CLASSICAL MECHANICS TFY4345 - Exercise 9

(1) A particle with mass m moves in an attractive potential $V(r)$. Show, on the basis of energy conservation, how the problem can be looked upon as a one-dimensional problem with effective potential $\tilde{V}(r)$. What is the condition for the particle to reach the scattering centre, $r = 0$?

(2) A hard sphere has radius a. For $r > a$, the sphere yields a Kepler potential $V = -k/r$, where $k > 0$. Particles coming in from infinity have mass m and original velocity v_0. The part of the particles having impact parameter $s \leq s_{\max}$, will hit the sphere's surface. Find $s_{\max}$ and the corresponding "effective" scattering cross section $\sigma_{\text{eff}} = \pi s_{\max}^2$.

(3) Find Lagrange's and Hamilton's equations in cylindrical coordinates $q_i = (r, \theta, z)$ for a particle with mass m in a potential $V = V(q_i)$.