TFY4205 Quantum Mechanics II

Problemset 10 fall 2022

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Problem 1 (density matrix and polarization)

Consider a beam of unpolarized light. We know that the polarization of light comprises a two-dimensional Hilbert space because an electromagnetic wave can only have two independent polarization directions. Let us for instance name the two polarization states $|1\rangle$ and $|2\rangle$. Physically, the polarization of light determines the direction of oscillation of the electric field component of the light.

First, compute the density matrix operator $\hat{\rho}$ for unpolarized light. Prove mathematically if this is a pure or mixed state. Secondly, consider a polarizing device which we send the unpolarized light beam through. Find the density matrix operator for the polarized light after it has passed through this device, and prove mathematically if this is a pure or mixed state.

Problem 2 (entangled states)

When combining two qubits, a basis for the possible combined states is the *singlet-triplet* basis. The three *triplet* states are

$$|T,1\rangle = |11\rangle \tag{1}$$

$$|T,0\rangle = \frac{1}{\sqrt{2}}\big[|01\rangle + |10\rangle\big] \tag{2}$$

$$|T,-1\rangle = |00\rangle \tag{3}$$

and the *singlet* state is defined as

$$|S\rangle = \frac{1}{\sqrt{2}}\big[|01\rangle - |10\rangle\big]. \tag{4}$$

1. Show that $|S\rangle$ and $|T,0\rangle$ states are not separable states, $\phi \otimes \psi$, for any single-qubit states ϕ and ψ.

2. Show that $|S\rangle$ is physically unchanged under a unitary operation U, such as a rotation of the qubit states. That is, for any unitary one-qubit operator U, show that

$$(U \otimes U)|S\rangle \tag{5}$$

   is physically equivalent to $|S\rangle$.