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Dynamic optimisation and control of energy storage systems

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Abstract

The aim of this project was to derive a model for a simple thermal energy storage system that consists of at least one supplier, one consumer, a storage unit, a direct heating source which is cheap and an expensive emergency heating source from the external market. This was done for a two plant system and four plant system. After obtaining the model and validating it through simulation tests, it was used for dynamic optimisation calculations focusing on control. The simulation tests showed that disturbances that occur in the supply side did have a reduced effect in the consumer side. This implies that the energy storage will ensure a steady supply to the consumer regardless of source unavailability at certain periods. The buffer action of the energy storage was clearly seen and the responses to disturbances and step changes in inputs from steady state, suggested that the model is valid. Then a scenario was formulated only for the simple two plant system to meet a predicted consumer energy demand while minimising the total energy cost function. A trade-off exists between purchasing extra energy to fulfill consumer requirements and using the limited storage efficiently. This resulted into an optimal control problem which was mathematically formulated, discretised and converted into a non-linear program using direct collocation method. The nonlinear program was defined using CasADi and solved by using IPOPT in MATLAB. Open loop optimisation calculations over a prediction horizon of one day resulted in optimal controls and states that the system should be operated in. The solution is valid and practical since it ensures that the energy storage is heated up extra or “fully charged” before a peak demand. A brief discussion on how model predictive control could be implemented on the system is also done. The report also mentions how machine learning approaches could be used to improve the control of the system.

Preface

This report is written for the fulfillment of the requirements of the TKP4580 Specialization Project carried out at the Norwegian University of Science and Technology (NTNU). This work was carried out during the Fall of 2018 at the Department of Chemical Engineering.

I would like to thank my supervisor Professor Johannes Jäschke for the guidance and feedback he gave for my work. My sincere gratitude also goes to Mandar Thombre who has been giving me close assistance throughout the semester.

Declaration of Compliance

I declare that this is an independent work according to the exam regulations of the Norwegian University of Science and Technology (NTNU).

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1 — Introduction

The contemporary world has developed in its entirety due to technological advancement. Advanced technology has enabled human development in its ability to utilize natural resources and acquire surplus to basic requirements. Industries are entities that develop this technology and apply it to convert natural resources into valuable and useful products. While mass production has been achieved, flooding cheaper and useful products into the market to ease life, there has been an increased burden to the environment. The demand for raw materials and energy sources threatens future sustainability [3]. Therefore, current research is focused on rethinking to develop technology with less environmental footprint [4]. Achieving this requires continuous improvement of industrial processes for efficient conversion of materials and replacing high energy processes with optimal ones. Moreover, minimization of waste products and energy streams is achieved by recycling and energy integration. The urge for efficient use of resources gave birth to the idea of industrial clusters.

Industrial clusters are a group of plants that are placed in close proximity to easily share and communicate resources. They are characterized by having common utility supply and integration of energy and product streams. That is to say one plant may produce main products, by-products and wastes that are inputs to another plant process. Likewise, some processes generate surplus energy while others have higher energy demand. Creating a cluster consisting of a number of such plant processes and integrating them accordingly will result in greater efficiency of resource utilization. The ideal situation of an industrial cluster is when each waste stream of a unit is connected to another unit creating an "ecosystem", and it becomes an *industrial ecosystem* [5,6].

To achieve industrial ecosystem we must aim at utilizing energy and materials, including raw materials, products or by-products to completely eliminate waste and reduce economic losses. Industrial clusters are far from becoming ecosystems, but to achieve this, one must design an optimal and efficient process. Exchange of by-products and waste streams can be easily done provided the facilities are in close proximity. Efficient thermal energy integration between plant streams is a challenging task because energy is transferred by temperature differences between streams. The quality and amount of energy transfer depends on the temperature of the hottest stream. Therefore, it is more efficient to transfer heat at the hottest point of the source. Moreover, it is a challenge to meet energy demands at all periods especially at peak demands. A solution is to have a central energy storage that collects heat from sources and acts as an energy buffer to supply the sinks. It is also advantageous from a planning perspective, as it becomes easier when a plant wants to change its process [1].

Energy storage systems consist of at least one plant with surplus energy that acts as a source, plant(s) with demand (customers) that become heat sinks, an energy storage unit, energy market where energy can be bought to meet peak demands, a cheaper heating source such as hot flue gases and also extra sink usually district heating to release heat.

1.1 Motivation

Energy storages are quite beneficial since they provide a buffer to hold and release thermal energy between energy integrated units. The storage holds thermal energy when the source streams supply more energy and there is no demand in the sink streams matching it. It then cools to release the stored energy when the demand in the sinks is much higher and hence ensuring that process requirements are fulfilled at all times. A common practice in these systems is to purchase energy externally from the market, for example heating by burning fossil fuels, which are expensive sources of energy and this has influence on the operation costs. This strategy creates a need for optimisation of the system, since there is always a trade-off between costs involved and meeting process requirements at times of low supply or high demand . It is obviously cheaper to utilize the stored energy first but these storage units have limitations in the amount they can store. An optimal operation strategy that ensures cost minimisation without violating operational constraints at any time is required. In order to operate the storage to meet the energy demands in the system, one requires a model-based strategy to predict the next best actions that will satisfy system requirements. The benefits that come with such an optimised system are:

1. Higher peak capacity. This means that the energy storage has extra heating just before peak demands. Then the system could handle peak demands better.
2. Exploit energy market prices. The system heats extra from the market when the prices are lower and decides not to at times when it is expensive.
3. Utilize favourable environment conditions: for example heating in the day and cooling at night [7].

1.2 Objective

The aim of this project is to establish a working dynamic model for a simple energy storage system which consists of at least two chemical plants that may have varying energy supply and demand profiles. The model must be used for open loop dynamic optimization and for model predictive control implementation on the system.

The following are the specific objectives of this project:

1. Study about thermal energy systems, heat exchanger modeling, dynamic optimisation and model predictive control.
2. Develop a dynamic model for a simple energy system with two plants i.e. a source and a sink. Extend the model to contain more than two plants.

3. Perform model simulations to analyse system behaviour and validate them.
4. Open-loop optimisation of a simple two plant system with constant and varying demand profile
5. Implementation of model-predictive control for the system.

This work was based on ideal scenarios which may resemble practical cases with parameters that are similar to real processes.

1.3 Report structure

The report has seven other chapters. First, the background theory necessary for modeling, optimisation and control is covered. Then the assumptions and steps taken to write the governing equations are listed for two-plant and four-plant system scenarios. The next section describes the approach used in this work to perform simulations for the systems and simulation results are shown and discussed. This is followed by the open loop dynamic optimisation section where the optimal solution computed for the two plant system with varying customer demand profile is shown and discussed. This followed by a brief discussion on MPC principle and how it could be implemented to control the system. Finally, a general discussion of the work is done followed by a conclusion with recommendations for further work.

2 — Theory

This chapter includes all the necessary background knowledge used in this work. To begin with a basic knowledge about energy storage and thermal energy storage systems is written. Then heat transfer modeling in heat exchangers is explained to create a foundation for thermal energy storage system modeling. Also there is a brief description on dynamic optimisation and use of direct collocation method to formulate a non-linear program.

2.1 Energy Storage Systems

Energy storage (ES) systems are an important development towards a successful intermittent energy source in meeting demand. The ES has recently developed to create a significant impact in modern technology. With the varying availability in renewable energy sources such as solar power, energy storage plays a huge role in its implementation when availability is lowest. The ES systems also contribute towards realising efficient environmentally friendly energy use in many applications including heating and cooling systems in industrial processes. The benefits associated with use of ES systems are reduced energy costs and consumption, increased operation flexibility and reliability, reduced equipment size, efficient use of process equipment, minimal use of fossil fuels and reduced pollutant emissions [8].

In the modern industrial world, abundant and reliable supply of energy is desired. Moreover, this should be achieved at the minimal costs possible. Usually humanity derives raw energy from a source through heat release. Even electricity that is favoured as a power source is produced from generators that are driven by burning fuel directly or indirectly by heated steam. These heating processes are inefficient and plenty of energy is lost in the waste streams. A glance on the other side, renewables have a characteristic of unsteady supply. The need for efficiency, minimising waste energy and ensuring reliable supply of energy to meet peak demand requirements has elicited investment in energy storage. The energy storage concept is not only limited to industries but also in centralised cooling and heating systems for households and buildings. Moreover, the sharp increase in fuel costs - especially fossil fuels - in the previous decades calls for processes that are efficient and this is possible with energy storage.

2.1.1 Energy Demand

Energy demand in industrial and generally all sectors fluctuates on a time basis. The normal practice is to have energy supply systems that produce by adapting to the demand at peak times. This can be termed as external market energy. Peak demands usually were met by running extra gas turbines or oil generators that add up significantly to the costs due to the scarcity of fossil fuel resources. Energy storage is an alternative way of meeting peak energy demands and are flexible to incorporate renewable energy sources and waste energy streams. The ES strategy based on the sector can be applied as follows:

- Utility: a cheaper utility such as electricity at base load or waste stream such as flue gases can be used to "charge" the energy storage systems during low demand periods. The energy stored is released in peak demand times, reducing reliance on purchased energy from the market.
- Industry: in industries, some plant processes have high temperature waste heat streams that preheat the storage for use when peak demand arrives.
- Co-generation: when there are several types of energy sources energy storage play a big role to ensure reliability. For example with energy storage it is possible to combine thermal energy and solar energy sources to meet energy demands.

2.1.2 Categories of Energy Storage Methods

Energy storage requires a medium that has a capacity to store energy in a certain form. Due to differences in which energy can be stored there are several energy storage methods. These methods are classified as mechanical, chemical, biological, magnetic and thermal energy storage. Thermal energy storage is discussed here, other forms of energy storage are out of the scope of this work.

2.1.3 Thermal Energy Storage Systems

Thermal energy storage (TES) systems have the ability to store heat to be used at a later time depending on temperature, time, place, power demand or price conditions. This storage eliminates the discrepancy between thermal energy supply and demand throughout system operation. The concept is similar to a battery with a synonymous storage cycle of charging, storage and discharging processes.

A TES system requires a high energy density storage material, that is high heat capacity medium. A good heat transfer between it and the heat transfer fluid (HTF) or the supply and demand streams is desirable. The design should also consider the chemical and mechanical stability of the storage medium at the thermal storage range of temperature. Moreover, the most important design criteria as explained in [9] are the operation strategy, the maximum load needed, the nominal temperature and enthalpy drop, and the integration into the whole application system.

Types of TES

TES systems can either be sensible heat storage, latent heat storage or thermochemical heat storages.

1. **Sensible heat storage**: Sensible heat storage or heat capacity storage, is done by altering the temperature of a material without phase change or chemical composition change. The amount of thermal energy stored in a sensible heat storage is given by equation 2.1

$$E = \int_{T_{\text{ref}}}^{T} \left(\frac{\partial H}{\partial T} \right)_{p,n} \mathrm{d}T \tag{2.1}$$

   where E = thermal energy stored, H = enthalpy, T = absolute temperature, T_{ref} = reference temperature. The equation can be written in form of specific heat capacity at constant pressure, c_p in equation 2.2a

$$E = m \int_{T_{\text{ref}}}^{T} c_p(T)\,\mathrm{d}T \tag{2.2a}$$

   where

$$c_p(T) = \left(\frac{\partial h}{\partial T} \right)_{p,n} \tag{2.2b}$$

   m = mass of stored material and h is the specific enthalpy

(2.2c)

   When the assumption is made that the specific heat capacity is independent of temperature then it is constant. The energy stored can be written simply as equation 2.3

$$E = mc_p(T - T_{\text{ref}}) \tag{2.3}$$

   For solids and liquids, the specific heat capacities at constant volume and constant pressure are always equal ($c_p = c_v$). Therefore from equation 2.2a, the amount of thermal storage stored in a material due to raising its temperature can be determined. ([10])

2. **Latent heat storage**: These storage media store thermal energy in form of their latent heat during a constant temperature process like phase change. Solid-liquid phase change is the most common method used. Liquid-gas phase change has the highest latent heat of phase change but the huge volume change of the storage material is a problem and hence is not used in general [11]. The thermal energy stored by latent heat can be expressed as in equation 2.4.

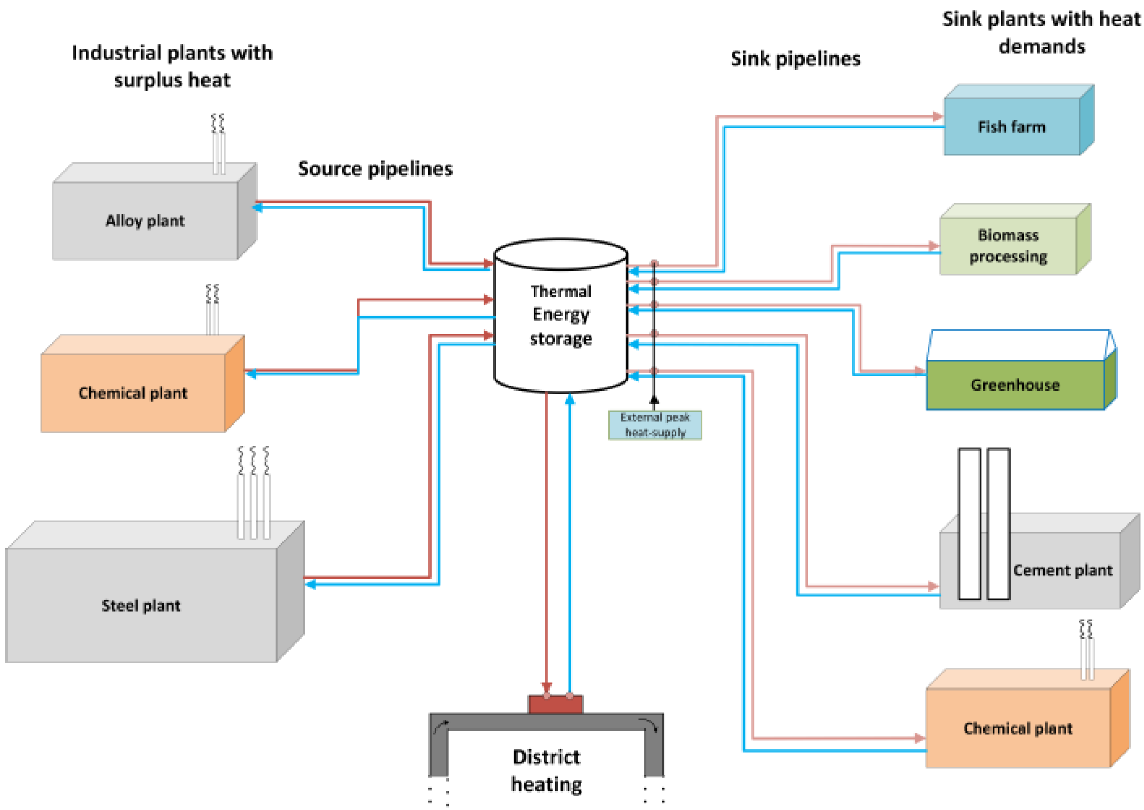

Figure 2.1: Conceptual ilustration of energy or heat exchange in industrial cluster with heat exchange and thermal storage. The red lines indicate hot supply water and blue lines cold return water [1]

$$E = mL \tag{2.4}$$

where m = the mass of stored material and L = latent heat of phase change.

3. **Thermochemical heat storage**: Thermochemical energy storage is a result of a high energy chemical reaction that stores energy. The reaction products must be stored and the heat generated during the reaction must be available when backward reaction occurs. Therefore, this storage methods involves only reversible reactions. In the charging process, injected heat is used to drive an endothermic chemical reaction; the chemical products are later used to restore thermal energy by performing the reverse (exothermic) reaction [12].

2.2 Heat Transfer Modeling in Heat exchangers

A heat exchanger is a process unit that is used to transfer thermal energy between two fluid streams without mixing them. The equipment has two parts which are the hold and cold sides that are separated by a solid and is designed in such a way that the heat transfer area between the two streams, and hold-up is maximised.

Heat exchangers are important in thermal energy storage systems because they are used to transfer heat from the source to the heat transfer fluid and storage medium. They are consequently used to release the heat energy from the storage medium and heat transfer fluid to the thermal sinks of the system. They are simply fluid cooling and heating equipment.

Heat exchangers can be classified in terms of flow arrangement or type of construction. In this project, there is minimum focus of the construction design aspects of the heat exchangers hence flow arrangements will be discussed. Based on flow arrangement, the simplest heat exchangers with double-pipes according to [13], can be classified as:

- *Cross flow heat exchangers*: the flow of cold and hot streams are arranged in a way that one stream flows perpendicular to the other.
- *Parallel flow heat exchangers*: the cold and hot fluid enter the heat exchanger in the same end and therefore flow in the same direction.
- *Counter-current flow heat exhangers*: the cold and hot fluids enter at different ends and have opposite directed flows.

