FY3403 Particle physics
Problemset 10 fall 2012

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Problem 18. e^+e^--production due to the background radiation in the universe.
It is well known that the universe is filled with thermal photons of temperature $T \approx 2.7$ K (the background radiation), close to isotropic in our Lorentz frame of reference. The precense of this radiation means that photons with *very* high energy cannot propagate over long distances in the universe. Instead they will collide with photons of the background radiation to produce electron-positron pairs.

Given: The Boltzmann constant $k_B \approx 8.6 \times 10^{-5}$ eV/K.

a) Give an estimate of how high energy, $\hbar\omega$, a photon must have to be able to create an electron-positron pair by collision with the background radiation.

b) As a much simplified model for the production process we treat the photon as a massless neutral scalar particle, and the electron (positron) as a charged scalar particle with mass m_e. The interaction vertex beween a photon and two charged (electron or positron) particles is given a value $\mathrm{i}em_e$. Draw the Feynman diagrams for the pair production process.

c) Find in this simplified model the algebraic expression for the scattering amplitude $\mathcal{M}_{fi}$. You may use units where $\hbar = c = 1$.

d) Find in this simplified model, in the center-of-mass system, the differential and the total crossection for electron-positron pair production.

Problem 19. Two-component model for particle decay, and the quantum mechanical Xeno's paradox.
In this problem we will consider an extremely simplified model of particle decay, in which we let the quantum state $\Psi_i = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ describe the situation where a particle (f.i. a neutron) still exists, and $\Psi_f = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ the situation where it has disintegrated. We assume the hamiltonian for the system to be $H = H_0 + H_1$, where $H_0 = \begin{pmatrix} E & 0 \\ 0 & E \end{pmatrix}$ determines the time evolution for "free" system, and $H_1 = \begin{pmatrix} 0 & \epsilon \\ \epsilon & 0 \end{pmatrix}$ is the origin of the decay process.

a) Find the eigenvalues and eigenvectors of H.

b) Assume that $\Psi(t_i) = \Psi_i$, and determine $\Psi(t_i + \Delta t)$. Use units where $\hbar = 1$.

c) At time $t_i + \Delta t$ we observe whether the particle has decayed or not. What is the probability $p(\Delta t)$ that is still exist?

d) According to the standard quantum mechanical postulates the system after the observation can be described by the state Ψ_i if we find the particle to still exist (e.g. with probability $p(\Delta t)$), and by the state Ψ_f if it has disintegrated (i.e. with probability $1 - p(\Delta t)$).

We now want to survey the system over a time interval $T = \frac{\pi}{4\epsilon}$, and do this by making N repeated observations with interval $\Delta t = T/N$. What is the probability that we dont observe any particle disintegration in the time interval T? Investigate how this probability depend on N. Consider in particular the behaviour as $N \to \infty$ with T fixed.