^2\phi^2$ and $g^2\phi^4$ as perturbations on equal footing.
  - Calculate physical quantities and set $m_1 = m$ at the end
  - Feynman rules for SPT
- Mass prescription

Screened perturbation theory

- Feynman graphs through four loops

Screened perturbation theory

- Prescriptions for mass parameter m
  - Debye mass

$$p^2 + m^2 + \Sigma(0,p) \quad = \quad 0\,.$$

  - Tadpole mass

$$\begin{aligned} m_t^2 &= g^2 \frac{\partial \mathcal{F}}{\partial m^2}\bigg|_{m_1=m} \\ &= g^2 \langle \phi^2 \rangle \end{aligned}$$

  - Variational mass

$$\frac{\partial \mathcal{F}}{\partial m^2} \quad = \quad 0$$

- Prescriptions for mass parameter m cont'd
  - All gap equations are the same at one loop:

$$
\begin{aligned}
m^2 &= \frac{1}{2}g^2 \sum_{p_0} \int \frac{d^3p}{(2\pi)^3} \frac{1}{P^2+m^2} \\
&= \frac{1}{2}\alpha(\mu^*) \left[J_1(\beta m) T^2 - \left(2\log\frac{\mu}{\mu^*} + 1 \right) m^2 \right] \\
J_1(\beta m) &= 8\beta^2 \int_0^\infty \frac{dp\, p^2}{(p^2+m^2)^{1/2}} \frac{1}{e^{\beta(p^2+m^2)^{1/2}} - 1} \\
J_1(0) &= \frac{4\pi^2}{3}
\end{aligned}
$$

  - Gap equations differ at two loop and higher

- Prescriptions for mass parameter m cont'd
  - Debye mass not well defined in nonabelian gauge theories beyond leading order.
  - Tadpole mass not well defined in nonabelian gauge theories since $\langle\phi^2\rangle \to \langle A_\mu A_\mu\rangle$
  - Variational mass well defined for gauge theories
  - Screening mass and tadpole mass do not have solutions for all values of g

Miscellaneous

- Two-loop SPT-result for the pressure, the gap equation and the entropy are the same as those for the two-loop 2PI effective action (but different renormalization!) [5]

$$
\begin{aligned}
TS_2 &= \frac{1}{(4\pi)^2}\left[2J_0T^4 + J_1m^2T^2\right] \\
J_0(\beta m) &= \frac{16}{3}\beta^4\int_0^\infty \frac{dp\, p^4}{(p^2+m^2)^{1/2}}\frac{1}{e^{\beta(p^2+m^2)^{1/2}}-1} \\
J_0(0) &= \frac{16\pi^4}{45}
\end{aligned}
$$

- Entropy of ideal gas of massive particles.

[5] Blaizot, Iancu, and Rebhan PRD **63**, 065003 (2001), JOA, Braaten and Strickland PRD **63**, 105008 (2001)

Mass expansions

- At four loops, we cannot calculate sum-integrals (semi)-analytically
- Carry out an m/T expansion, where m is of order gT
- Expansions converges reasonably fast
- One-loop example:

$$\sum_{p_0}\int\frac{d^3p}{(2\pi)^3}\log(p_0^2+p^2+m^2) = \int\frac{d^3p}{(2\pi)^3}\log(p^2+m^2) + {\sum_{p_0}}'\int\frac{d^3p}{(2\pi)^3}\left[\frac{m^2}{P^2}-\frac{1}{2}\frac{m^4}{P^4}+\ldots\right]$$

- More involved for multiloop diagrams

Numerical results

- m/T expansions of two, three, and four-loop approximations. Weak-coupling expansion for comparison

Numerical results

- m/T expansions of two, three, and four-loop

Conclusions and outlook

- We have calculated the free energy to order g^7 at weak-coupling
- Can use evolution equation for parameters in $\mathcal{L}_{\text{eff}}$ to sum up $g^{2n+3}\log^n(g)$ etc
- Complicated calculations of loop diagrams in four dimensions necessary to get hard g^6 contribution in QCD
- Screened perturbation theory shows remarkable stability
- We are working on three-loop hard-thermal-loop perturbation theory for QED and QCD