TFY4205 Quantum Mechanics II
Problemset 1 fall 2022

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Problem 1 (eigenvalues)
A particle experiences a one dimensional potential $V(x)$. The minimum value of the potential is V_{min}, and for large positive and negative values of the coordinate x the potential is constant, $V(x) = V_+$.

Use the stationary Schrödinger equation to argue that there are no energy eigenfunctions with a finite norm for $E < V_{\text{min}}$ or for $E > V_+$. In other words, if normalizable energy eigenfunctions exist, then the corresponding discrete energy eigenvalues E_n are in the interval (V_{min}, V_+).

Problem 2 (Hilbert space)
Describe what a Hilbert space is both in terms of its key mathematical properties and which role it plays in quantum mechanics.

Problem 3 (bra-ket algebra)
Let the (ket) vector $|a\rangle$ be normalized to 1, so that $\langle a|a|\rangle = 1$. If $|b\rangle = (1+i)|a\rangle$, what is then:

- $\langle b|$
- $\langle a|b\rangle$
- $\langle b|a\rangle$
- $\langle b|b\rangle$

Problem 4 (repetition of expectation values and probabilites
The electron in a hydrogen atom is in the normalized state

$$R_{21}(r)\left[\sqrt{1/3}Y_{10}\chi_+ + \sqrt{2/3}Y_{11}\chi_-\right], \tag{1}$$

where $\chi_\pm$ are the spin states with the z-axis as the axis of quantization, and

$$Y_{10} = \sqrt{\frac{3}{4\pi}}\cos\theta,\; Y_{11} = -\sqrt{\frac{3}{8\pi}}\sin\theta e^{i\phi}. \tag{2}$$

The angular part of the state, which can be written in terms of angular momentum eigenvectors $|l,m\rangle$,

$$|v\rangle = \sqrt{1/3}|1,0\rangle|\uparrow\rangle + \sqrt{2/3}|1,1\rangle|\downarrow\rangle, \tag{3}$$

is all you need for the following questions.

If you measure the quantity F, what values can you obtain, and with what probability, when F is the quantity specified below:

1. $\boldsymbol{L}^2$.
2. L_z.
3. S_z.
4. $\boldsymbol{J}^2$, where $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$. You are provided the information that $\langle \boldsymbol{J}^2 \rangle = 41\hbar^2/12$.
5. J_z.