A common configuration in practice is a *shell-and-tube heat exchanger* that has a single shell and tube passes inside the shell. The shell has baffles installed inside to maximise convective heat transfer by increasing turbulence. In this shell and tube construction, the flow configuration can not be clearly identified as any of the three discussed above.

2.2.1 Overall Heat Transfer Coefficient

The heat transfer between the two streams is proportional to the differences in absolute temperature of the two streams at that point. The constant of proportionality is the total thermal resistance to heat transfer between the two fluids. This parameter is the overall heat transfer coefficient which is a function of the convective and conductive fluid properties, and conductive solid material properties. The determination of this value is important to analyse heat exchanger performance but is often the most uncertain parameter.

2.2.2 Log Mean Temperature Difference

To model the performance of a heat exchanger one can relate the inlet and outlet temperatures of exchanging streams, overall heat transfer coefficient and total heat transfer surface area. This is shown in equation 2.5.

$$q = UA\Delta T_m \tag{2.5}$$

where U = overall heat transfer coefficient, A = heat transfer area and ΔT_m = appropriate *mean* temperature difference.

The appropriate mean temperature difference can be found by performing energy balances across an infinitesimal transfer area and integrating w.r.t the distributed stream temperatures throughout all the positions (length), assuming parallel flow or counter-current flow. After some derivation steps in [13,14], the appropriate average temperature difference is a *log mean temperature difference*, ΔT_{LM}. Therefore 2.5 can be written as 2.6

$$q = UA\Delta T_{LM} \tag{2.6}$$

where

$$\Delta T_{LM} = \frac{\Delta T_2 - \Delta T_1}{\ln\left(\Delta T_2/\Delta T_1\right)} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\Delta T_1/\Delta T_2\right)} \tag{2.7}$$

where positions 1 and 2 are the inlet or outlet positions of the double-pipe heat exchanger.

It may be noted that, for the same inlet and outlet temperatures, the LMTD for counter-current flow surpasses that for parallel flow. Hence, for the same value of U the heat transfer area A required to satisfy the same heat transfer rate q is smaller for counter-current flow than for parallel flow arrangements. It is also possible for the outlet stream temperature of the cold stream to be greater than the outlet temperature of the hot stream in a counter-current flow setting but never in parallel flow. [13]

2.2.3 Approximation to the LMTD

The log mean temperature difference is an accurate expression for the mean temperature difference between the hot and cold streams provided that the overall heat transfer coefficient is not a function of the position. However, it has been globally accepted that the logarithmic mean causes inconvenience to chemical engineering programmers. Zavala-Río et al [15], Paterson [16] and Chen [17], communicated the difficulties associated with performing heat exchanger calculations with the log mean function. It is common practice in iterative equation solving schemes that equality of stream temperatures is assumed as a starting value. This will result into an indeterminate form of the log mean. Moreover, at that limit, the derivatives of LMTD (which are needed in Newton iterative methods) are undefined.

Paterson [16] derived a new expression that overcomes the aforementioned difficulties, to replace the log mean. The expression is a good enough approximation to LMTD that can be used in calculations for practical purposes. He obtained the *new mean* as a weighted arithmetic mean (linear combination) of *geometric mean* and *arithmetic mean* temperature differences shown in eq. (2.8).

$$\Delta T_{NM} \equiv \frac{2}{3}\Delta T_{GM} + \frac{1}{3}\Delta T_{AM} \simeq \Delta T_{LM} \tag{2.8}$$

where

$$\Delta T_{AM} \equiv \frac{\Delta T_1 + \Delta T_2}{2} \tag{2.9a}$$

$$\Delta T_{GM} \equiv \sqrt{\Delta T_1 \Delta T_2} \tag{2.9b}$$

The arithmetic mean is considered a useful approximation for mean temperature difference in economic analysis but the new mean has refined it for more practical uses.

Underwood [18, 19] and then Chen [17] both derived an approximation that is a weighted geometric mean of the arithmetic mean and geometric mean. Their new means are a polynomial of the temperature differences at the ends of the exchanger ΔT_1 and ΔT_2.

$$\Delta T_{UM}^{1/3} = \frac{1}{2}(\Delta T_1^{1/3} + \Delta T_2^{1/3}) \tag{2.10}$$

$$\Delta T_{CM}^{0.3275} = \frac{1}{2}(\Delta T_1^{0.3275} + \Delta T_2^{0.3275}) \tag{2.11}$$

Generally,

$$\Delta T_m = \left[\frac{1}{2}(\Delta T_1^n + \Delta T_2^n)\right]^{1/n} \tag{2.12}$$

such that the n values will determine the type of approximation used. This has been summarised in table 2.1.

Table 2.1: Values of n and their respective approximations

n	Approximation (ΔT_m)
1	Arithmetic mean (ΔT_{AM})
$\frac{1}{3}$	Underwood's mean (ΔT_{UM})
0.3275	Chen's mean (ΔT_{CM})

2.3 Dynamic Optimisation

A mathematical optimisation problem includes three main parts which are an *objective function, decision variables and constraints.* The objective function is a scalar function which describes the quantity to be minimised or maximised. Decision variables can either be real numbers, integers or binary numbers. Usually decision variables are vectors of real numbers. The constraints are divided into equality and inequality constraints. All these elements together define an optimisation problem. When the objective function or constraints are non-linear functions then the optimisation problem becomes a *non-linear program* (NLP) [20].

$$\min_{z \in \mathbb{R}^n} f(z) \tag{2.13a}$$

subject to

$$c_i(z) = 0, \quad i \in \mathcal{E} \tag{2.13b}$$

$$c_i(z) \geq 0, \quad i \in \mathcal{I} \tag{2.13c}$$

where, z is the decision variable, $\mathcal{E}$ is the equality constraint index set and $\mathcal{I}$ is the inequality constraints index set

Usually optimization problems are stated as minimisation problems. To obtain an optimal point, either a local or global minimum must satisfy the *Karush-Kuhn-Tucker (KKT) conditions* which are conditions for optimality. This has been discussed properly in [21].

Dynamic systems have the property of possessing variables that change in the course of time. Therefore, dynamic optimisation involves solving for optimal decision variables at every point in time. In simple words the optimal solution is a function of time. It is vital to perform dynamic optimisation rather than static optimisation when dynamics have a significant effect in the systems operating conditions.

The time varying variables of a dynamic system are described by differential equations whose specific solution is mostly impossible to calculate analytically. Therefore the systeme has to be converted into a disrete time system where the variables are sampled at discrete points in time. It is common to have such sample points equally spaced in time. Therefore for example if there are $(n_x + n_u)$ decision variables in the system and the time space is discretised into N intervals then the discretised model has a total of $N(n_x + n_u)$ decision variables.

2.4 Direct Collocation

This is a method that is employed to make it possible to solve optimal control problems. The method converts an optimal control problem to a non-linear problem (NLP) that could be solved by SQP or interior-point methods. It has been regularly applied in state trajectory optimisation and parameter optimisation problems or a combination of the two. The ability of a optimal control problem solver to find a solution is improved when a problem is further constrained even if it increases the dimensionality. In addition, direct collocation better relates the states to the augmented objective function. [22]

Collocation method is an approach of solving ordinary differential equations numerically. It is an implicit variation of Runge-Kutta method [22]. Runge-Kutta allows calculation of states at each discrete time element by forward integration. In direct collocation, the states are expressed implicitly as the function of states and their derivatives. The method employs polynomials to interpolate state variables [23] inside the discrete time element.

2.4.1 Polynomial Interpolation

In the temporal domain $\{t_{k,0}, \ldots, t_{k,K}\} \in [t_k, t_{k+1}]$ it is possible to express the state trajectory within a discrete time interval as a function of time points within the interval. The function used are Lagrange polynomials whose order K depends on the number of points taken inside the interval.

$$P_{k,1}(t) = \prod_{j=0,j\neq i}^{K} \frac{t - t_{k,j}}{t_{k,i} - t_{k,j}} \in \mathbb{R} \tag{2.14}$$

In order to interpolate the state trajectory the linear combination of all these polynomials is used.

$$\mathbf{s}(\theta_k, t) = \sum_{i=0}^{K} \theta_{k,i} P_{k,i}(t) \tag{2.15}$$

where

$$\mathbf{s}(\theta_k, t_{k,j}) = \theta_{k,j} \tag{2.16}$$

The main idea of collocation method is using eq. (2.15) to interpolate the state trajectory by adjusting the parameters $\theta_{k,i}$ and approximate system dynamics (ODE) $\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x}, \mathbf{u})$. In that case we obtain $K + 1$ degrees of freedom per state. The constraints used in collocation are given by eq. (2.17) and the states $\mathbf{x}_k$ and inputs $\mathbf{u}_k$ are degrees of freedom from the NLP.

$$\mathbf{s}(\theta_k, t_k) = \theta_{k,0} = \mathbf{x}_k \tag{2.17a}$$

$$\frac{\partial}{\partial t}\mathbf{s}(\theta_k, t_{k,j}) = \mathbf{F}(\mathbf{s}(\theta_k, t_{k,j}, \mathbf{u}_k)), \quad j = 1, \ldots, K \tag{2.17b}$$

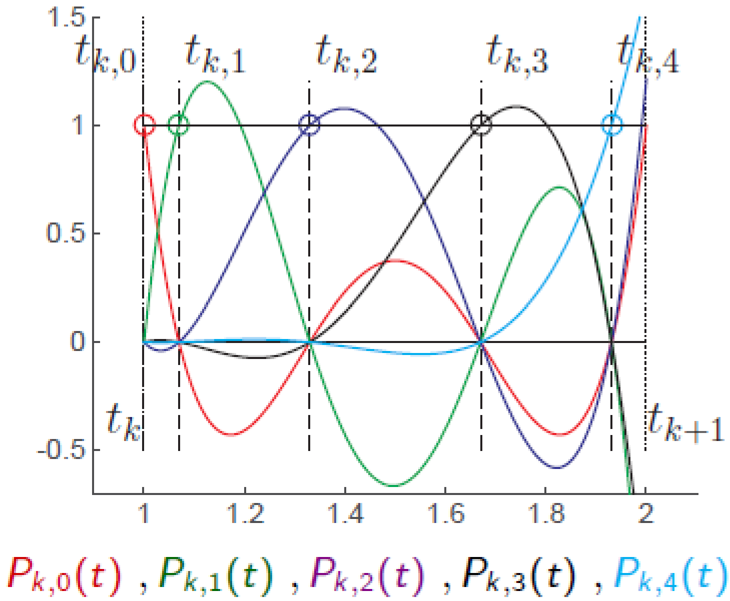

Figure 2.2: Possible Lagrange polynomials in an interval with 4 interpolation points [2]

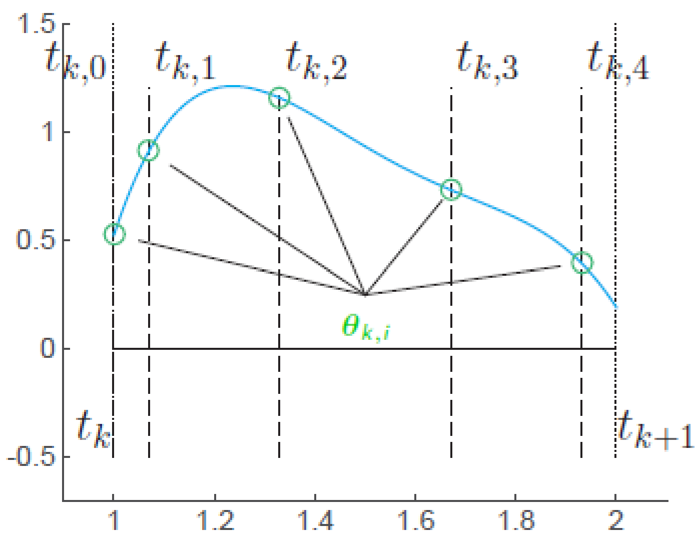

Figure 2.3: Interpolated state trajectory as a function of parameters $\theta_{k,i}$ [2]

eq. (2.17b) can further be modified to give eq. (2.18)

$$\sum_{i=0}^{K} \theta_{k,i} \dot{P}_{k,i}(t_{k,j}) = \mathbf{F}(\theta_{k,j}, \mathbf{u}_k) \tag{2.18}$$

The parameters $\theta_{k,i}$ can be solve for using Newton method from the constraints at each collocation point.

2.4.2 Selection of Time Grid

As discussed above, there are many possibilities of selecting collocation points within the interpolation interval. However, there are proven set of collocation points which deliver an exact integration solution for any polynomial of order $< 2K$ (Legendre) and $< 2K-1$ (Radau) [2].

2.4.3 Error and Stability

Collocation methods are A-stable: They can handle stiff model equations. This implies that even larger time steps can be used to predict steady state and slow dynamics correctly in the presence of very fast dynamics [2].

Radau collocation is L-stable: In addition to A-stability, Radau collocation handles eigenvalues at $-\infty$.

Order of integration error: It depends on K, integration error is $O(h^{2K})$ for Legendre and $O(h^{2K-1})$ for Radau. Runge-Kutta schemes have an order of $O(h^4)$. Moreover, the error only occurs to the end-state (t_k) of the integrator but not the intermediate points [2, 24].

2.4.4 NLP Solving with Direct Collocation

Solving and NLP is done by passing the constraints and the objective function through a solver. Direct collocation approximates the dynamics in intervals and gives constraint equations for the states and inputs at all collocation points and end-points. The optimal trajectory is found by solving for all the decision variables such that the objective function is minimum. The vector of decision variables $\mathbf{w}$ is equal to $N(n_x(K+1)+n_u)$ where n_x is the number of state variables, n_u is the number of input variables and N is the number of discrete time elements. The NLP is transformed into a form as shown in equation 2.19.

$$\min_{\mathbf{w}} \quad \Phi(\mathbf{w}) \tag{2.19a}$$

$$\text{subject to} \quad \mathbf{g}(\mathbf{w}) = \begin{bmatrix} \theta_{0,0} - \overline{\mathbf{x}}_0 \\ s(\theta_0, t_1) - \theta_{1,0} \\ \mathbf{F}(\theta_{0,i}, \mathbf{u}_0) - \sum_{j=0}^{K} \theta_{0,j} \dot{P}_{0,j}(t_{0,i}) \\ \vdots \\ s(\theta_k, t_{k+1}) - \theta_{k+1,0} \\ \mathbf{F}(\theta_{k,i}, \mathbf{u}_k) - \sum_{j=0}^{K} \theta_{k,j} \dot{P}_{k,j}(t_{k,i}) \\ \vdots \end{bmatrix} = 0 \tag{2.19b}$$

where $\mathbf{w} = \{\theta_{0,0}, \ldots, \theta_{0,K}, \mathbf{u}_0, \ldots, \theta_{N-1,K}, \mathbf{u}_{N-1}\}$ are the decision variables.

Since, the parameter $\theta_{i,k}$ are solved together with the optimisation variables $\mathbf{x}_k$ and $\mathbf{u}_k$, then direct collocation is a fully simultaneous approach. The integration and optimisation actions are performed together in the NLP solver.

The input is usually chosen piecewise-constant within each discrete time element. Picking a different input at different collocation times can also be done but is not common and may cause problems with finding a solution.

3 — Modeling

In this chapter, governing equations for energy storage systems are derived. First, a simple scenario with two plants where one is a source and another a sink is considered. After that, dynamics for a system with four plants are derived. These equations are derived from principles of mass and energy conservation together with assumptions. The equations are necessary before simulation to investigate the system dynamics, and dynamic optimisation.

3.1 Two Plants Storage System

The topology for a two plant system scenario is shown in figure 3.1. The system is made of two plants, plant 1 is a thermal source and plant 2 is a thermal sink. The storage tank contains storage material that is heated up by either heat exchanger 1 consisting of L1 and R1 sides or flue gas stream from D. It loses heat to plant 2 via heat exhanger 2 consisting of L2 and R2 and to the surroundings E.

