CLASSICAL MECHANICS TFY4345 - Solution Exercise Set 8

(1) *See figure in the Norwegian version of the solution*. The scattering angle Θ satisfies $2\Psi + \Theta = \pi$. From the figure, we see that the impact parameter is given by $s = a\cos\Theta/2$, so that

$$|ds/d\Theta| = (a/2)\sin(\Theta/2). \tag{1}$$

. Therefore, the differential scattering cross section is

$$\sigma(\Theta) = s|ds/d\Theta|/\sin\Theta = a^2/4. \tag{2}$$

The total cross section is obtained by integration over Θ, so that

$$\sigma = 2\pi\int_0^\pi \sigma(\Theta)\sin\Theta d\Theta = \pi a^2. \tag{3}$$

This is physically sensible since it is the actual cross-sectional area of the sphere.

(2) *See figure in the Norwegian version of the solution*. Let us use as a starting point

$$\Theta = \pi - 2\int_0^{u_m} \frac{s\cdot du}{\sqrt{1 - V/E - s^2u^2}} \tag{4}$$

as shown in the lectures. Since $f = k/r^3$ we have $V = ku^2/2$. Above, $u_m = 1/r_m = 1/\sqrt{s^2 + k/(2E)}$ is the inverse minimum distance of approach. Performing the integration with our explicit form of V, one obtains

$$\Theta = \pi - 2su_m[\arcsin(u/u_m)]_0^{u_m} = \pi(1 - su_m) \tag{5}$$

Introduce $x = \Theta/\pi$ and reexpress the above equation as follows:

$$s = s(x,E) = \sqrt{k/(2E)}\frac{1-x}{\sqrt{2x - x^2}} \tag{6}$$

We then have:

$$\sigma(\Theta) = \frac{s}{\sin\Theta}|ds/d\Theta| = \frac{s}{\pi\sin\pi x}|ds/dx| \tag{7}$$

which using the above expression for $s(x,E)$ gives:

$$\sigma(\Theta) = \frac{k}{2\pi E}\frac{1-x}{x^2(2-x)^2\sin\pi x} \tag{8}$$