Suggested solutions TEP4170 Heat and combustion technology 10 August 2019

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1)
Homogeneous turbulence: Mean turbulence field is translatorically invariant. That is, no change when you move the coordinate system in any direction; no spatial gradients of turbulence mean quantities.

Isotropic turbulence: Mean turbulence fields is rotationally invariant. That is, no change when you rotate the coordinate system in any direction.

In isotropic turbulence, normal stresses are equal, and shear stresses are zero.

The equations of the k-ε model is reduced to change (storage) and dissipation/destruction:

$$\frac{dk}{dt} = -\varepsilon \quad \text{and} \quad \frac{d\varepsilon}{dt} = -C_{\varepsilon 2}\frac{\varepsilon^2}{k}$$

2)
A possible procedure is first to divide one equation by the other, obtaining $\dfrac{dk}{k} = \dfrac{1}{C_{\varepsilon 2}}\dfrac{d\varepsilon}{\varepsilon}$

Solving this gives $\dfrac{k}{\varepsilon^{1/C_{\varepsilon 2}}} = \text{constant} = \dfrac{k_0}{\varepsilon_0^{1/C_{\varepsilon 2}}}$, where subscript 0 denote initial values (t=0).

Introducing this into the first equation gives, $\dfrac{dk}{dt} = -\varepsilon_0\dfrac{k^{C_{\varepsilon 2}}}{k_0^{C_{\varepsilon 2}}}$, which can be solved as

$$k = k_0\left[1 + (C_{\varepsilon 2}-1)\frac{\varepsilon_0}{k_0}t\right]^{\frac{-1}{C_{\varepsilon 2}-1}} .$$

Next, this is used in the 2nd equation, which is solved to $\varepsilon = \varepsilon_0\left[1 + (C_{\varepsilon 2}-1)\dfrac{\varepsilon_0}{k_0}t\right]^{\frac{-C_{\varepsilon 2}}{C_{\varepsilon 2}-1}} .$

(For more details, see the solution to the exercise (Exercise 5 this year.))

The given experimental results means that $1,25 = 1/(C_{\varepsilon 2}-1)$ or $C_{\varepsilon 2} = 1/1,25 + 1 = 1,80$

3)
The energy spectrum tensor is defined as the Fourier transform of the two-point correlation of the turbulent velocity field. The trace of this tensor is the 3-dimensional energy spectrum.
Sketch: see textbook.

The relation: $k = \int_0^\infty E(\kappa)d\kappa$; that is, the area under the curve.

Decaying turbulence means that the area is shrinking (less turbulence energy, k), that the length scales of "energy containg" eddies are increased (lower wave number), and that the ratio of small to large scales is reduced (width of the spectrum reduced). Thus the curve will move towards lower wavenumbers and become smaller.
(see sketches in textbook).

4)
Given: $\bar{u} = 10$ m/s and $d\bar{u}/dy = 5$ s^{-1}

Estimates can be: $L \approx \bar{u} / (d\bar{u}/dy) = (10 \text{ m/s}) / (5 \text{ s}^{-1}) = 2 \text{ m}$, $u' \approx 0,03\bar{u} = 0,3 \text{ m/s}$,

$k \approx u'^2 \approx 0,1 \text{ m}^2/\text{s}^2$, $v_t \approx u'L \approx (0,3 \text{ m/s})(2 \text{ m}) = 0,6 \text{ m}^2/\text{s}$,

$\varepsilon \approx u'^3 / L \approx (0,3 \text{ m/s})^3 / (2 \text{ m}) \approx 0,01 \text{ m}^2/\text{s}^3$, $\alpha_t \approx v_t \approx 0.6 \text{ m}^2/\text{s}$,

$\text{Re}_t = \dfrac{k^2}{v\varepsilon} \approx \dfrac{u'L}{v} \approx \dfrac{0,6}{10^{-5}} \approx 6 \cdot 10^4$; $\tau = (v/\varepsilon)^{1/2} \approx (10^{-5}/0,01)^{1/2} \text{ s} \approx 3 \cdot 10^{-4} \text{ s}$

5)
See sketch in the textbook (Turns, Ch.8)
Can assume the simple reaction 1 kg F $+ v$ kg Ox $\rightarrow (1+v)$ kg Pr
(however, distinguishing O_2, N_2, CO_2, H_2O is not wrong).

6)
(Following the notation of Turns)

Continuity: $\dfrac{d(\rho v_x)}{dx} = 0$ or $\rho v_x = \text{ constant} = \dot{m}''$

Momentum: not used/needed, as mass flow rate is specified and the pressure is constant.

Species mass: $\dfrac{d}{dx}\left(\rho v_x Y_i\right) = \dfrac{d}{dx}\left(\rho D \dfrac{dY_i}{dx}\right) + \dot{m}_i'''$ for $i = \text{F, Ox, Pr}$

For the one-step reaction $\dot{m}_{Ox}''' = v\dot{m}_F'''$ and $\dot{m}_{Pr}''' = -(v+1)\dot{m}_F'''$

Energy: $\rho v_x C_p \dfrac{dT}{dx} = \dfrac{d}{dx}\left(\rho D C_p \dfrac{dT}{dx}\right) - \dot{m}_F''' \Delta h_c$

Boundary conditions,
far upstream, $x \rightarrow -\infty : T = T_u$; $\dfrac{dT}{dx} = 0$; far downstream, $x \rightarrow \infty : T = T_b$; $\dfrac{dT}{dx} = 0$

With assumptions on complete combustion, thermal diffusivity equal to mass diffusivity, etc, the species mass equations need not be solved (mass fractions follow from solution of the energy/temperature equation).

