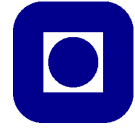


FY3403 Particle physics
Problemset 4 fall 2013

PAGE 1 OF 1

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Problem 1. The symmetry group of a square

In the lectures we studied the symmetry group of an equilateral triangle. In this problem you should repeat this for a square.

- a) How many elements are there in this group?
- b) Find the multiplication table for this group.
- c) Is this group abelian or non-abelian?
- d) Find all subgroups of this group.

Problem 2. Rotation of spin- $\frac{1}{2}$ states

The rotation operator $R(\theta, \hat{n})$ for spin- $\frac{1}{2}$ wave functions (spinors) $\chi = \begin{pmatrix} a \\ b \end{pmatrix}$ is

$$R(\theta, \hat{n}) = e^{i\theta \hat{n} \cdot \boldsymbol{\sigma}/2}, \quad (1)$$

where θ is the angle rotated and \hat{n} is the direction of the rotation axis.

- a) Explain geometrically why we should have $R(-\theta, -\hat{n}) = R(\theta, \hat{n})$.
- b) Show, f.i. by series expansion of the exponential, that

$$R(\theta, \hat{n}) = \cos(\theta/2) + i \sin(\theta/2) \hat{n} \cdot \boldsymbol{\sigma}. \quad (2)$$

- c) For a state with spin along the z -axis the corresponding spinor is $\chi = \chi_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$. Rotate this state an angle $-\theta$ about the y -axis, and show that the rotated spinor becomes $\chi_\theta = \begin{pmatrix} \cos \theta/2 \\ \sin \theta/2 \end{pmatrix}$.
- d) Show that χ_θ is an eigenstate of $\sigma_{\hat{n}} \equiv \hat{n} \cdot \boldsymbol{\sigma}$, where \hat{n} is the vector obtained by rotating the unit vector \hat{e}_z an angle θ about the y -axis. Draw a figure.