ISE jan.16

Department of Energy and Process Engineering
TEP4170 Heat and combustion technology

Exercise 1: Introduction and repetition

Tensor notation

Problem 1:

Write in Cartesian tensor notation:

a)

$$\rho\frac{\partial u}{\partial x}+u\frac{\partial \rho}{\partial x}+\rho\frac{\partial v}{\partial y}+v\frac{\partial \rho}{\partial y}+\rho\frac{\partial w}{\partial z}+w\frac{\partial \rho}{\partial z}=0$$

b)

$$\varepsilon_{ij}=\begin{cases}\frac{2}{3}\varepsilon & \text{when } i=j\\ 0 & \text{otherwise}\end{cases}$$

c)

$$\begin{aligned}\Phi_{11}&=-C_2(P_{11}-\tfrac{2}{3}P) \qquad & \Phi_{12}&=-C_2P_{12}\\ \Phi_{22}&=-C_2(P_{22}-\tfrac{2}{3}P) & \Phi_{23}&=-C_2P_{23}\\ \Phi_{33}&=-C_2(P_{33}-\tfrac{2}{3}P) & \Phi_{13}&=-C_2P_{13}\end{aligned}$$

where $P=\frac{1}{2}(P_{11}+P_{22}+P_{33})$

Problem 2:

Write in normal notation:

a) The momentum equation

$$\frac{\partial}{\partial t}(\rho u_i)+\frac{\partial}{\partial x_j}(\rho u_i u_j)=-\frac{\partial p}{\partial x_i}+\frac{\partial \tau_{ij}}{\partial x_j}+\rho f_i$$

with the stress tensor

$$\tau_{ij}=\mu\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)+(\mu_B-\tfrac{2}{3}\mu)\frac{\partial u_k}{\partial x_k}\delta_{ij}$$

b) The dissipation function

$$\Phi=\tau_{ij}\frac{\partial u_i}{\partial x_j}$$

Problem 3:

Show that

$$C_\varphi=\frac{\partial}{\partial t}(\rho\varphi)+\frac{\partial}{\partial x_j}(\rho u_j\varphi)$$

also can be formulated as

$$C_\varphi=\rho\frac{\partial \varphi}{\partial t}+\rho u_j\frac{\partial \varphi}{\partial x_j}$$

Are there assumptions that have to be made in doing this?

Problem 4:

Why do we use tensor notation?

Basic equations

Problem 5:

In Thermodynamics you have learned that the 1st law can be formulated mathematically as

$$\dot{m}_{\mathrm{in}}\left(h+\tfrac{1}{2}u^2+gz\right)_{\mathrm{in}}-\dot{m}_{\mathrm{out}}\left(h+\tfrac{1}{2}u^2+gz\right)_{\mathrm{out}}+\dot{Q}-\dot{W}=\frac{d}{dt}\left(m\cdot(e+e_{\mathrm{kin}}+e_{\mathrm{pot}})\right)_{\mathrm{cv}} \qquad (1)$$

The notation is as in the textbook: u is velocity, e is internal energy.

In the textbook this equation is written on differential form, Eq. (A.42) p. 207:

$$\frac{\partial}{\partial t}(\rho e_{\mathrm{t}})+\frac{\partial}{\partial x_j}(\rho e_{\mathrm{t}}u_j)=-\frac{\partial q_j}{\partial x_j}+\dot{Q}-\frac{\partial}{\partial x_j}(pu_j)+\frac{\partial}{\partial x_j}(\tau_{ij}u_i)+\rho\sum_k Y_k f_{k,i}(u_i+V_{k,i}), \qquad (2)$$

where $e_{\mathrm{t}}=e+\frac{1}{2}u_iu_i$ is (specific) internal energy and kinetic energy.

– What is the relation between the two equations? Which terms correspond to each other?

Notice: The terms with gravitational acceleration are not straightforward to reformulate. Do not spend much time on that part of the problem.

Problem 6:

Do as in Problem 5 with the mass balance (continuity equation) and the mass balance for species k.

Hint for Problems 5-6: the solutions are found in books on fluid mechanics and heat and mass transfer.

Thermodynamics and thermochemistry

Problems from Turns: “An introduction to combustion”:

2.2, 2.4, 2.8, 2.9, 2.11, 2.20

11. Describe the effect of increasing pressure on the equilibrium composition of combustion products.

12. Why does flue-gas recirculation decrease flame temperatures? What happens if the flue gas recirculated is at the flame temperature?

for Exercise 1

Turns: "An introduction to Combustion", 3rd ed, Chpt. 2

PROBLEMS

2.1 Determine the mass fraction of O_2 and N_2 in air, assuming the molar composition is 21 percent O_2 and 79 percent N_2.

