FY3403 Particle physics
Problemset 3 fall 2013

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Problem 1. Kinematics of Compton scattering
In an experiment carried out by A.H. Compton in 1923 it was demonstrated that light scattered from a charged particle at rest will change its wavelength according to the formula

$$\lambda' = \lambda + \frac{h}{mc}(1 - \cos\theta) \tag{1}$$

where λ is the wavelength of the incoming light, λ' the wavelength of the scattered light, θ the scattering angle, and m the mass of the particle. This can be viewed as the definite proof of the particle nature of the photon.

a) Consider the incoming light as a massless particle with momentum $p = \frac{h}{\lambda}$ and energy $E_\gamma = pc$ (where h is the Planck constant), and write down the conservation laws for energy an momentum for the scattering process.

b) Solve the conservation laws with respect to the momentum p' of the scattered photon.

c) Show that the result from the previous point is in accordance with equation (1).

Hint: A pedestrian way to attack the problem is[1] to first eliminate $p_y^{(\mathrm{m})}$, then $p_x^{(\mathrm{m})}$ (where $\boldsymbol{p}^{(\mathrm{m})}$ is the momentum of the particle after the scattering process), and finally solve for p' (the absolute value of the photon momentum after the scattering process).

Given: The relativistic connection between energy and momentum for a particle with mass m is

$$E = \sqrt{(pc)^2 + (mc^2)^2}. \tag{2}$$

Problem 2. Unstable particles produced by cosmic rays
Cosmic rays will produce muons, $\mu^\pm$, high in the atmosphere, assume at a height of 8 km. Assume that the muons have a kinetic energy T and a velocity directed towards the center of the earth. What is the probability that a muon will reach the surface of the earth if

a) T = 20 MeV

b) T = 20 GeV

c) Cosmic rays will also produce pions, e.g. $\pi^\pm$, at the same height. Repeat the calculation for such particles

In all these calculations you may neglect interactions between the produced particles and the atmosphere.
Given:
$m_\mu = 105.66$ MeV, $\tau_\mu = 2.197 \times 10^{-6}$ s
$m_{\pi^\pm} = 139.57$ MeV, $\tau_{\pi^\pm} = 2.603 \times 10^{-8}$ s

[1] Assume the incoming photon is moving in the positive x-direction and that the scattering process happens in the xy-plane.

Problem 3. Kinematics of π^0–decay

A neutral pion may decay into two photons

$$\pi^0 \to \gamma_1 + \gamma_2 \tag{3}$$

a) Use conservation of four-momentum to find the energy of the two photons (measured in MeV) when the pion is at rest before the decay.

b) Next assume that the pion has a momentum $p_\pi = 100$ MeV/c when it decays. One of the photons emerge at an angle $\theta_1 = 60^\circ$ relative to $\boldsymbol{p}_\pi$. Find the energy of this photon. Also find the energy and direction of motion of the other photon.

c) Neutral pions with known energy E_π may decay into two photons. In each decay we observe the two photons and measure the angle θ_{12} between them. Show that we may determine the mass m_{π^0} of the pion if we know the energy E_π, and have found the least possible value θ_{min} of the opening angle θ_{12}.