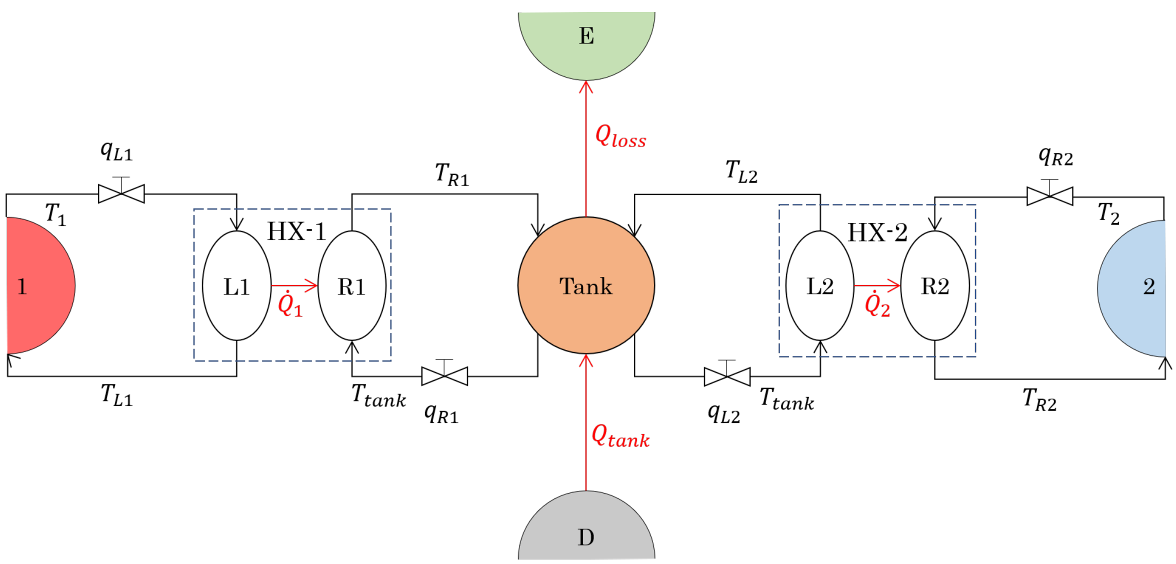

Figure 3.1: Schematic for modeling a two plant thermal storage system

The following assumptions have been taken in order to arrive to a reasonable system model:

1. Source and sink plants (plant 1 and 2) are reservoirs and emanating streams have same intensive property i.e. the temperature of the stream is a disturbance and only depends on the plant.
2. Heat exchangers are modeled as a system of two lumps that transfer heat between each other.
3. Heat losses from heat exchangers are neglected and the overall heat transfer coefficient is constant i.e. $(UA)_{\text{hex}} = \text{constant}$.
4. Storage temperature is uniform throughout the volume. The level of the tank is controlled at maximum. Therefore, it has a constant volume V_{tank}.
5. The fluids flowing in each of the streams has a constant specific heat capacity c_p.
6. All fluid streams are assumed to have properties of water in the simulations. Density and specific heat capacity values are equal to those of water.
7. The conventional energy flow direction is from left to right as illustrated in fig. 3.1.

Energy and Mass Balances

Following assumption 1, the sources and sinks have temperatures T_1 and T_2 respectively that are not affected by the system dynamics. They will be considered constant or given.

Source:

$$\frac{\mathrm{d}H_1}{\mathrm{d}t} = 0, \quad T_1 = \text{constant (given)} \tag{3.1}$$

Sink:

$$\frac{\mathrm{d}H_2}{\mathrm{d}t} = 0, \quad T_2 = \text{constant (given)} \tag{3.2}$$

Lumps:

1. **Heat exchanger 1:** Starting with the mass balances across the heat exchanger gives equation 3.3.

$$\frac{\mathrm{d}(\rho V_{\text{hex}})}{\mathrm{d}t} = \rho q_{1|L1} - \rho q_{L1|1} \tag{3.3a}$$

$$\frac{\mathrm{d}V_{\text{hex}}}{\mathrm{d}t} = q_{1|L1} - q_{L1|1} \tag{3.3b}$$

While in operation, the heat exchanger is filled with fluid so it is always constant i.e. $\frac{\mathrm{d}V_{\text{hex}}}{\mathrm{d}t} = 0$. Therefore, the inlet and outlet flows of the heat exchanger are equal. For simplicity, the following notations in eq. (3.4) will be used from now on in this report instead.

$$q_{1|L1} = q_{L1|1} = q_{L1} \tag{3.4a}$$
$$q_{\text{tank}|R1} = q_{R1|\text{tank}} = q_{R1} \tag{3.4b}$$
$$q_{\text{tank}|L2} = q_{L2|\text{tank}} = q_{L2} \tag{3.4c}$$
$$q_{2|R2} = q_{R2|2} = q_{R2} \tag{3.4d}$$

The outlet temperatures of the left and right side of the heat exchanger 1 (HX-1) are denoted as T_{L1} and T_{R1} respectively. The temperature of all the storage tank's outlet streams are equal and denoted as T_{tank}. Energy is conserved across each side of the heat exchanger. Starting with the left side L1 the energy conservation equation is eq. (3.5a).

$$\frac{\mathrm{d}H_{L1}}{\mathrm{d}t} = \sum H_{\text{in}} - \sum H_{\text{out}} + Q_{\text{net}} - W_s \tag{3.5a}$$

Adiabatic conditions are assumed and there is no shaft work done by the system i.e. $Q_{\text{loss}} = 0$ and $W_s = 0$, then

$$\frac{\mathrm{d}(\rho c_p V_{\text{hex}} T_{L1})}{\mathrm{d}t} = \rho c_p q_{L1} T_1 - \rho c_p q_{L1} T_{L1} - Q_{L1|R1} \tag{3.5b}$$
$$\rho c_p V_{\text{hex}} \frac{\mathrm{d}T_{L1}}{\mathrm{d}t} = \rho c_p q_{L1}(T_1 - T_{L1}) - Q_{L1|R1} \tag{3.5c}$$
$$\tag{3.5d}$$

Where $Q_{L1|R1}$ stands for energy transfer rate from lump L1 to R1. Hence, equation for temperature dynamics of stream q_{L1} given by equation 3.7.

$$\frac{\mathrm{d}T_{L1}}{\mathrm{d}t} = \frac{1}{V_{\text{hex}}} \left\{ q_{L1}(T_1 - T_{L1}) - \frac{Q_{L1|R1}}{\rho c_p} \right\} \tag{3.6}$$

The right side, R1 dynamics can be found with the same steps done in the left side L1 to give equations 3.7.

$$\frac{\mathrm{d}T_{R1}}{\mathrm{d}t} = \frac{1}{V_{\text{hex}}} \left\{ q_{R1}(T_{\text{tank}} - T_{R1}) + \frac{Q_{L1|R1}}{\rho c_p} \right\} \tag{3.7}$$

2. **Heat exchanger 2:** The design parameters of every heat exchanger is the same so the energy balances for the second heat exchanger will result to similar equations with different variables shown in equation 3.8 and 3.9. The outlet temperatures of the left and right sides of the heat exchanger 2 (HX-2) are denoted as T_{L2} and T_{R2} respectively.

$$\frac{\mathrm{d}T_{L2}}{\mathrm{d}t} = \frac{1}{V_{\text{hex}}} \left\{ q_{L2}(T_{\text{tank}} - T_{L2}) - \frac{Q_{L2|R2}}{\rho c_p} \right\} \tag{3.8}$$

$$\frac{\mathrm{d}T_{R2}}{\mathrm{d}t} = \frac{1}{V_{\text{hex}}}\left\{q_{R2}(T_2 - T_{R2}) + \frac{Q_{L2|R2}}{\rho c_p}\right\} \tag{3.9}$$

3. **Heat transferred from hot to cold side:** Heat transfer model between cold and hot streams in heat exchangers is given by 2.5.

$$Q = U_{\text{hex}}A_{\text{hex}}\Delta T_m \tag{3.10}$$

where, ΔT_m is the mean temperature difference between the cold and hot streams. The appropriate value for this is the LMTD, (ΔT_{LM}) but a polynomial approximation is suited for practical purposes instead. These approximations are by Chen ΔT_{CM}, Underwood ΔT_{UM}. The ΔT_m can simply be expressed as the differences between the inlet temperatures of hot and cold sides or as an arithmetic mean ΔT_{AM}. The simple approximations are suitable for cases when the ratio of flow rates of either streams is close to 1. Otherwise when one of the flows in very large compared to another then the approximation becomes very poor.

$$Q_{L1|R1} = U_{\text{hex}}A_{\text{hex}}\Delta T_{m,1} \quad \text{and} \quad Q_{L2|R2} = U_{\text{hex}}A_{\text{hex}}\Delta T_{m,2}$$

where,

$$\Delta T_{m,1}^n = 0.5\big[(T_1 - T_{R1})^n + (T_{L1} - T_{\text{tank}})^n\big], \quad n = 1,\ \frac{1}{3}\ \text{or}\ 0.3275$$

$$\Delta T_{m,2}^n = 0.5\big[(T_{L2} - T_2)^n + (T_{\text{tank}} - T_{R2})^n\big], \quad n = 1,\ \frac{1}{3}\ \text{or}\ 0.3275$$

Using the simpler approximation the mean temperature differences can be estimated as equations 3.11.

$$\Delta T_{m,1} = T_1 - T_{R1} \tag{3.11a}$$

$$\Delta T_{m,2} = T_{\text{tank}} - T_{R2} \tag{3.11b}$$

4. **Storage tank:** Beginning with mass balances we obtain equation 3.12.

$$\frac{\mathrm{d}(\rho V_{\text{tank}})}{\mathrm{d}t} = \rho q_{R2} + \rho q_{L2} - \rho q_{R2} - \rho q_{R2} = 0 \tag{3.12}$$

Energy balance across the tank following the general energy balance equation 3.5a provided the enthalpies and the net heat flow in equations 3.13 leads up to equation 3.14.

$$\sum H_{\text{in}} = \rho c_p(q_{R1}T_{R1} + q_{L2}T_{L2}) \tag{3.13a}$$

$$\sum H_{\text{out}} = \rho c_p(q_{R1}T_{\text{tank}} + q_{L2}T_{\text{tank}}) \tag{3.13b}$$

$$Q_{\text{net}} = Q_{D|\text{tank}} - Q_{\text{tank}|E} \tag{3.13c}$$

$$\frac{\mathrm{d}H_{\text{tank}}}{\mathrm{d}t} = \rho c_p(q_{R1}(T_{R1} - T_{\text{tank}}) + q_{L2}(T_{L2} - T_{\text{tank}})) + Q_{D|\text{tank}} - Q_{\text{tank}|E} - 0 \tag{3.14}$$

Let $Q_{D|\text{tank}} = Q_{\text{tank}}$ and $Q_{\text{tank}|E} = Q_{\text{loss}}$

$$\frac{\mathrm{d}T_{\text{tank}}}{\mathrm{d}t} = \frac{1}{V_{\text{tank}}}\left\{q_{R1}(T_{R1} - T_{\text{tank}}) + q_{L2}(T_{L2} - T_{\text{tank}}) + \frac{Q - Q_{\text{loss}}}{\rho c_p}\right\} \tag{3.15}$$

Hence storage tank temperature dynamics.

The following set of ordinary differential equations eq. (3.16) describe the energy system with two plants.

$$\frac{\mathrm{d}T_{L1}}{\mathrm{d}t} = \frac{1}{V_{\text{hex}}}\left\{q_{L1}(T_1 - T_{L1}) - \frac{U_{\text{hex}}A_{\text{hex}}}{\rho c_p}\Delta T_{m,1}\right\} \tag{3.16a}$$

$$\frac{\mathrm{d}T_{R1}}{\mathrm{d}t} = \frac{1}{V_{\text{hex}}}\left\{q_{R1}(T_{\text{tank}} - T_{R1}) + \frac{U_{\text{hex}}A_{\text{hex}}}{\rho c_p}\Delta T_{m,1}\right\} \tag{3.16b}$$

$$\frac{\mathrm{d}T_{L2}}{\mathrm{d}t} = \frac{1}{V_{\text{hex}}}\left\{q_{L2}(T_{\text{tank}} - T_{L2}) - \frac{U_{\text{hex}}A_{\text{hex}}}{\rho c_p}\Delta T_{m,2}\right\} \tag{3.16c}$$

$$\frac{\mathrm{d}T_{R2}}{\mathrm{d}t} = \frac{1}{V_{\text{hex}}}\left\{q_{R2}(T_2 - T_{R2}) + \frac{U_{\text{hex}}A_{\text{hex}}}{\rho c_p}\Delta T_{m,2}\right\} \tag{3.16d}$$

$$\frac{\mathrm{d}T_{\text{tank}}}{\mathrm{d}t} = \frac{1}{V_{\text{tank}}}\left\{q_{R1}(T_{R1} - T_{\text{tank}}) + q_{L2}(T_{L2} - T_{\text{tank}}) + \frac{1}{\rho c_p}(Q - Q_{\text{loss})}\right\} \tag{3.16e}$$

where,

$$Q_{\text{loss}} = (UA)_{\text{tank}}(T_{\text{tank}} - T_{\text{surr}}) \tag{3.16f}$$

The two plant energy storage system model in eq. (3.16) contains five ODEs. The variables in the equations can be classified as follows:

- States ($\mathbf{x}$): there are 5 states variables in the system.

$$\mathbf{x} = \{T_{L1},\, T_{R1},\, T_{L2},\, T_{R2},\, T_{\text{tank}}\} \tag{3.17}$$

- Inputs ($\mathbf{u}$): there are 5 input variables in the system.

$$\mathbf{u} = \{q_{L1},\, q_{R1},\, q_{L2},\, q_{R2},\, Q_{\text{tank}}\} \tag{3.18}$$

- Disturbances ($\mathbf{d}$): there are at least 3 disturbances in the system.

$$\mathbf{d} = \{T_1,\, T_2,\, T_{\text{surr}}\} \tag{3.19}$$

- Parameters: the remaining variables are parameters as long as they are constant w.r.t time. They depend on the design of the system, material properties of the storage fluid and heat exchanger.

Therefore, the systems model can be written simply as: $\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x}, \mathbf{u}, \mathbf{d})$

3.2 Four Plant Storage System

A thermal storage system with four plants is illustrated in fig. 3.2. Plant 1 and 3 with temperatures T_1 and T_3 are sources while Plant 2 and 4 with temperatures T_2 and T_4 are sinks. The system dynamics will be similar to the two plants systems only that there are more states and inputs. The number of states increases to 9 and the number of control inputs is also 9. There are a total of 18 variables.

- States ($\mathbf{x}$): there are 9 states variables in the system.

$$\mathbf{x} = \{T_{L1},\, T_{R1},\, T_{L2},\, T_{R2},\, T_{L3},\, T_{R3},\, T_{L4},\, T_{R4},\, T_{\text{tank}}\} \tag{3.20}$$

- Inputs ($\mathbf{u}$): there are 9 input variables in the system.

$$\mathbf{u} = \{q_{L1},\, q_{R1},\, q_{L2},\, q_{R2},\, q_{L3},\, q_{R3},\, q_{L4},\, q_{R4},\, Q_{\text{tank}}\} \tag{3.21}$$

- Disturbances ($\mathbf{d}$): there are at least 5 disturbances in the system.

$$\mathbf{d} = \{T_1,\, T_2,\, T_3,\, T_4,\, T_{\text{surr}}\} \tag{3.22}$$

Using the same approach and assumptions as in section 3.1 the system dynamics are listed in

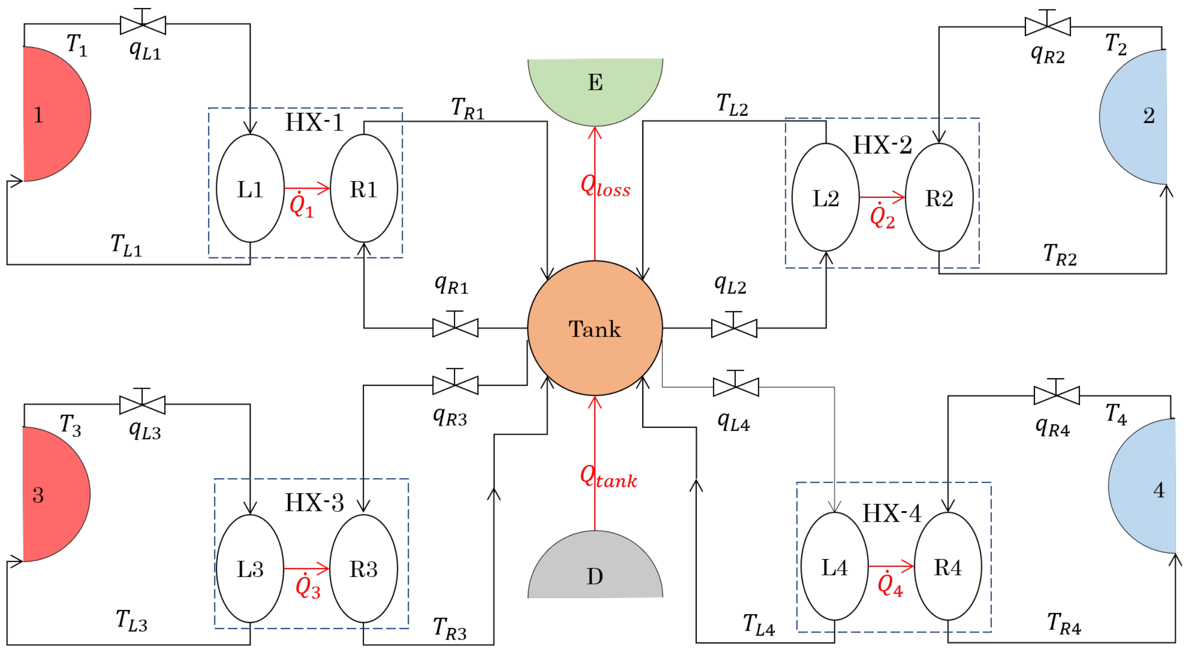

Figure 3.2: Schematic for modeling a four plant thermal storage system

equations 3.23.

