CLASSICAL MECHANICS TFY4345 - Solution Exercise Set 3

(1a) This problem is treated in detail in the compendium.

(1b) Choose the reference level for potential energy $(V=0)$ at $x=0$. In that case, we have $v^2 = v_0^2 + 2gx$ due to energy conservation. The condition for the Brachistochrone-curve is as before that

$$\int ds/v = \int \frac{\sqrt{1+(y')^2}}{\sqrt{v_0^2+2gx}}dx \tag{1}$$

should have its minimum value. The Euler-Lagrange equations then give us $\partial f/\partial y = 0$ where f is the integrand such that:

$$\frac{d}{dx}\Big[\frac{y'}{\sqrt{v_0^2+2gx}\sqrt{1+(y')^2}}\Big] = 0. \tag{2}$$

The quantity inside the brackets then has to be a constant C. We find thus:

$$(y')^2 = \frac{C^2(v_0^2+2gx)}{1-C^2(v_0^2+2gx)} \tag{3}$$

The initial condition is that $y'(0)=1$, which allows us to determine C namely $C^2 = 1/(2v_0^2)$. Inserting this into the expression for $(y')^2$, we find that:

$$(y')^2 = \frac{h_0+x}{h_0-x} \tag{4}$$

with $h_0 = v_0^2/(2g)$.