CLASSICAL MECHANICS TFY4345 - Exercise 4

(1a) Two particles 1 and 2, each with mass m, are connected with a massless line of constant length s. The line can move without friction through a small opening in the horiztonal table ABCD. Particle 2 can move vertically, whereas particle 1 moves horizontally (without friction) on the table. The movement of particle 1 is in general composed of a radial and azimuthal part. Let the potential energy be zero in the plane of the table. Write down the Lagrangian of the system.

(1b) Show that the z-component of the angular momentum, $l_z = l$, is constant. Moreover, show that

$$2m\ddot{r} - l^2/(mr^3) + mg = 0 \qquad (1)$$

One solution of this equation is that particle 1 is in circular motion, $r = r_0$. Find r_0.

(1c) Assume that the circular orbit $r = r_0$ is exposed to a small radial perturbation, $r \to r_0 + x$ where $x = x(t)$. Find the equation of motion for x. Include only first order terms in x/r_0. If the intitial conditions are $x(t=0) = x_0$, $\dot{x}(t=0) = 0$, the solution will have the form $x = x_0 \cos \omega t$. Find the angular frequency ω.

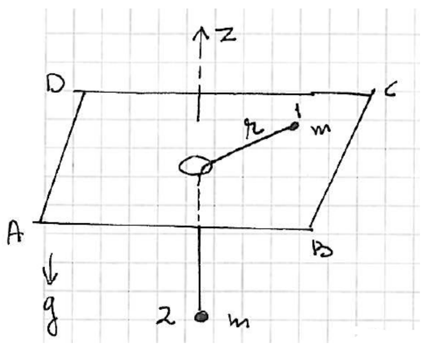

FIG. 1: (Color online). The system under consideration.

(2) A particle with mass m and constant velocity $\mathbf{v_0}$ is incident from a region $x < 0$ with zero potential, $V = 0$. At the position $x = 0$ there is a potential jump, such that for $x > 0$ the potential is constant and equal to $V = V_0$. Assume that $V_0 < E$ where E is the energy of the particle. Comment on whether energy and momentum are conserved or not. Assume that the particle is incident from $x < 0$ at an angle α with respect to the normal of the interface region and that it propagates with an angle β inside the region $x > 0$. Show that one may write

$$\sin\alpha / \sin\beta = f(E, V_0) \qquad (2)$$

and identify the function $f(E, V_0)$.

(3) The particle m_2 moves vertically without friction on a rod coincident with the z-axis. There are four massless arms, each

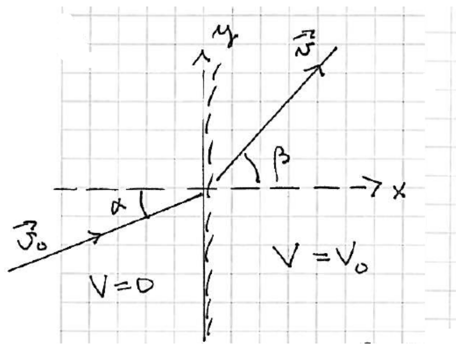

FIG. 2: (Color online). The system under consideration.

of length a. The suspension point A is fixed. The whole system rotates around the z-axis with a constant angular frequency Ω. Find the Lagrangian of the system. Explain qualitatively how $\theta = \theta(t)$ evolves with time under the assumption that the initial condition is $\theta(t=0) = 0$. Also write down Lagrange's equations and determine for which angle $\theta = \theta_0$ the system is at equilibrium. Comment on how you arrived at this result and show that the same result for θ_0 can be obtained by balancing the forces acting on the system. Finally, consider the problem as equivalent to a 1D problem with θ as the variable and with effective potential

$$\tilde{V}(\theta) = -ma^2(\Omega^2 \sin^2\theta + 2\omega_0^2 \cos\theta) \qquad (3)$$

under the assumption that $m_1 = m_2 = m$. We have introduced $\omega_0^2 = 2g/a$. Assume now that $\Omega > \omega_0$ and show how $\tilde{V}$ has a minimum when $\theta = \theta_0$.

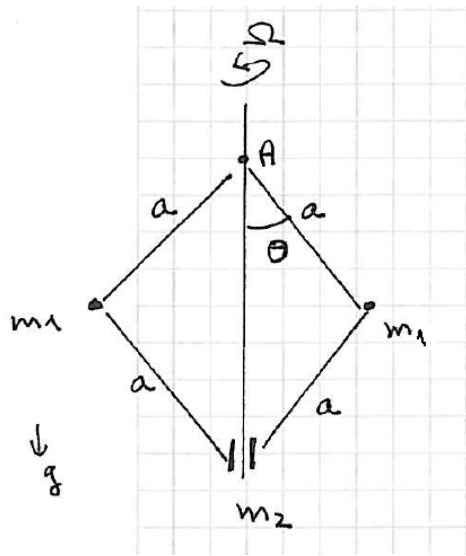

FIG. 3: (Color online). The system under consideration.