$$\frac{\mathrm{d}T_{L1}}{\mathrm{d}t} = \frac{1}{V_{\text{hex}}}\left\{q_{L1}(T_1 - T_{L1}) - \frac{U_{\text{hex}}A_{\text{hex}}}{\rho c_p}\Delta T_{m,1}\right\} \tag{3.23a}$$

$$\frac{\mathrm{d}T_{R1}}{\mathrm{d}t} = \frac{1}{V_{\text{hex}}}\left\{q_{R1}(T_{\text{tank}} - T_{R1}) + \frac{U_{\text{hex}}A_{\text{hex}}}{\rho c_p}\Delta T_{m,1}\right\} \tag{3.23b}$$

$$\frac{\mathrm{d}T_{L2}}{\mathrm{d}t} = \frac{1}{V_{\text{hex}}}\left\{q_{L2}(T_{\text{tank}} - T_{L2}) - \frac{U_{\text{hex}}A_{\text{hex}}}{\rho c_p}\Delta T_{m,2}\right\} \tag{3.23c}$$

$$\frac{\mathrm{d}T_{R2}}{\mathrm{d}t} = \frac{1}{V_{\text{hex}}}\left\{q_{R2}(T_2 - T_{R2}) + \frac{U_{\text{hex}}A_{\text{hex}}}{\rho c_p}\Delta T_{m,2}\right\} \tag{3.23d}$$

$$\frac{\mathrm{d}T_{L3}}{\mathrm{d}t} = \frac{1}{V_{\text{hex}}}\left\{q_{L3}(T_3 - T_{L3}) - \frac{U_{\text{hex}}A_{\text{hex}}}{\rho c_p}\Delta T_{m,3}\right\} \tag{3.23e}$$

$$\frac{\mathrm{d}T_{R3}}{\mathrm{d}t} = \frac{1}{V_{\text{hex}}}\left\{q_{R3}(T_{\text{tank}} - T_{R3}) + \frac{U_{\text{hex}}A_{\text{hex}}}{\rho c_p}\Delta T_{m,3}\right\} \tag{3.23f}$$

$$\frac{\mathrm{d}T_{L4}}{\mathrm{d}t} = \frac{1}{V_{\text{hex}}}\left\{q_{L4}(T_{\text{tank}} - T_{L4}) - \frac{U_{\text{hex}}A_{\text{hex}}}{\rho c_p}\Delta T_{m,4}\right\} \tag{3.23g}$$

$$\frac{\mathrm{d}T_{R4}}{\mathrm{d}t} = \frac{1}{V_{\text{hex}}}\left\{q_{R4}(T_4 - T_{R4}) + \frac{U_{\text{hex}}A_{\text{hex}}}{\rho c_p}\Delta T_{m,4}\right\} \tag{3.23h}$$

$$\frac{\mathrm{d}T_{\text{tank}}}{\mathrm{d}t} = \frac{1}{V_{\text{tank}}}\left\{q_{R1}(T_{R1} - T_{\text{tank}}) + q_{L2}(T_{L2} - T_{\text{tank}}) + q_{R3}(T_{R3} - T_{\text{tank}})\right. \tag{3.23i}$$

$$\left. + q_{L4}(T_{L4} - T_{\text{tank}}) + \frac{1}{\rho c_p}(Q - Q_{\text{loss}})\right\} \tag{3.23j}$$

where,

$$Q_{\text{loss}} = (UA)_{\text{tank}}(T_{\text{tank}} - T_{\text{surr}}) \tag{3.23k}$$

4 — Simulation

This chapter includes the description of the method used to solve the system of ordinary differential equations (ODEs) for the two plant and four plant energy storage systems. Simulation plots for all system states are generated to observe their dynamics. A discussion on the simulation results is done together with the figures.

4.1 Methodology

The system of ODEs could be solved by an integrator for discretised time steps to obtain the approximate system dynamics. In this project, `ode45` and `ode15s` solvers in MATLAB were used to solve the differential equations. These are MATLAB functions that allow passing the system of ODEs and the required time span it should solve in order to obtain the state values at every time step. There are other ODE solvers in MATLAB such as `ode23s` that are used to solve problems with high degree of stiffness. `ode45` is a fast ode solver and is enough to solve these system of equations [25]. The parameters used in the model are approximately realistic and are listed in table 4.1

Table 4.1: Parameters and values used

Parameter	Description	Value	Units
U_{hex}	Overall heat transfer coefficient for heat exchanger	0.5	$\text{kW/m}^3\text{K}$
A_{hex}	Heat transfer area for heat exchanger	300	m^2
U_{tank}	Overall heat loss coefficient for heat exchange	0.05	$\text{kW/m}^3\text{K}$
A_{tank}	Heat loss area for storage tank	1000	m^2
V_{tank}	Volume of the tank	100	m^3
c_p	specific heat capacity	4.186	kJ/kgK
ρ	density	1000	kg/m^3

4.2 Two Plant System Dynamics

The two plant system model equations 3.16 were written in MATLAB, see appendix B.1.1. Then an integrator (`ode45`) was used to simulate the system from an initial state until it arrives at steady state using nominal inputs described in table 4.2. The simulation results were plotted as shown in figure 4.1. The initial conditions used in the ode solver are random values. The system will converge from any starting point to its steady state.

Table 4.2: Nominal inputs

Input	Symbol	Value	Units
Volumetric flows	q_i	0.5	m^3/s
Direct tank heating	Q_{tank}	5×10^3	kW

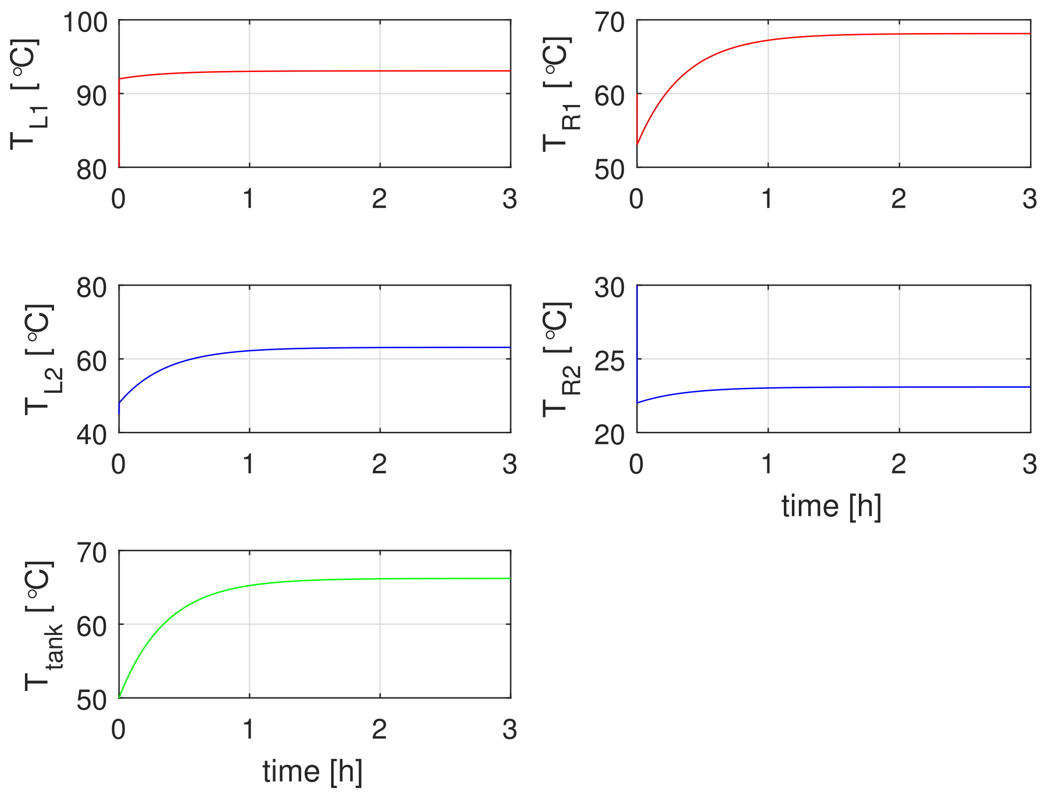

Figure 4.1: Two plant simulation to steady state with nominal inputs

Figure 4.1 shows that apart from the tank temperature, all the other states have initial fast dynamics, that is either a sharp rise or drop in stream temperature followed by slower dynamics before approaching steady state. These different dynamics appear because the accumulation in the heat exchanger is accounted for in the model. Since V_{hex} is smaller than V_{tank} consequently the residence time in the storage tank is way higher in comparison to the former, and in a different time-scale. Therefore, the fast dynamics are due to the heat exchanger accumulation followed by the storage tank accumulation dynamics which are slower. The steady state values for the system are shown in table 4.3. The steady state values at nominal inputs were used as initial values before introducing step changes in the volumetric flows and direct tank heating to test the behaviour of the model.

Table 4.3: Steady state temperatures for two plant system

State	T_{L1}	T_{R1}	T_{L2}	T_{R2}	T_{tank}
°C	93.07	68.14	63.12	23.09	66.21

4.2.1 Step Test in Volumetric Flows

The system is simulated at steady state before introducing a step in the value of all volumetric flows. The step change was introduced in both directions and the plots of the response in the system states can be seen in figure 4.2.

When q is increased there is an increase in temperature of hot side streams of the heat exchangers and a decrease in the cold side streams. In other words, the mean temperature difference across the heat exchanger increases with increased stream flows. This is because since there is no change in the source and sink temperatures the amount of thermal energy that can be transferred is the same, so in order to keep that constant the mean temperature difference must increase. This implies that the temperature of the coldest stream from plant 2 will drop however, the thermal energy will be supplied at the same rate.

When q is decreased there is an decrease in temperature of hot side streams of the heat exchangers and an increase in the cold side streams. This is a consequence of a decrease in the mean temperature difference across the heat exchangers. This is because since there is no change in the source and sink temperatures the amount of thermal energy that can be transferred is the same, so in order to keep that constant the mean temperature difference must increase. It implies that the temperature of the coldest stream from plant 2 will increase. However, the thermal energy will be supplied at the same rate.

The storage tank temperature increases when the flows are low. This is expected because the rate of energy transferred from the tank depends on the rate of storage fluid that is used to heat the demand streams. At higher flows the rate of heat lost from the storage fluid is higher thus resulting to a drop in the storage tank temperature. The drop is not as large as the rise because of the effect of direct heating (Q_{tank}).

4.2.2 Step Test in Direct Tank Heating

Another input that could be manipulated is direct heating Q_{tank}. Simulations were done where a step change in Q_{tank} was introduced from steady state and results plotted as shown in fig. 4.3.

Figure 4.3 shows step changes where Q_{tank} is increased and decreased. It can be seen that in all states, direct heating has a positive effect. This is expected since by injecting extra energy externally then the sink streams will be heated extra as well and the source streams would not lose heat to the tank to maintain a higher temperature.

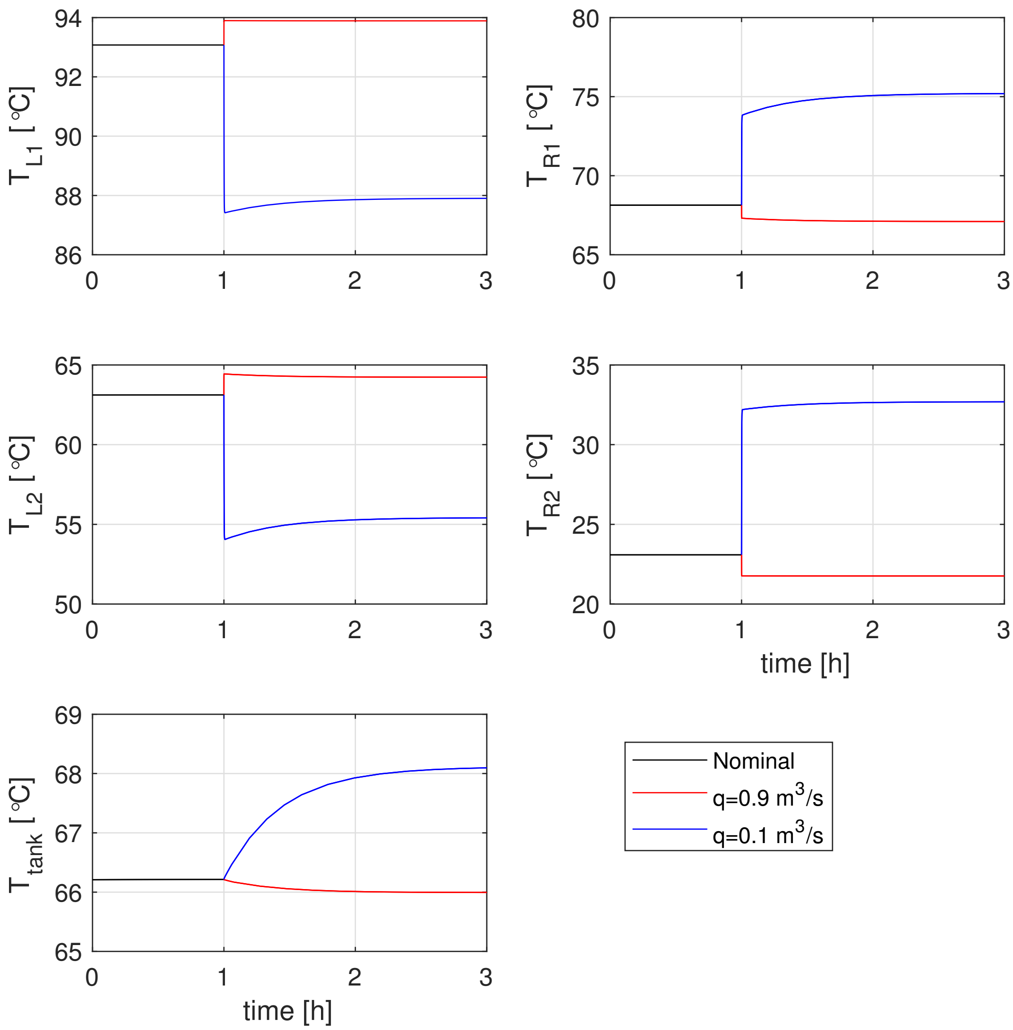

Figure 4.2: Step test with fast and slow fluid flows ($q_i = 0.9m^3/s$ and $q_i = 0.1m^3/s$)