2.2 A mixture is composed of the following number of moles of various species:

Species	*No. of moles*
CO	0.095
CO_2	6
H_2O	7
N_2	34
NO	0.005

A. Determine the mole fraction of nitric oxide (NO) in the mixture. Also, express your result as mole percent, and as parts-per-million.

B. Determine the molecular weight of the mixture.

C. Determine the mass fraction of each constituent.

2.3 Consider a gaseous mixture consisting of 5 kmol of H_2 and 3 kmol of O_2. Determine the H_2 and O_2 mole fractions, the molecular weight of the mixture, and the H_2 and O_2 mass fractions.

2.4 Consider a binary mixture of oxygen and methane. The methane mole fraction is 0.2. The mixture is at 300 K and 100 kPa. Determine the methane mass fraction in the mixture and the methane molar concentration in kmol of methane per m^3 of mixture.

2.5 Consider a mixture of N_2 and Ar in which there are three times as many moles of N_2 as there are moles of Ar. Determine the mole fractions of N_2 and Ar, the molecular weight of the mixture, the mass fractions of N_2 and Ar, and the molar concentration of N_2 in kmol/m^3 for a temperature of 500 K and a pressure of 250 kPa.

2.6 Determine the standardized enthalpy in J/$kmol_{mix}$ of a mixture of CO_2 and O_2 where $\chi_{CO_2} = 0.10$ and $\chi_{O_2} = 0.90$ at a temperature of 400 K.

2.7 Determine the molecular weight of a stoichiometric ($\Phi = 1.0$) methane–air mixture.

2.8 Determine the stoichiometric air–fuel ratio (mass) for propane (C_3H_8).

2.9 Propane burns in a premixed flame at an air–fuel ratio (mass) of 18:1. Determine the equivalence ratio Φ.

2.10 For an equivalence ratio of $\Phi = 0.6$, determine the associated air–fuel ratios (mass) for methane, propane, and decane ($C_{10}H_{22}$).

2.11 In a propane-fueled truck, 3 percent (by volume) oxygen is measured in the exhaust stream of the running engine. Assuming "complete" combustion without dissociation, determine the air–fuel ratio (mass) supplied to the engine.

2.12 Assuming "complete" combustion, write out a stoichiometric balance equation, like Eqn. 2.30, for 1 mol of an arbitrary alcohol $C_xH_yO_z$. Determine the number of moles of air required to burn 1 mol of fuel.

2.13 Using the results of problem 2.12, determine the stoichiometric air–fuel ratio (mass) for methanol (CH_3OH). Compare your result with the stoichiometric ratio for methane (CH_4). What implications does this comparison have?

2.14 Consider a stoichiometric mixture of isooctane and air. Calculate the enthalpy of the mixture at the standard-state temperature (298.15 K) on a per-kmol-of-fuel basis (kJ/$kmol_{fuel}$), on a per-kmol-of-mixture basis (kJ/$kmol_{mix}$), and on a per-mass-of-mixture basis (kJ/kg_{mix}).

2.15 Repeat problem 2.14 for a temperature of 500 K.

2.16 Repeat problem 2.15, but now let the equivalence ratio $\Phi = 0.7$. How do these results compare with those of problem 2.15?

2.17 Consider a fuel which is an equimolar mixture of propane (C_3H_8) and natural gas (CH_4). Write out the complete stoichiometric combustion reaction for this fuel burning with air and determine the stoichiometric fuel–air ratio on a molar basis. Also, determine the molar air–fuel ratio for combustion at an equivalence ratio, Φ, of 0.8.

2.18 Determine the enthalpy of the products of "ideal" combustion, i.e., no dissociation, resulting from the combustion of an isooctane–air mixture for an equivalence ratio of 0.7. The products are at 1000 K and 1 atm. Express your result using the following three bases: per kmol-of-fuel, per kg-of-fuel, and per kg-of-mixture. *Hint:* You may find Eqns. 2.68 and 2.69 useful; however, you should be able to derive these from atom-conservation considerations.

2.19 Butane (C_4H_{10}) burns with air at an equivalence ratio of 0.75. Determine the number of **moles** of air required per mole of fuel.

2.20 A glass melting furnace is burning ethene (C_2H_4) in pure oxygen (not air). The furnace operates at an equivalence ratio of 0.9 and consumes 30 kmol/hr of ethene.

A. Determine the energy input rate based on the LHV of the fuel. Express your result in both kW and Btu/hr.

B. Determine the O_2 consumption rate in kmol/hr and kg/s.