CLASSICAL MECHANICS TFY4345 - Exercise 7

(1a) A plane pendulum has mass m_2. The rod has length l. The suspension point (mass m_1) can slide without friction along a horizontal line in the x direction. Find the Lagrangian $L = L(\dot{x}, \theta, \dot{\theta})$ for the system.

(1b) Suppose that the CM is at rest in the x-direction. Show that the time $t = t(\theta)$ can be written as:

$$t = l\sqrt{\frac{m_2}{2(m_1+m_2)}}\int \sqrt{\frac{m_1+m_2\sin^2\theta}{E+m_2 gl\cos\theta}}\mathrm{d}\theta \tag{1}$$

where E is the total energy. Choose the potential to be zero at the height of m_1.

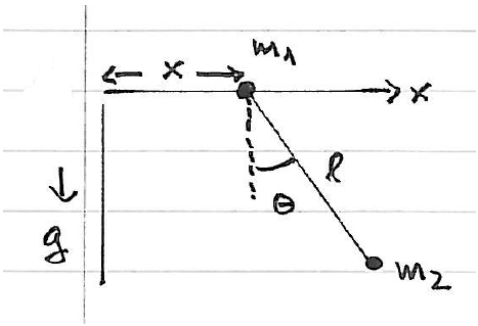

FIG. 1: (Color online). The system under consideration.

(2) A charged particle in an electromagnetic field has the Hamiltonian

$$H = \frac{1}{2m}(\mathbf{p} - q\mathbf{A})^2 + q\varphi \tag{2}$$

Find Hamilton's equations. Use these to find the expression for the Lorentz force

$$\mathbf{F} = q\mathbf{E} + q\mathbf{v}\times\mathbf{B} \tag{3}$$

Use, for instance, the Levi-Civita symbol to rewrite $[\mathbf{v}\times(\nabla\times\mathbf{A})]_i$.