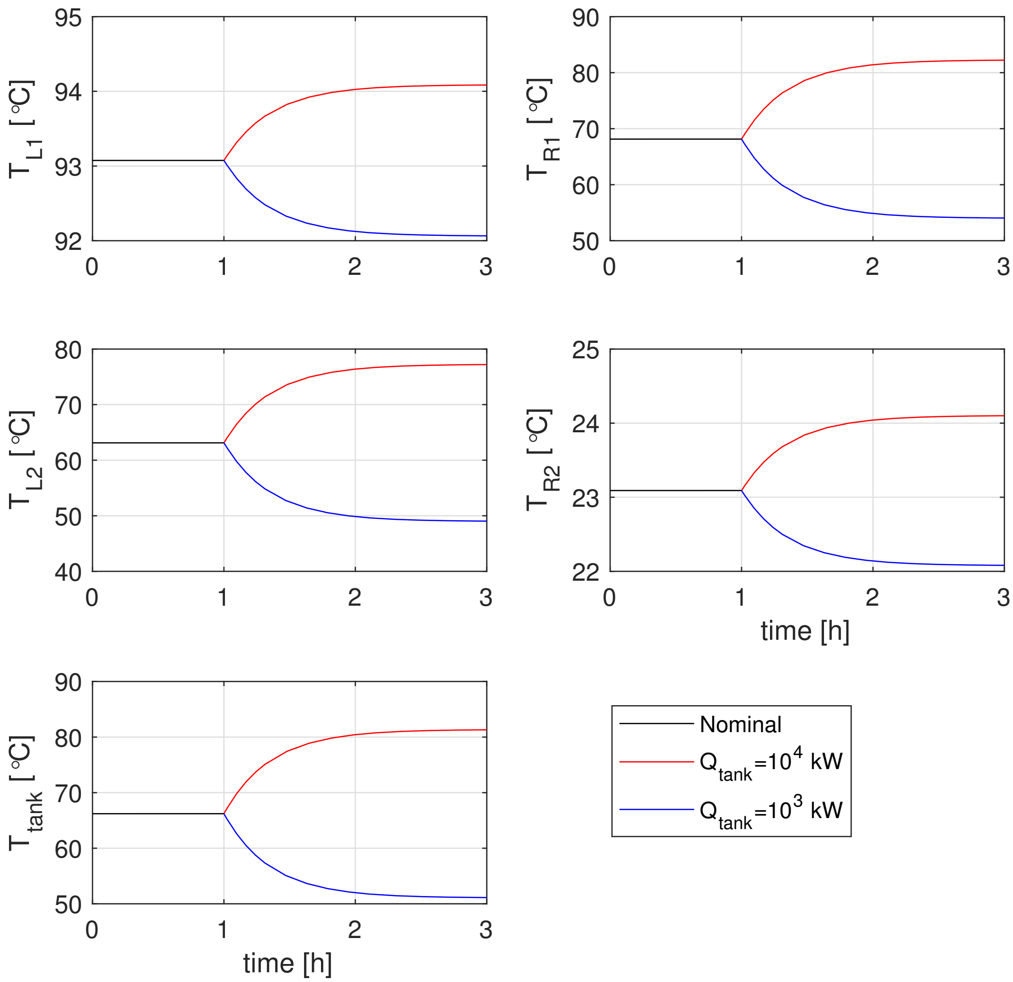

Figure 4.3: Step test in direct heating ($Q_{tank} = 10^4$kW and $Q_{tank} = 10^3$kW)

4.2.3 Step Test in Source Temperature

It is interesting to observe what happens when suddenly the supply has a lower heating potential. A drop in T_1 is introduced in the system after steady state operation and the simulation plots are shown in 4.4

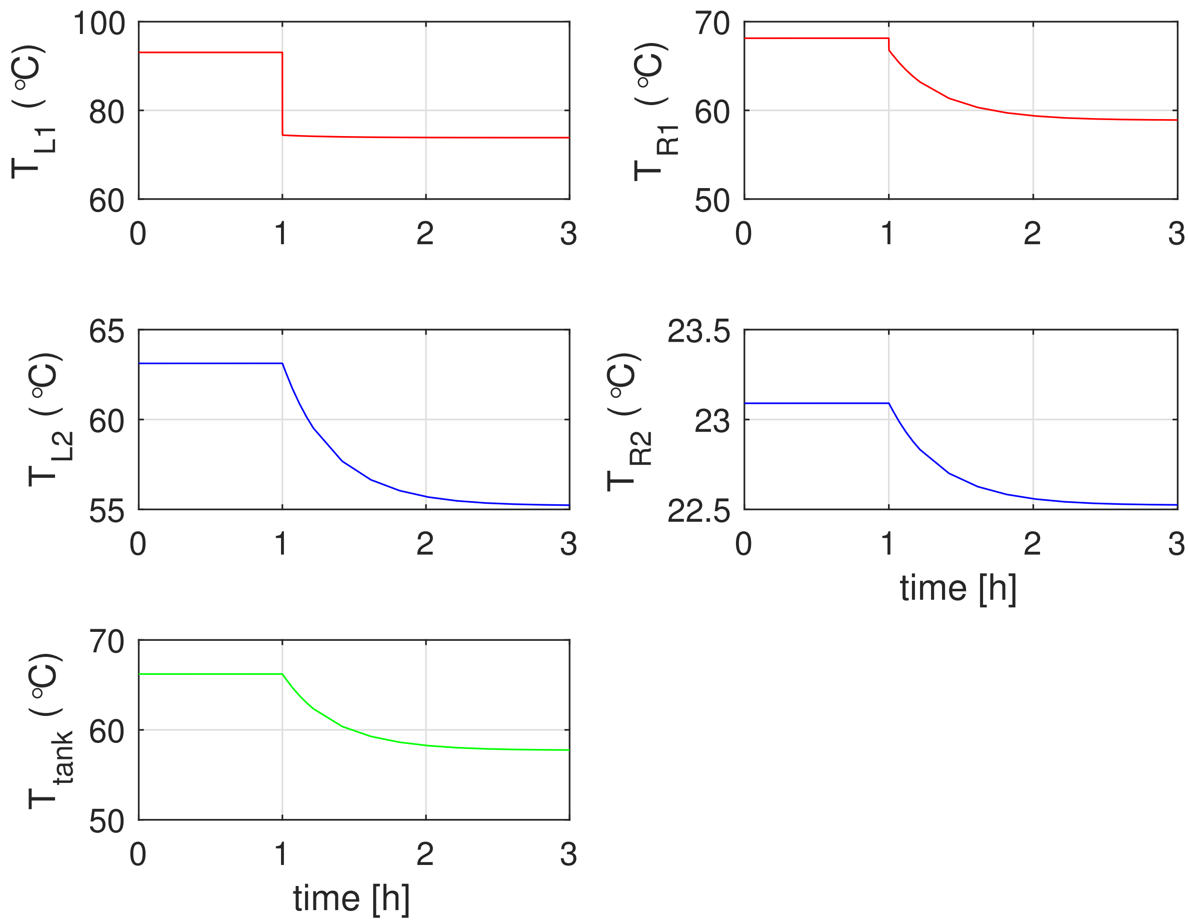

Figure 4.4: Step change in T_1 for two plant system ($\Delta T_1 = -20°C$)

It is evident that the drop in temperature of the source has a greater effect on the temperature of the streams in heat exhanger HX-1 than in those of heat exchanger HX-2. The simulations show that there is a 20°C drop in T_{L1} but only a 0.6°C drop in T_{R2}. This shows that even if the source supply changes, the consumer will still receive an almost steady supply with very little effect on the quality. The storage acts as a buffer to smoothen the effect of the disturbance in the source.

4.3 Four Plant System Dynamics

The simulation of the four plant system describe by equation 3.23 was also done. Source temperatures T_1 and T_3 were assumed as 90°C and 60°C respectively. The sink temperatures

T_2 and T_4 were set as 5°C and 20°C respectively.

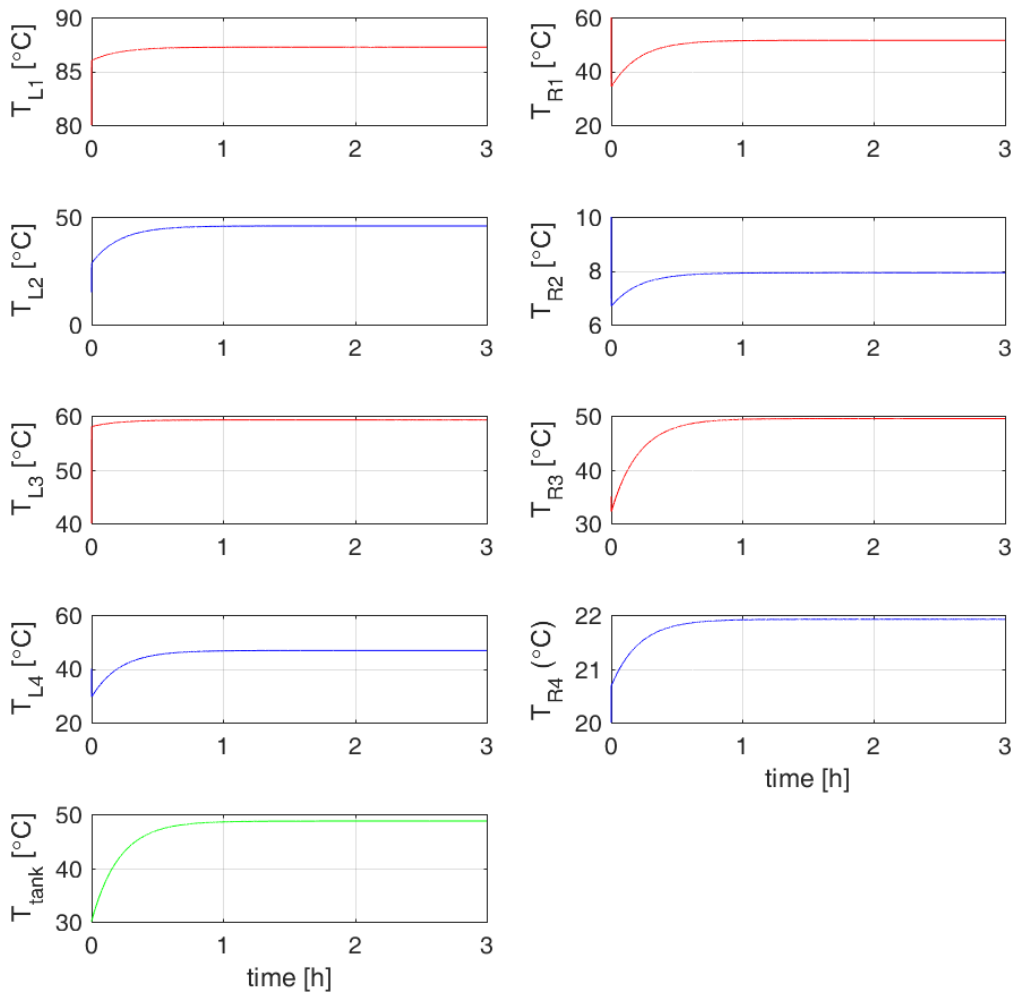

Figure 4.5: Four plant system simulation with nominal inputs

It can be seen in figure 4.5 that the dynamics of four plant system are similar to that of a two plant system. The difference is the steady state values of the system. The steady state tank temperature is slightly lower compared to the two plant case when provided with the same amount of direct heating. This is expected and it is due to an increased number of sinks which cool ("discharge") the storage tank streams faster.

From initially steady state operation, a positive and negative step in temperature of plant 1 was introduced. The negative step in T_1 caused a drop in all stream temperatures because the drop in temperature of the source implies less thermal energy available to be transferred to the sinks (see fig. 4.6). In addition, the effect of the temperature drop is significantly reduced at the sink streams T_{R4} and T_{R2}, similar to what was observed in a two plant system . A positive step in T_1 causes a rise in temperature of all the streams and in the same way, the effect is significantly reduced at the sinks.

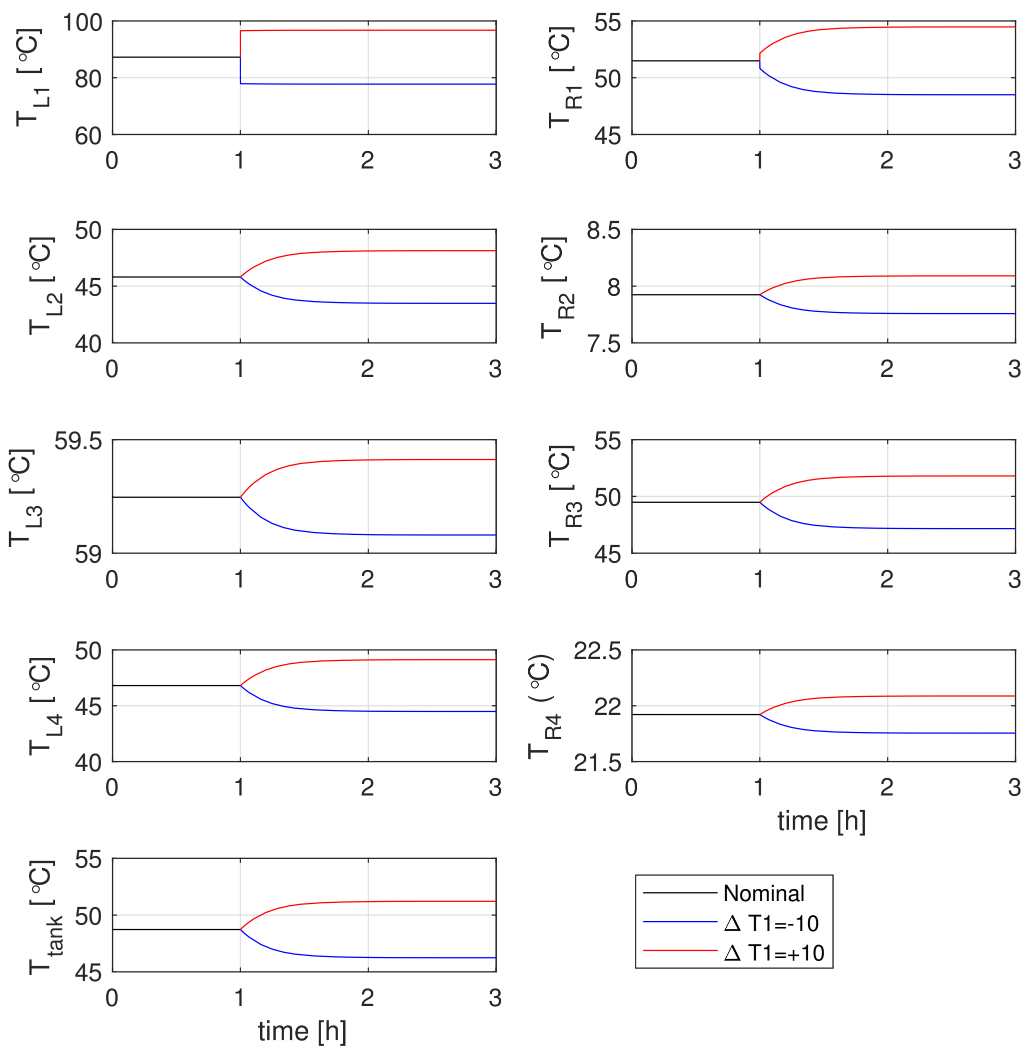

Figure 4.6: Four plant system simulation with step in T_1

The simulation plots with step test in the inputs and disturbances of the two plant and four plant system models show expected responses. This means that they are correct representations of the systems and can be used for system analysis, dynamic optimisation and control purposes.

5 — Open Loop Optimisation

In this chapter, an optimisation scenario is considered when there is a required thermal energy demand in the sink that must be satisfied by the energy storage system. The consumer's energy demand must be satisfied by manipulating the stream flows and direct tank heating. Only when necessary, at peak demands, the system may purchase energy from an expensive external source. The trade-off between the use of stored thermal energy and purchase is described by minimising a cost function. A non-linear program solver is used to solve the optimisation problem once over a prediction horizon of 24 hours to obtain an optimal control strategy with the associated optimal state trajectories. The location of energy demand and external peak supply from market for a simple two plant system is illustrated in figure 5.1.

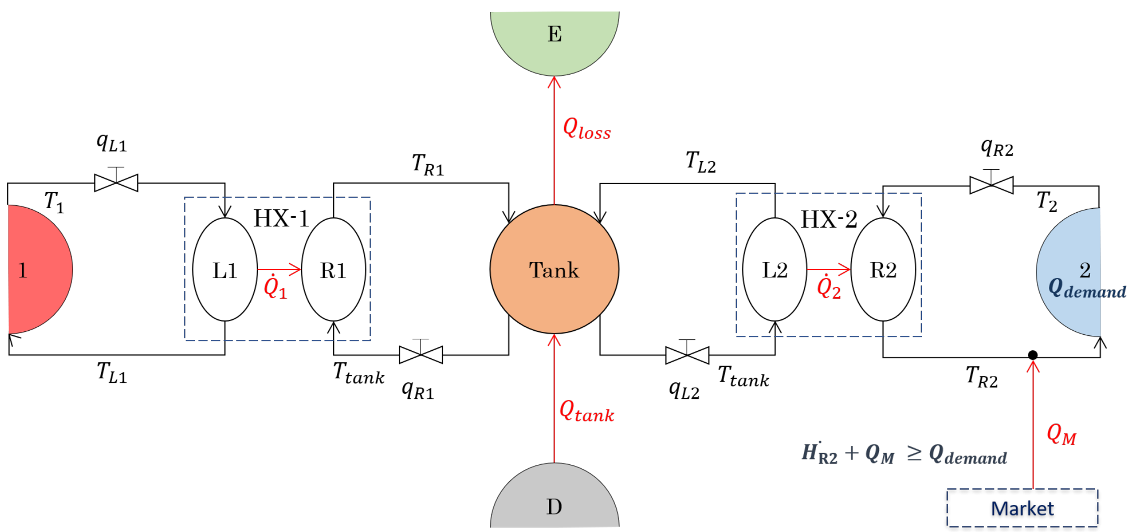

Figure 5.1: Illustration of a plant thermal storage system with demand on plant 2 and extra peak supply from market

5.1 The Optimal Control Problem

The objective is to control the system over a certain demand period, called prediction horizon, in order to ensure that the cost of purchasing energy from market is minimised while satisfying consumer demand at all times. The objective can be mathematically presented as an objective function in eq. (5.1).

$$\phi(\mathbf{x}, \mathbf{u}) = p_T Q_{\text{tank}} + p_M Q_M \tag{5.1a}$$

$$\min_{(\mathbf{x}, \mathbf{u})} \quad \phi(\mathbf{x}, \mathbf{u}) \tag{5.1b}$$

where p_T is the unit cost of direct tank heating, p_M is the unit price of energy purchased from the market. In this particular case we wish to penalise Q_M and favour Q_{tank}. Hence, p_M must be much greater than p_T.

