TFY4205 Quantum Mechanics II
Problemset 2 fall 2022

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Problem 1 (harmonic oscillator)

In this problem, we consider various aspects of a 1D harmonic oscillator with mass m and angular frequency ω.

1. Calculate the average value of q^5 in the nth energy eigenstate $|n\rangle$.
2. Calculate the average value of q^4 in the ground state $|0\rangle$ using the raising and lowering operators.
3. Why is the average value in 2. equal to the square of the absolute value of the vector $\hat{q}^2|0\rangle$? Calculate $\hat{q}^2|0\rangle$ and deduce from this the average value of q^4 in the ground state.
4. Show that the matrix elements of the position operator are
$$\langle k|\hat{q}|l\rangle = q_0(\sqrt{l}\delta_{k,l-1} + \sqrt{l+1}\delta_{k,l+1}) \tag{1}$$
where δ_{nm} is a Kronecker delta and
$$q_0 \equiv \sqrt{\hbar/2m\omega}. \tag{2}$$
5. Let the state vector be in a superposition of two stationary states so that at $t = 0$ it reads:
$$|\Psi(t=0)\rangle = \frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle. \tag{3}$$
Calculate the average position $\langle q\rangle$ in this state (use the result from 4.)
6. For the oscillator which at $t = 0$ was in the state given by Eq. (3), calculate the average value of q at a later time t.

Problem 2 (time-independent perturbation theory)

A harmonic oscillator with $H_0 = p_x^2/2m + \frac{1}{2}m\omega^2x^2$ is perturbed by a force acting in the positive x-direction. This corresponds to adding a term $H_1 = -Fx$ to the Hamiltonian, so that $H = H_0 + H_1$.

1. Using time-independent perturbation theory, compute the energy eigenvalues E_n up to second order in the perturbation and also compute the state vector $|\psi_n\rangle$ to first order in the perturbation.
2. Show that by rewriting the total Hamiltonian H, it is possible to solve this problem *exactly* and identify what the exact energy eigenvalues $E_n = E_n(F)$ are.

Problem 3 (time-independent perturbation theory)

A nonlinear two-dimensional harmonic oscillator has the Hamiltonian:
$$H = \frac{1}{2m}(p_x^2 + p_y^2) + \frac{1}{2}m\omega^2(x^2+y^2)(1+\lambda xy), \tag{4}$$
where λ is a small parameter.

1. Find the three lowest states when $\lambda = 0$.
2. Using first order perturbation theory, calculate the energy eigenvalues for the three lowest energy states when $\lambda \neq 0$.