7)
$\dfrac{d[CO]}{dt} = -k_1[CO][O_2] - k_3[CO][OH]$

$\dfrac{d[OH]}{dt} = 2k_2[O][H_2O] - k_3[CO][OH] + k_4[H][O_2]$

Reaction (1) is chain initiating (producing one radical from stable reactants); Reactions (2) and (4) are chain branching (producing two radicals, while consuming one radical).
(Not asked about: Reaction 3 is chain propagating, producing one radical and consuming one.)

Starting from a mixture of stable species (CO, O_2, H_2O), Reaction (1) –when it occurs – produces one radical (O), which react in Reaction (2) to produce two radicals (OH), which in Reaction (3) produces one radical (H) each, and consume one CO each; and then one O each in Reaction (4). These O go into

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Reaction 2, and so on. As soon as Reaction 1 has occurred, the remaining reactions can go on and convert more and more CO in Reaction 3.

9)
The lower/upper flammability limits are the leanest/richest mixture of a fuel and an oxidizer that will allow steady flame propagation.
The oxidizer is usually atmospheric air, but can be specified as some other (O2-containing) mixture.
Basically, the temperature and pressure should be specified (usually some standard).
The limits can be specified in terms of equivalence ratio or air-excess ratio or in terms of fuel content (molar fraction/percentage) of the mixture.

Propane reacting in air can be described as $C_3H_8 + \lambda \cdot 5(O_2 + 3,76N_2) \rightarrow$ products ,

where λ is the excess air ratio. Thus, the mole fraction of propane in the mixture is
$X_{C_3H_8} = 1/(1 + \lambda \cdot 5(1 + 3,76)) = 1/(1 + \lambda \cdot 23,8)$, which can be reformulated to
$\lambda = (1/X_{C_3H_8} - 1)/23,8$. The equivalence ratio is the reciprocal of this, $\phi = \lambda^{-1}$.
The lower limit of 2,1% corresponds to $\lambda = 1,96$, while the upper limit of 10,1% corresponds to $\lambda = 0,37$

The mass of propane ("p" for simplicity) required to reach the lower limit will be

$$m_p = M_p n_p = M_p X_p n = M_p X_p \frac{pV}{R_u T} = 44 \frac{kg}{kmol} 0,021 \frac{100 \text{ kPa} \cdot 60 \text{ m}^3}{8,314 \text{ (kJ/(kmol} \cdot \text{K))} \cdot 295,15 \text{ K}} = \underline{2,26 \text{ kg}}$$

10)
The overall (global) reaction balance can be written as:
$$C_{12}H_{24} + 18\lambda(O_2 + 3,76N_2) \rightarrow 12CO_2 + 12H_2O + (\lambda\text{-}1)18O_2 + \lambda 18 \cdot 3,76N_2$$
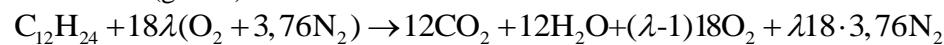
The air-to-fuel ratio can be expressed as $\frac{n_{air}}{n_{fuel}} = \frac{m_{air}}{m_{fuel}} \frac{M_{fuel}}{M_{air}} = 21 \cdot \frac{168}{28,9} = 122,1$,

and also as $\frac{n_{air}}{n_{fuel}} = 18\lambda(1 + 3,76) = 85,7\lambda$; thus $\lambda = 1,42$

The amount of flue gas ("wet", i.e. with H2O included in the mixture when other species are specified) is

$\frac{n_{fg}}{n_{fuel}} = 12 + 12 + (1,42 - 1) \cdot 18 + 1,42 \cdot 18 \cdot 3,76 = 127,7$,

The mole fraction of HC in the flue gas $X_{HC} = \frac{n_{HC}}{n_{fg}} = 120 \text{ ppm} = 120 \cdot 10^{-6}$

The molar mass of HC (same H-C-ratio as the fuel): $M_{HC} = M_{C_{12}H_{24}}/12$

Now, we can express the emission index:

$$EI_{HC} = \frac{m_{HC,emitted}}{m_{fu,burned}} = \frac{n_{HC,emitted}M_{HC}}{n_{fu,burned}M_{fu}} = \frac{n_{HC,emitted}}{n_{fg}} \cdot \frac{n_{fg}}{n_{fu,burned}} \cdot \frac{M_{HC}}{M_{fu}} = 120 \cdot 10^{-6} \cdot 127,7 \cdot \frac{1}{12} \frac{kg}{kg} = \underline{1,28 \cdot 10^{-3} \frac{kg}{kg}}$$

The mass-specific emission
$$(MSE)_{HC} = \frac{\dot{m}_{HC,emitted}}{\dot{W}} = \dot{m}_{fu} EI_{HC} \frac{1}{\dot{W}} = 4,9 \cdot 10^{-3} \frac{kg}{s} \cdot 1,28 \cdot 10^{-3} \frac{kg}{kg} \frac{3600 \text{ s/h}}{80 \text{ kW}} = \underline{2,8 \cdot 10^{-4} \frac{kg}{kWh}}$$