The economic objective in 5.1 is subjected to constraints that limit the search domain for the solution. The constraints include:

1. Equality constraints: The states and controls are related by the system dynamic model and they can not violate it. The model of the system is an equality constraint in this optimal problem.

$$\dot{\mathbf{x}} = \mathbf{g}(\mathbf{x}, \mathbf{u}, \mathbf{d}) \tag{5.2}$$

2. Inequality constraints: These are relaxed conditions that the system must satisfy. They include:

   (a) State bounds: The system is assumed to have a storage fluid with properties of water. Therefore, in this case it assumed that the storage fluid does not exceed a temperature of 200 °C and does not go below zero. The tank temperature is not allowed to go below 30 °C. Hence eq. (5.3).

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 30 \end{bmatrix} \leq \begin{bmatrix} T_{L1} \\ T_{R1} \\ T_{L2} \\ T_{R2} \\ T_{\text{tank}} \end{bmatrix} \leq \begin{bmatrix} 200 \\ 200 \\ 200 \\ 200 \\ 150 \end{bmatrix} \tag{5.3}$$

   (b) Input bounds: The manipulated variables have limits to which they can be changed. In a practical case one would want to avoid having an optimal solution above or below a MV saturation limit range. In this scenario the flows can be adjusted between 0 and 1 m^3/s and Q_{tank} cannot exceed 5×10^3 kW. Hence eq. (5.4).

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \leq \begin{bmatrix} q_{L1} \\ q_{R1} \\ q_{L2} \\ q_{R2} \\ Q_{\text{tank}} \\ Q_{\text{M}} \end{bmatrix} \leq \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 5000 \\ +\infty \end{bmatrix} \tag{5.4}$$

(c) Consumer demand satisfaction: The scenario assumes that a customer requires a specified amount of energy at a specific time. If the demand profile is constant then the demand is independent of time. However, it is almost always the case that there is a varying demand in the sink which are usually periodic - depending on the time of the day or season of the year. In this case, a varying demand was considered assuming that it is periodic over a single day of operation. Therefore, the consumer stream must be heated enough by the storage to satisfy the demand. If the storage is not enough to cater the needs of the customer, energy is purchased from the market. It is strictly desired at any time t we must have eq. (5.5) satisfied.

$$Q_{\text{demand}}(t) = Q_{\text{M}} + \rho c_p q_{R2}(T_{R2} - T_2) \tag{5.5}$$

But the equation eq. (5.5) is a very hard constraint. IPOPT solver will encounter problems to find the optimal solution because of that. Implementing the energy demand constraint as eq. (5.6) helps IPOPT by widening the search area. It does not affect the solution since eq. (5.6) remains active at optimal solution to ensure feasibility.

$$Q_{\text{demand}}(t) - [Q_{\text{M}} + \rho c_p q_{R2}(T_{R2} - T_2)] \leq 0 \tag{5.6}$$

5.2 Results for Open Loop Optimisation with Periodic Demand

The OCP was transformed into a non-linear program (NLP) using direct collocation. The time grid selected for collocation points is Radau order 3. The reasons of using direct collocation are mentioned in section 2.4. Since there are ODEs as constraints, CasADi [26] was used to easily write them in MATLAB. After listing all constraints for the NLP, it was passed through an NLP solver `IPOPT` [27], to find an optimal solution. The iterations converged to an optimal solution. The MATLAB code used for optimisation is included in appendix B.1.3.

The demand profile for consumer plant 2 considered over the prediction horizon is shown in fig. 5.2. It has a period of 1 day (24 h) and has a peak demand of 9000 kW and lowest demand of 1000 kW. The mean energy demand is about 5000 kW. This is a typical consumer energy demand profile that there is one point in the day that it is highest and it varies throughout the day to cycle back to the peak demand after a constant time interval [28].

`IPOPT` was able to find an optimal solution to the problem. The source code for optimisation is in appendix B.1.3. The optimal states trajectory and optimal inputs were obtained and plotted against the horizon time in figure 5.3 and 5.5 respectively.

From the optimal states trajectory in fig. 5.3 it is observed that at the beginning of the prediction horizon the tank is heated up and its temperature rises to match the high demand at that point in time. After that T_{tank} remains constant before cooling because it does not need to maintain a high temperature when approaching the point of lowest demand.

As the time of lowest demand $t = 12$ h approaches the tank temperature rises. This is due to very small flow of the sink stream q_{R2} at that point. It can be seen at this time there is a

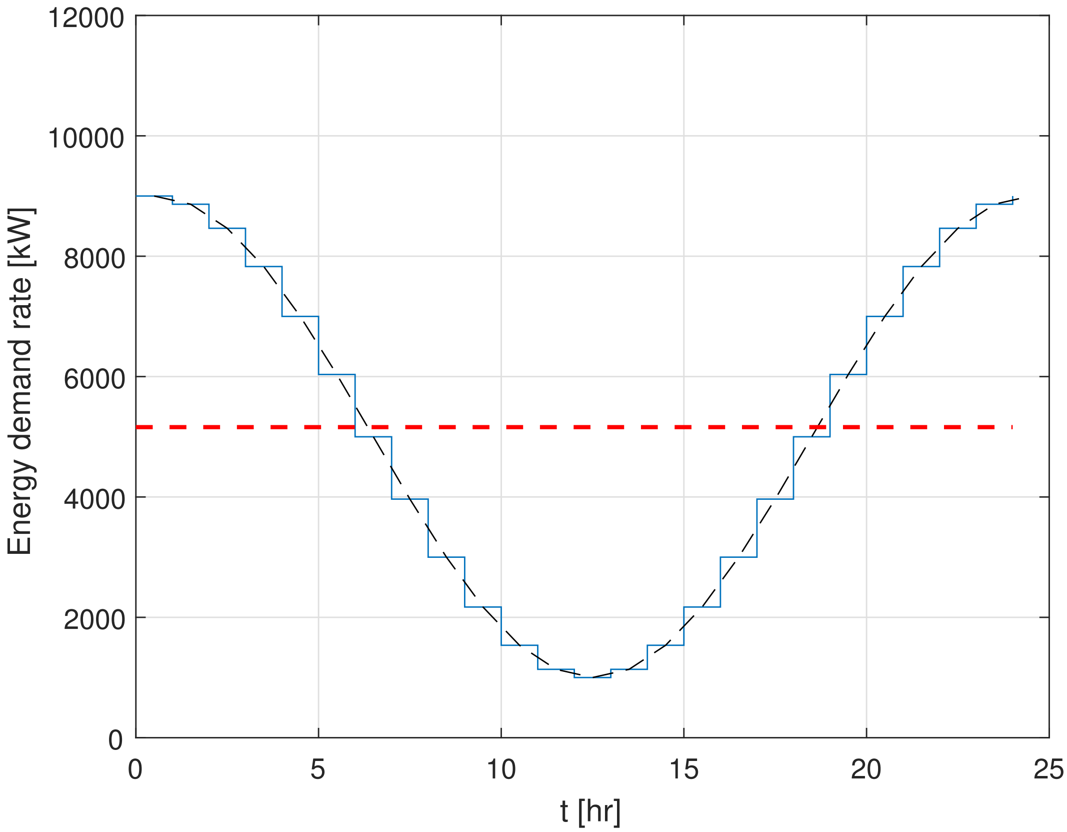

Figure 5.2: Periodic demand profile for plant 2

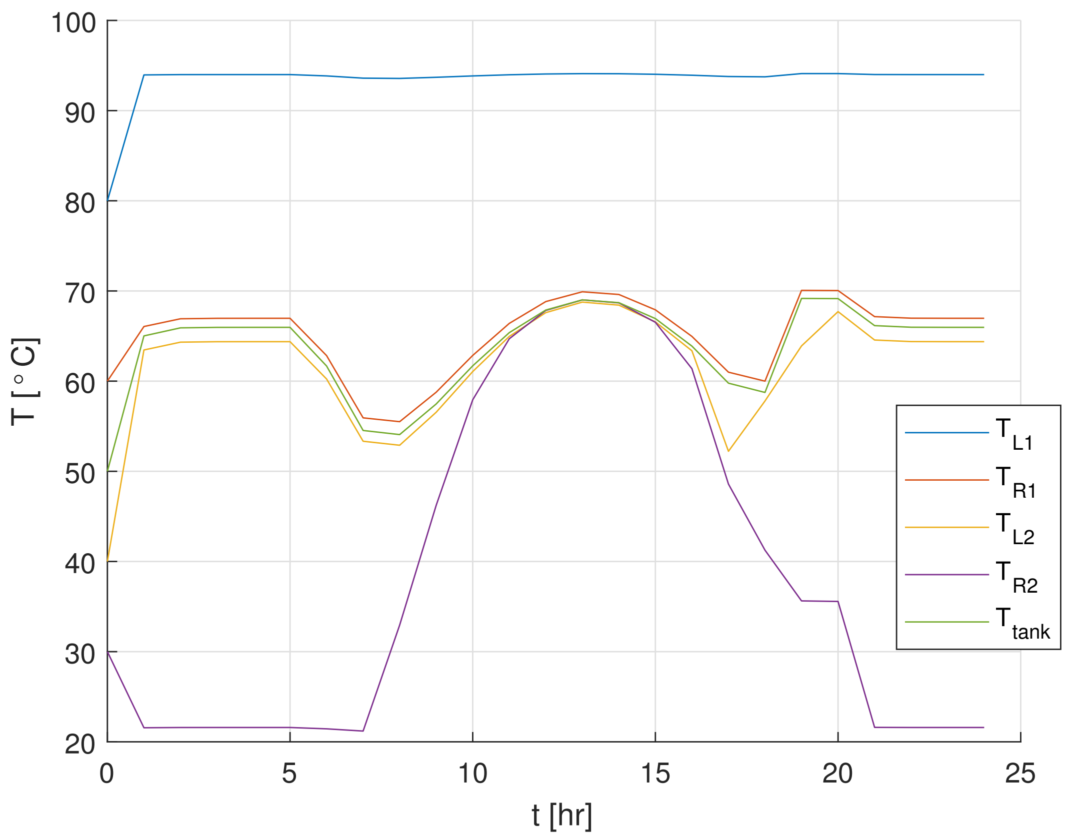

Figure 5.3: Optimal trajectory for the states

temperature pinch point across all heat exchangers and therefore the cooling rate from tank outlet stream is very low resulting in a rise of temperature.

Past the 12 hour mark the tank cools down because the demand starts rising again to another peak. However, before the peak is reached the tank is heated up and its temperature rises so that it is prepared to satisfy the peak demand.

The states trajectory does not violate laws of thermodynamics thus the optimal solution is physically correct. The stream temperatures in the heat exchangers do not "cross-over" at any point. It is only at the pinch point when the heat capacity of stream R2 is very low compared to L2 that the temperatures are very close approaching each other but T_{R2} never crosses above T_{tank}. So as to see this clearly, a zoomed plot at that pinch point is shown in fig. 5.4. The Underwood mean approximation does not allow temperature crossover even at large ratio between the heat exchanger flow streams. Use of arithmetic mean gives poor results as the optimal solution has temperature crossover (see optimisation with arithmetic mean in appendix A.2).

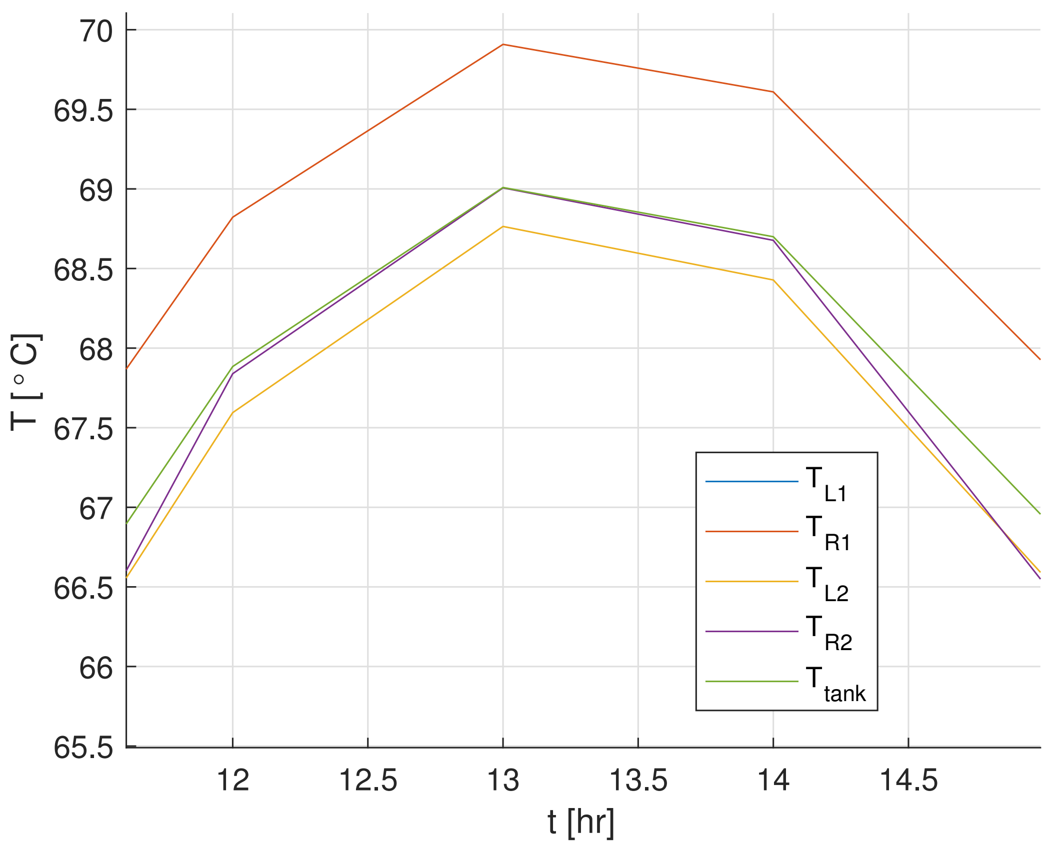

Figure 5.4: Illustration of temperature pinch point without crossover in heat exchangers with Underwood mean

From the optimal inputs in fig. 5.5 it is seen that the flows q_{L1} and q_{R1} remain at a maximum

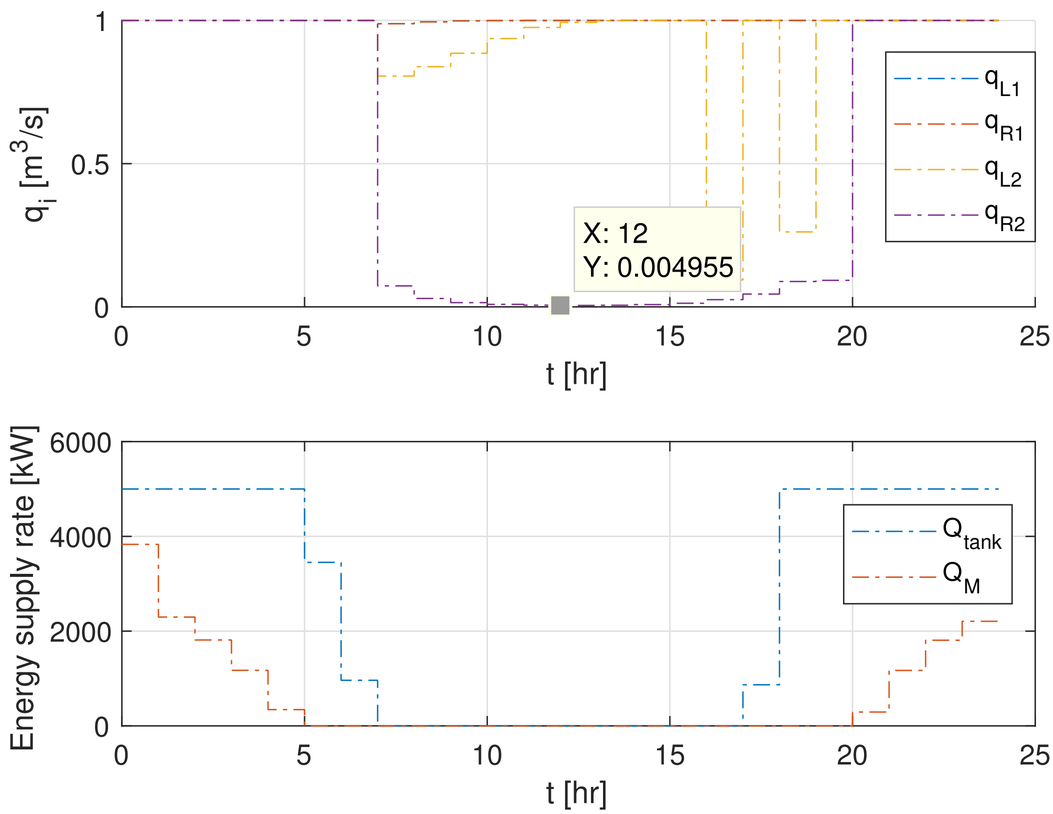

Figure 5.5: Optimal inputs for optimal trajectory

value of 1 m^3/s throughout the time horizon. This is because there are no constraints in the supply side and it desired to extract as much heat as possible from the source that coud be stored in the tank.

q_{L2} and Q_{tank} both start at their maximum limits to satisfy the peak demand at the beginning. After that both start decreasing as the demand drops significantly. q_{L2} decreases slightly as the demand starts to drop but as Q_{tank} decreses significantly, q_{L2} rises again in order to ensure that the energy demand requirement is met.

q_{R2} which is the flow stream from the consumer starts at maximum as well before dropping to a very low flow when the demand is lowest. At the 12 hour mark q_{L2} is about 0.004955m^3/s which is comparatively lower. This is because the flow is still large enough to satisfy the consumer requirements at that time. Thereafter, when the demand is approaching another peak, q_{L2} rises back to maximum.

Extra direct tank heating (Q_{tank}) is zero when the demand is lowest but it rises to maximum before an expected peak demand in order to heat the storage tank enough to prepare for peak period.

The system purchases extra energy from the market only when the demand stream flow q_{R2}, and Q_{tank} are at their maximum limits and still more energy is required to meet demand. Therefore, Q_{M} is highest at the peak demand time but quickly drops to zero as the demand decreases.

The optimal solution found for the two plant system is practical because the system is operated to always satisfy the demand at that point in time with minimum cost. The system also uses the thermal storage to its advantage and decides on the optimal way to heat the tank in the predetermined time horizon so as to make sure a future expected demand requirement is met. However, the control inputs from open loop optimization can not be implemented to efficiently control the system because:

- There are always deviations in the model from the real system. Open loop optimisation is based entirely on the model calculations but it is clear that no model is perfect. This model has been derived from a number of assumptions which do not exactly hold in a real case. Therefore, after some time of operation there will be significant deviations and the plant would be far from an optimal state resulting to losses.
- The demand profile is an uncertain and there is some degree of randomness. It is not sufficient to assume a fixed demand profile as done in this simple case study.

A more practical control strategy will be to obtain *plant measurement feedback* that identifies the states at fixed intervals within the horizon, and then reoptimise the open loop problem using feedback values as initial guesses to account for the model imperfections. In this case the prediction horizon becomes a moving horizon, moving forward after every reoptimisation loop. This control strategy is known as the model-predictive control (MPC) strategy.

6 — Model Predictive Control

In the previous chapter, dynamic optimisation was performed on a discretised model within a fixed time horizon without measurement feedback feedback from the system. It was an open loop optimisation problem, where the solution is calculated once at time $t = 0$ and the optimal inputs solution found are implemented throughout the prediction horizon. An alternative is to include feedback, that is *closed loop optimisation* where the optimal solution is recalculated at every time step. It is known as the model predictive control (MPC) principle. This chapter describes how MPC principle can be implemented in the simple case of a two plant thermal storage system.

6.1 The MPC Algorithm

The MPC strategy is described by Mayne [29] as:

> Model predictive control is a form of control in which the current control action is obtained by solving, at each sampling instant, a finite horizon open loop optimal control problem, using the current state of the plant as the initial state; the optimization yields an optimal control sequence and the first control in this sequence is applied to the plant.

The basic MPC algorithm is shown in table 6.1. It must be noted that once an optimal solution for the first step is found, the prediction horizon for the next optimisation calculations will shift one step forward. Therefore, for the simple two plant system the same optimal problem eq. (5.1) is solved over and over again at each time step [20].

Table 6.1: Basic MPC algorithm [20]

Algorithm:	State feedback MPC procedure
for	$t = 0, 1, 2, \ldots$ **do** Get the current state x_t. Solve a dynamic optimization problem on the prediction horizon from t to $t + N$ with x_t as initial condition Apply the first control move u_t from the solution above.
end for	

This case involves a non-linear program is solved in every time step the problem is specifically known as a nonlinear MPC or NMPC optimisation problem. The idea of implementation for the two plant system is to obtain the current state from a “simulator plant”. The simulator plant is the ODE system but with some Gaussian measurement noise in the states. This noise will model the similar deviations that are expected from a real system. Then the “plant” is simulated for one time step and the final value is set as the initial condition for the optimisation computations in the next interval. It is also possible to have the simple model with mean temperature difference as the difference of the inlet stream temperatures and obtain reasonable optimal operation. The implementation of this idea to the energy storage system is at an infant stage and concrete results on NMPC have not been covered in this report. However it is planned to be done in the future regarding this project.

7 — Discussion

Included in this chapter is a general discussion about the work that has been done. Some issues that occured while conducting the project work have been explained. Suggestions on future work and possible improvements of results have been highlighted.

7.1 Issues with initial guess to IPOPT

The optimisation solver IPOPT employs iterative interior-point algorithms which require an initial guess or starting point from which its starts to search for the optimal point. The selection of a starting point is usually not trivial, at least not in this project. The system model had the mean temperature difference term as an nth root polynomial - Underwood approximation which is highly non-linear. Some starting points would cause errors since CasADi can not evaluate the derivatives at those points. A good initial point would be the nominal or steady state values calculated from the model which is feasible and will avoid complex roots in computations.

7.2 Improvement of the Optimal Problem

The optimal control problem defined in eq. (5.1) is based on the linear cost function of purchased inputs. The optimal solution allows very large and sudden movements in the volumetric flows which may not be desired in operation. A way to avoid such control moves would be to include the flow terms in the objective and penalise them.

7.3 Further work on MPC Implementation

MPC control as discussed in chapter 6 is a more reliable approach since it includes feedback and will account for the model imperfections. Implementation of NMPC is recommended as a further step towards dynamic optimal control of the system. The real plant can be replaced by the accurate model which includes the Underwood mean approximation plus Gaussian noise while the optimiser can solve the model with a simple temperature difference between the two input streams of the heat exchangers.

7.4 Machine Learning Approaches

The parameters of the system keep changing with time. For example the heat transfer coefficients will change to time due to fouling conditions and stream quality. On top of that, the consumer demand profile is uncertain. Machine learning approaches could be applied to "learn" about the consumer behaviour from previous data and generate better demand profile predictions for the prediction horizons. It could also be used to adjust the changing system model parameters to obtain better predictions or even simpler models.

7.5 Other Practical Scenarios

More work could be done with systems that have more than one source and more than one sink. For example a scenario with two supplier but one has an availability constraint. Such a scenario is practical with renewable energy sources such as solar energy. Solar heating is available only in the day time and therefore it has a periodic supply profile.

For the case of two sinks it would be interesting to see how a control system could be designed to prioritize consumers based on their importance. Supply of energy to some plants could be given up quite easily because their processes are not critical. Another criteria would be meeting demands of a consumer whose process generates more profit.

8 — Conclusion

To begin with, a dynamic model for a two plant and four plant thermal energy storage system has been found. The heat exchanger mean temperature difference has been approximated using Underwood or Chen new mean. After simulation of the models with nominal inputs using `ode15s` function in MATLAB, the steady-state values for the system were obtained. From the steady-state, step responses from step changes in the inputs and disturbance variables were generated to check validity of the model. The simulation results showed the buffer action of the storage, that it ensured the consumer had minimal effect due to changes in the supply side. The model behaves as expected of the system and is suitable for optimisation calculations.

An optimisation scenario of two plant system with an expected energy demand profile has been investigated. The objective was to satisfy consumer energy demands at all times in the prediction horizon using the least cost possible. The decision variables has included the states and the inputs. The problem was discretised by direct collocation method and written in CasADi. The optimal solution to the dynamic control problem was found using an IPOPT solver in MATLAB. However, special care must be taken in selecting an initial guess for the optimiser in order to converge to the optimal solution. Not all initial conditions will give an optimal solution. The use of Underwood approximation in the model creates a high degree of non-linearity in the problem. I suggest the use of steady state values from model simulations as initial conditions.

The open loop optimisation of the system suggests a control sequence where the tank heats up before demand peaks to prepare for expected consumer requirements. The optimal operation allows the system to purchase extra energy from the market when the cheap heating source has reached maximum limit without satisfying demand. Moreover, when the demand is lowest, the energy stored in the tank is enough to satisfy the consumer. It might be possible to implement NMPC control strategy on the energy storage system which has feedback that will reduce the effect of model imperfections. On top of that, machine learning is a promising approach towards better forecasts of consumer demand and correcting system model parameters.

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A — Simulation data

A.1 Steady state values

The following table summarises the steady state values when the system inputs are set to their high and low values one at a time. Where $q^{low} = 0.1\text{m}^3/\text{s}$, $q^{high} = 0.9\text{m}^3/\text{s}$, $Q^{low}_{\text{tank}} = 10^3\text{kW}$, $Q^{high}_{\text{tank}} = 10^4\text{kW}$. The nominal values are defined in section 4.2.

Table A.1: Steady state values from two plant system simulations

x	Nom	q^{low}	q^{high}	Q^{low}_{tank}	Q^{high}_{tank}
T_{L1}	93.07	93.89	87.91	92.26	94.09
T_{R1}	68.14	67.10	75.21	56.83	82.28
T_{L2}	63.12	64.23	55.42	51.81	77.26
T_{R2}	23.09	21.76	32.69	22.28	24.10
T_{tank}	65.99	66.21	68.11	54.09	81.37

A.2 Open loop optimisation using arithmetic mean

This is an extra case done when $n = 1$ that the mean temperature difference in the heat exchanger becomes an arithemtic mean. Using the same demand requirements as in section 5.2 open loop optimisation calculations were done and the optimal solution found. The optimal states trajectory is shown in fig. A.1 and the optimal inputs are in fig. A.2. The optimal state trajectory is not physically possible because the outlet of the cold stream in HX-2 T_{R2} cannot be hotter than the inlet of the cold stream T_{tank}. This crossover violates the laws of thermodynamics and therefore the solution provided when arithmetic mean is used is not acceptable.

It can be seen that this crossover starts when the demand approaches it lowest. The flow of the cold stream R2 is decreasing to $0.002\text{m}^3/\text{s}$ which is very low compared to the exchanging hot stream L2. The ratio between the two heat capacities of the hot and cold streams become very large and therefore the arithmetic mean is no longer suitable.

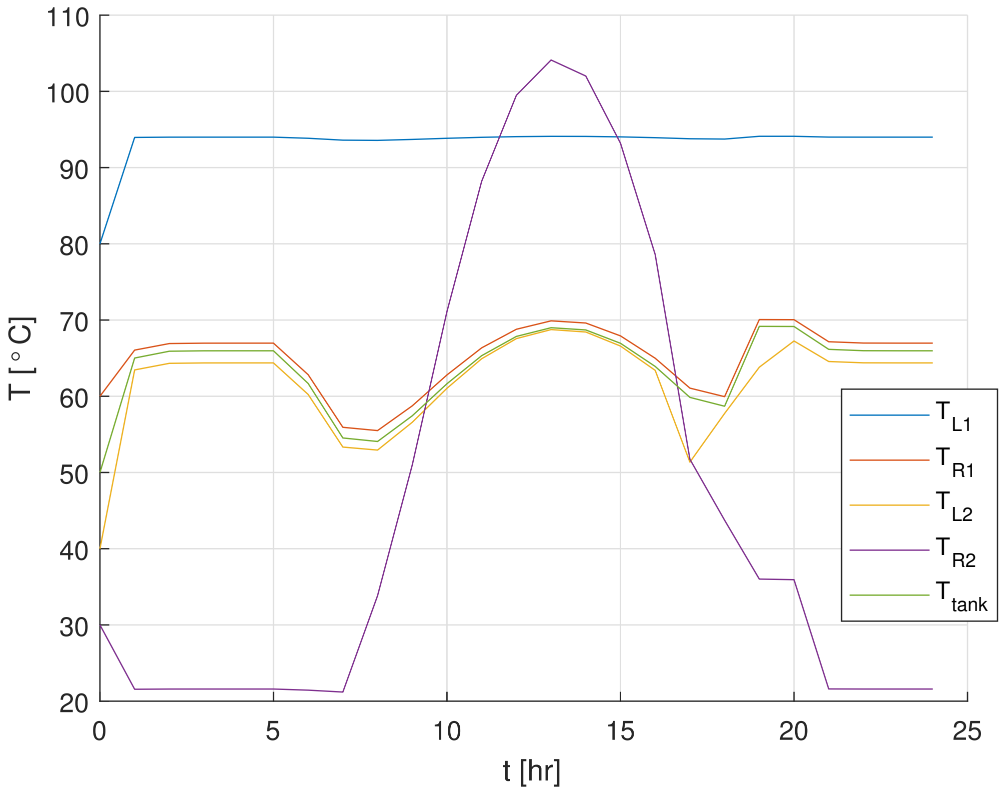

Figure A.1: Optimal state trajectory using arithmetic mean approximation

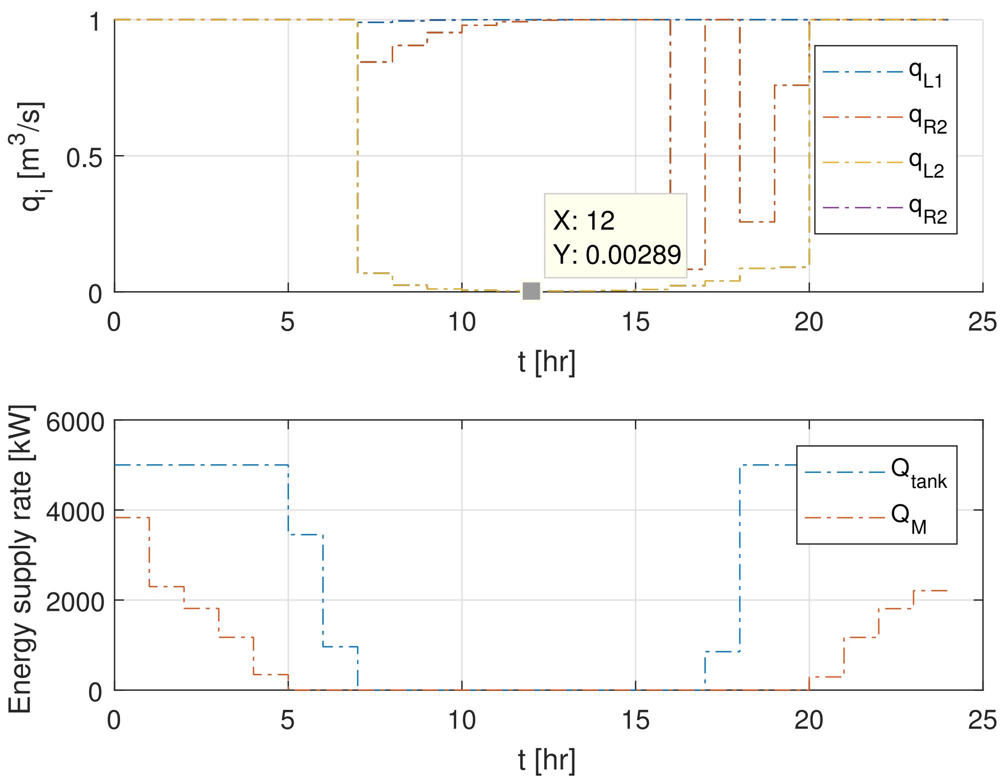

Figure A.2: Optimal control inputs using arithmetic mean approximation

B — Source codes

B.1 Two plant dynamics

B.1.1 ODE function

```
function dxdt = twoPlantModel(~,x,p)
% Model for an energy storage system with two plants
% a heat supplier and consumer, a storage tank and external heating.
%=========================================================================
% Author: Zawadi Mdoe
% Date: September 2018
%=========================================================================
% Description of the states:
% T_L1 = x(1)
% T_R1 = x(2)
% T_L2 = x(3)
% T_R2 = x(4)
% T_tank = x(5)
%========================================================================
%% Assignment of inputs and disturbances
% Input variables u's
q_L1 = p(1);
q_R1 = p(2);
q_L2 = p(3);
q_R2 = p(4);
Q = p(5);

% Distubances d's
T1 = p(6);
T2 = p(7);

% Design and physical parameters
V_hex = p(8);
V_tank = p(9);
U = p(10);
A_hex = p(11);
rho = p(12);
cp = p(13);
h_s = p(14);
A_tank = p(15);
```

```
T_s = p(16);
n = p(17);

%% ODEs
dxdt = zeros(5,1);

dxdt(1) = (1/V_hex)*(q_L1*(T1-x(1)) - sign(T1-x(5))...
    *(U*A_hex*0.5^(1/n)/(rho*cp))*((abs(T1-x(2)))^n + ...
    (abs(x(1)-x(5)))^n)^(1/n));

dxdt(2) = (1/V_hex)*(q_R1*(x(5)-x(2)) + sign(T1-x(5))...
    *(U*A_hex*0.5^(1/n)/(rho*cp))*((abs(T1-x(2)))^n + ...
    (abs(x(1)-x(5)))^n)^(1/n));

dxdt(3) = (1/V_hex)*(q_L2*(x(5)-x(3)) - sign(x(5)-T2)...
    *(U*A_hex*0.5^(1/n)/(rho*cp))*((abs(x(3)-T2))^n + ...
    (abs(x(5)-x(4)))^n)^(1/n));

dxdt(4) = (1/V_hex)*(q_R2*(T2-x(4)) + sign(x(5)-T2)*...
    (U*A_hex*0.5^(1/n)/(rho*cp))*((abs(x(3)-T2))^n + ...
    (abs(x(5)-x(4)))^n)^(1/n));

dxdt(5) = (1/V_tank)*(q_R1*(x(2)-x(5)) + q_L2*(x(3)-x(5)) ...
    + (Q-h_s*A_tank*(x(5)-T_s))/(rho*cp));

end
```

B.1.2 Simulation script

```
% TWO PLANT ENERGY STORAGE SYSTEM WITH STEP TEST
% ================================================================
% Script for simulation of energy storage system with two plants
% with a supplier and consumer, a storage tank and external heating.
%=================================================================
% Author: Zawadi N. Mdoe
% Date: September 2018
%=================================================================
clear all;
clc;

%% Declaration of model parameters
V_hex = 0.5;          %Heat exchanger volume (shell or tube side) [m^3]
V_tank = 100;         %Storage tank volume [m^3]
U_hex = 0.5;          %Overall heat transfer coefficient [kW/m^2K]
A_hex = 300;          %Heat transfer area [m^2]
rho= 1000;            %Density [kg/m^3]
cp = 4.186;           %Specific heat capacity of fluid [kJ/kgK]
h_s = 0.050;          %Heat loss coefficient [kW/m^2K]
n = 0.3275;           %Index (either 1,1/3 or 0.3275)
T_s = 15;             %Ambient temperature [C]
A_tank =1000;         %Tank heat loss area [m^2]

%% Initialize disturbances
```

```
T1 = 95;            %Source temperature [C]
T2 = 20;            %Sink Temperature [C]

T1_step = -20;      %Step disturbance in T1 [C]

%% Initilize inputs
q_L1 = 0.5;         %[m^3/s]
q_R1 = 0.5;         %[m^3/s]
q_L2 = 0.5;         %[m^3/s]
q_R2 = 0.5;         %[m^3/s]
Q = 5e3;            %[kW]

p1 = [q_L1 q_R1 q_L2 q_R2 Q T1 T2 V_hex V_tank ...
    U_hex A_hex rho cp h_s A_tank T_s n];

%% Time controls
t_final = 6*3600;   %[sec]
t_step = 10000;     %[sec]

%% Initial values for states
x0 = [80 60 45 30 50];

%% Introducing a time step in T1
T1_2 = T1 + T1_step;
p2 = [q_L1 q_R1 q_L2 q_R2 Q T1_2 T2 V_hex V_tank U_hex ...
    A_hex rho cp h_s A_tank T_s n];

%% Solving the system of ODEs using ode45
tspan = [0 t_step];     % simulation timespan
[t1,x1] = ode45(@(t,x) twoPlantModel(t,x,p1), tspan, x0);
trows = size(t1,1);     % number of rows of the time vector

% after the step is applied
tspan = [0 t_final-t_step];
x0 = x1(trows,:);
[t2,x2] = ode45(@(t,x) twoPlantModel(t,x,p2), tspan, x0);

t2 = t2 + t_step;       % time offsetting

% concatenation to obtain final output
t = vertcat(t1,t2);     %building time vector
x = vertcat(x1,x2);     %building states vector
t = t/3600;             %conversion from seconds to hours

%% Plotting results
figure(1)
subplot(321)
plot(t,x(:,1),'r-')
ylabel('T_{L1} (\circC)')
grid on

subplot(322)
plot(t,x(:,2),'r-')
ylabel('T_{R1} (\circC)')
grid on
```

```
subplot(323)
plot(t,x(:,3),'b-')
ylabel('T_{L2} (\circC)')
grid on

subplot(324)
plot(t,x(:,4),'b-')
xlabel('time (h)')
ylabel('T_{R2} (\circC)')
grid on

subplot(325)
plot(t,x(:,5),'g-')
xlabel('time (h)')
ylabel('T_{tank} (\circC)')
grid on
```

B.1.3 Open loop optimisation

```
% An implementation of direct collocation to open loop
% dynamic optimisation of a two plant
% Energy Storage System using CasADi
%% VARIABLE DEMAND!!!
% Note: Convergence of solution depends on the starting values x_init!
%=========================================================================
% Author: Zawadi Mdoe
% Date: October 2018
% ========================================================================
clear all;
clc;

addpath(...
  'C:\Users\Zawadi Mdoe\Desktop\Matlab\casadi-windows-matlabR2016a-v3.4.5')
import casadi.*

%% Collocation settings

% Degree of interpolating polynomial
d = 3;

% Get collocation points
tau_root = [0 collocation_points(d, 'radau')]; %can be 'legendre'

% Coefficients of the collocation equation
C = zeros(d+1,d+1);

% Coefficients of the continuity equation
D = zeros(d+1, 1);

% Coefficients of the quadrature function
B = zeros(d+1, 1);

% Construct polynomial basis
```

```
for j=1:d+1
  % Construct Lagrange polynomials to get the polynomial basis
  % at the collocation point
  coeff = 1;
  for r=1:d+1
    if r ~= j
      coeff = conv(coeff, [1, -tau_root(r)]);
      coeff = coeff / (tau_root(j)-tau_root(r));
    end
  end
  % Evaluate the polynomial at the final time to get the
  % coefficients of the continuity equation
  D(j) = polyval(coeff, 1.0);

  % Evaluate the time derivative of the polynomial at all collocation
  % points to get the coefficients of the continuity equation
  pder = polyder(coeff);
  for r=1:d+1
    C(j,r) = polyval(pder, tau_root(r));
  end

  % Evaluate the integral of the polynomial to get the coefficients
  % of the quadrature function
  pint = polyint(coeff);
  B(j) = polyval(pint, 1.0);
end

%% Initialization of model parameters
V_hex = 0.5;            %Heat exchanger volume (shell or tube side) [m^3]
V_tank = 100;           %Storage tank volume [m^3]
U_hex = 0.50;           %Overall heat transfer coefficient [kW/m^2K]
A_hex = 300;            %Heat transfer area [m^2]
rho= 1000;              %Density [kg/m^3]
cp = 4.186;             %Specific heat capacity of fluid [kJ/kgK]
h_s = 0.050;            %Heat loss coefficient [kW/m^2K]
n = 1/3;                %Approximation index
A_tank =1000;           %Tank heat loss area [m^2]
Pm =1000e-6;            %Price of market energy source
Pt =5e-6;               %Price of cheap heating from flue gas
Q_demand_0 = 5000;      %Mean demand [kW]

%% Initialize Disturbances (Boundary conditions)
T1 = 95;                %Plant 1 temperature (source)[degC]
T2 = 20;                %Plant 2 temperature (sink)[degC]
T_s = 15;               %Ambient temperature [degC]

%% Variable Bounds

%Upper bounds
T_L1ub = 200;
T_R1ub = 200;
T_L2ub = 200;
T_R2ub = 200;
T_tankub = 100;
q_L1ub = 1;
q_R1ub = 1;
```

```
q_L2ub = 1;
q_R2ub = 1;
Qub = 5000;
Qmub = Inf;

%Lower bounds
T_L1lb = 0;
T_R1lb = 0;
T_L2lb = 0;
T_R2lb = 0;
T_tanklb = 30;
q_L1lb = 0;
q_R1lb = 0;
q_L2lb = 0;
q_R2lb = 0;
Q_tanklb = 0;
Qmlb = 0;

%Create bound vectors
xub = vertcat(T_L1ub, T_R1ub, T_L2ub, T_R2ub, T_tankub);
xlb = vertcat(T_L1lb, T_R1lb, T_L2lb, T_R2lb, T_tanklb);
uub = vertcat(q_L1ub, q_R1ub, q_L2ub, q_R2ub, Qub, Qmub);
ulb = vertcat(q_L1lb, q_R1lb, q_L2lb, q_R2lb, Q_tanklb, Qmlb);

%% Initial conditions
x_init = [80; 60; 40;30; 50]; %[85; 75; 55;45; 65]; %[50; 40; 20; 10; 30];
u_init = [0.5;0.5;0.5;0.5; 5000; 0];

%% Time horizon
T = 24*60*60;           %Time horizon for optimization

%% State and Input variables
nx = 5;                 %number of state variables
nu = 6;                 %number of controls

% Declare model variables
x1 = SX.sym('x1');
x2 = SX.sym('x2');
x3 = SX.sym('x3');
x4 = SX.sym('x4');
x5 = SX.sym('x5');
x = [x1; x2; x3; x4; x5];
u1 = SX.sym('u1');
u2 = SX.sym('u2');
u3 = SX.sym('u3');
u4 = SX.sym('u4');
u5 = SX.sym('u5');
u6 = SX.sym('u6');
u = [u1; u2; u3; u4; u5; u6];

%% Model equations
xdot =  [(1/V_hex)*(u1*(T1-x1) - (U_hex*A_hex*0.5^(1/n)/(rho*cp))*...
    ((T1-x2)^n + (x1-x5)^n)^(1/n));

         (1/V_hex)*(u2*(x5-x2) + (U_hex*A_hex*0.5^(1/n)/(rho*cp))*(...
         (T1-x2)^n + (x1-x5)^n)^(1/n));
```

```
        (1/V_hex)*(u3*(x5-x3) - (U_hex*A_hex*0.5^(1/n)/(rho*cp))*(...
        (x3-T2)^n + (x5-x4)^n)^(1/n));

        (1/V_hex)*(u4*(T2-x4) + (U_hex*A_hex*0.5^(1/n)/(rho*cp))*(...
        (x3-T2)^n + (x5-x4)^n)^(1/n));

        (1/V_tank)*(u2*(x2-x5) + u3*(x3-x5) + (u5-h_s*A_tank*(x5-T_s)...
        )/(rho*cp))];

%% Set up OCP

% Objective term
L = Pm*u6 + Pt*u5;

% Continuous time dynamics
f = Function('f', {x, u}, {xdot, L});

% Control discretization
N = 24;             % number of control intervals
h = T/N;            % length of discrete element
period = N;         % Demand is cycling with a period of one day
Q_demand = zeros(N,1);

% Demand curve
for i=1:N+1
    Q_demand(i) = Q_demand_0*(1+0.8*sin(2*(pi/period)*(i-1)+pi/2));
end

% Start with an empty NLP
w={};
w0 = [];
lbw = [];
ubw = [];
J = 0;
g={};
lbg = [];
ubg = [];

% "Lift" initial conditions
Xk = MX.sym('X0', nx);
w = {w{:}, Xk};
lbw = [lbw; x_init];
ubw = [ubw; x_init];
w0 = [w0; x_init];

% Formulate the NLP
for k=0:N-1
    % New NLP variable for the control
    Uk = MX.sym(['U_' num2str(k)],nu);
    w = {w{:}, Uk};
    lbw = [lbw; ulb];
    ubw = [ubw;  uub];
    w0 = [w0; u_init];

    % State at collocation points
```

```
    Xkj = {};
    for j=1:d
        Xkj{j} = MX.sym(['X_' num2str(k) '_' num2str(j)],nx);
        w = {w{:}, Xkj{j}};
        lbw = [lbw; xlb];
        ubw = [ubw;  xub];
        w0 = [w0; x_init];
    end

    % Loop over collocation points
    Xk_end = D(1)*Xk;
    for j=1:d
       % Expression for the state derivative at the collocation point
       xp = C(1,j+1)*Xk;
       for r=1:d
           xp = xp + C(r+1,j+1)*Xkj{r};
       end

       % Add inequality constraint
       g = {g{:}, Uk(6) + Uk(4)*rho*cp*(Xkj{j}(4)-T2)};
       lbg = [lbg; Q_demand(k+1)];
       ubg = [ubg; Inf];

       % Append collocation equations
       [fj, qj] = f(Xkj{j},Uk);
       g = {g{:}, h*fj - xp};
       lbg = [lbg; zeros(nx,1)];
       ubg = [ubg; zeros(nx,1)];

       % Add contribution to the end state
       Xk_end = Xk_end + D(j+1)*Xkj{j};

       % Add contribution to quadrature function
       J = J + B(j+1)*qj*h;
    end

    % New NLP variable for state at end of interval
    Xk = MX.sym(['X_' num2str(k+1)], nx);
    w = {w{:}, Xk};
    lbw = [lbw; xlb];
    ubw = [ubw;  xub];
    w0 = [w0; x_init];

    % Add equality constraint
    g = {g{:}, Xk_end-Xk};
    lbg = [lbg; zeros(nx,1)];
    ubg = [ubg; zeros(nx,1)];
end

%% Create an NLP solver
prob = struct('f', J, 'x', vertcat(w{:}), 'g', vertcat(g{:}));
solver = nlpsol('solver', 'ipopt', prob);

%% Solve the NLP
sol = solver('x0', w0, 'lbx', lbw, 'ubx', ubw,'lbg', lbg, 'ubg', ubg);
w_opt = full(sol.x);
```

```
%% Plot the solution
x1_opt = w_opt(1:(nx+nu)+nx*d:end);
x2_opt = w_opt(2:(nx+nu)+nx*d:end);
x3_opt = w_opt(3:(nx+nu)+nx*d:end);
x4_opt = w_opt(4:(nx+nu)+nx*d:end);
x5_opt = w_opt(5:(nx+nu)+nx*d:end);
u1_opt = w_opt(6:(nx+nu)+nx*d:end);
u2_opt = w_opt(7:(nx+nu)+nx*d:end);
u3_opt = w_opt(8:(nx+nu)+nx*d:end);
u4_opt = w_opt(9:(nx+nu)+nx*d:end);
u5_opt = w_opt(10:(nx+nu)+nx*d:end);
u6_opt = w_opt(11:(nx+nu)+nx*d:end);
T = T/3600;
tgrid = linspace(0, T, N+1);
clf;

% Plot state trajectory
figure(1)
plot(tgrid, x1_opt, '-')
hold on
plot(tgrid, x2_opt, '-')
plot(tgrid, x3_opt, '-')
plot(tgrid, x4_opt, '-')
plot(tgrid, x5_opt, '-')
xlabel('t [hr]')
ylabel('T [\circ C]')
legend('T_{L1}','T_{R1}','T_{L2}','T_{R2}','T_{tank}')
hold off
grid on

%Plot optimal controls
figure(2)
subplot(211)
stairs(tgrid, [u1_opt; nan], '-.')
hold on
stairs(tgrid, [u2_opt; nan], '-.')
stairs(tgrid, [u3_opt; nan], '-.')
stairs(tgrid, [u4_opt; nan], '-.')
hold off
xlabel('t [hr]')
ylabel('q_{i} [m^3/s]')
legend('q_{L1}','q_{R2}','q_{L2}','q_{R2}')
grid on

subplot(212)
stairs(tgrid, [u5_opt; nan], '-.')
hold on
stairs(tgrid, [u6_opt; nan], '-.')
hold off
xlabel('t [hr]')
ylabel('Energy supply rate [kW]')
legend('Q_{tank}','Q_M')
grid on

% Plot demand profile
```

```
figure(3)
stairs(tgrid, Q_demand, '-')
hold on
plot(tgrid+0.5, Q_demand, 'k--')
plot(tgrid, mean(Q_demand)*ones(size(tgrid,2),1),'r--','LineWidth',1.5)
hold off
xlabel('t [hr]')
ylabel('Energy demand rate [kW]')
ylim([0 12000])
grid on
```