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Interactions between acoustic waves and turbulent combustion with the Eddy Dissipation Concept

Carmine L. Iandoli [1], Ivar S. Ertesvåg [2], and Enrico Sciubba [3]

Department of Energy and Process Engineering
Norwegian Institute of Science and Technology,
N-7491 Trondheim, Norway

Introduction

Lean premixed, prevapourised technology has tremendous potential to reduce NO_x emissions, but is proving highly susceptible to self-excited oscillations. The interaction between sound and combustion can lead to self-excited oscillations of such large amplitudes that structural damage is done. The present work is aimed at the study of the physical and numerical behaviour of pressure waves observed in combustion instabilities. The acoustic interaction between heat release and acoustic modes happens in fact to be at the core of the growth of combustion instabilities.
Unstable heat release affects the flow by suddenly changing fluid temperature and density. These changes give rise to pressure waves travelling through the field. Since the effect of the heat release in premixed flames can be approximated as a sequel of pulses, it will induce waves with different wave lengths (spanning, in the ideal case, the whole spectrum) flowing at the same speed. In numerical solutions, the wave speed (close to the speed of sound for small Mach number) will, unlike the physical waves, not be independent on the frequency [13]. The effects of pressure waves in turbulence models and in Magnussen's Eddy Dissipation Concept (EDC) [7-8,4,2] was investigated in order to observe whether the models were qualitatively and quantitatively sensible to such effects.

The effect of the phase speed

The differences between the numerical solution and the real behaviour of an unsteady velocity field can be studied by performing a Fourier analysis of the linearised viscous advection equation [13]

$$\frac{\partial U}{\partial t} + c\frac{\partial U}{\partial x} - \nu\frac{\partial^2 U}{\partial x^2} = 0$$

This linear equation does not completely represent the problem at study, but it can be analysed to gather information about the numerical speed of sound in such a simpler case. Some insight will hopefully be gained for the non-linear case, and numerical calculations performed with FLUENT™ will show which discretization method must be used in order to satisfactorily improve accuracy.

[1] Carmine.L.Iandoli@mtf.ntnu.no
[2] Ivar.S.Ertesvåg@mtf.ntnu.no
[3] University of Roma 1 "La Sapienza"

It turns out that even for a conservative scheme the numerical speed of sound will differ from the physical one: in particular, for 3 point central-finite differences, the discretised linear advection equation becomes:

$$\frac{du_n}{dt} = -c\left(\frac{u_{n+1} - u_{n-1}}{2\Delta x}\right)$$

in this case, the ratio between the numerical (phase velocity) and physical speed is:

$$\frac{c^*}{c} = \frac{\sin(\lambda\Delta x)}{\lambda\Delta x}$$

It is clear that the proper mesh length Δx must be very small to obtain a good accuracy. It is also clear that the numerical speed of the waves for a given mesh depends on their frequency. By adopting a different discretization method, e.g. a 2-point semi discretisation, the phase velocity is found to be [13]:

$$c^*(\lambda) = c\left(\frac{\tan(\lambda\Delta x/2)}{\lambda\Delta x/2}\right)$$

It is again apparent that the numerical speed of wave propagation depends on the frequency, and that a fine mesh can improve the accuracy and reduce the frequency dependency. It is difficult to predict how fine a mesh needs to be, because this analysis is based on a simplified form of the equation and cannot be extended to a case with non-linear regions. Hence, specific mesh sensitivity tests have to be performed.

Simulation and tests

To test the wave-speed dependency in FLUENT™, a simple 2D channel has been simulated. A rectangular field $0.5\times0.05m$ has been discretised with a mesh consisting of 500×50 square volumes. The fluid is assumed to follow the ideal-gas law, and the turbulence is described by a second order Reynolds-stress-equation (RSE) model [12].
The inlet boundary condition is a sinusoidally varying in time total pressure. Various frequencies and amplitudes have been tested to assess their influence on the wave speed in the range observed in real combustor (*100 - 10000 Hz*). The outlet pressure is fixed.
The time integration is performed by a second order implicit algorithm, with a second order upwind stabilization method to discretise the space coordinate. The time step adopted is $2.5\cdot10^{-6}$ *s* and corresponds to a Courant number of *0.1*.
The speed of the waves is slightly dependent on their frequency and it is actually increasing with it. As expected the numerical velocity is close to the local speed of sound (330 m/s).

Frequency (Hz)	c^* (m/s)	c^*/c
100	360	1.091
1000	370	1.121
2000	380	1.151
4000	384	1.163
10000	390	1.181

Table 1

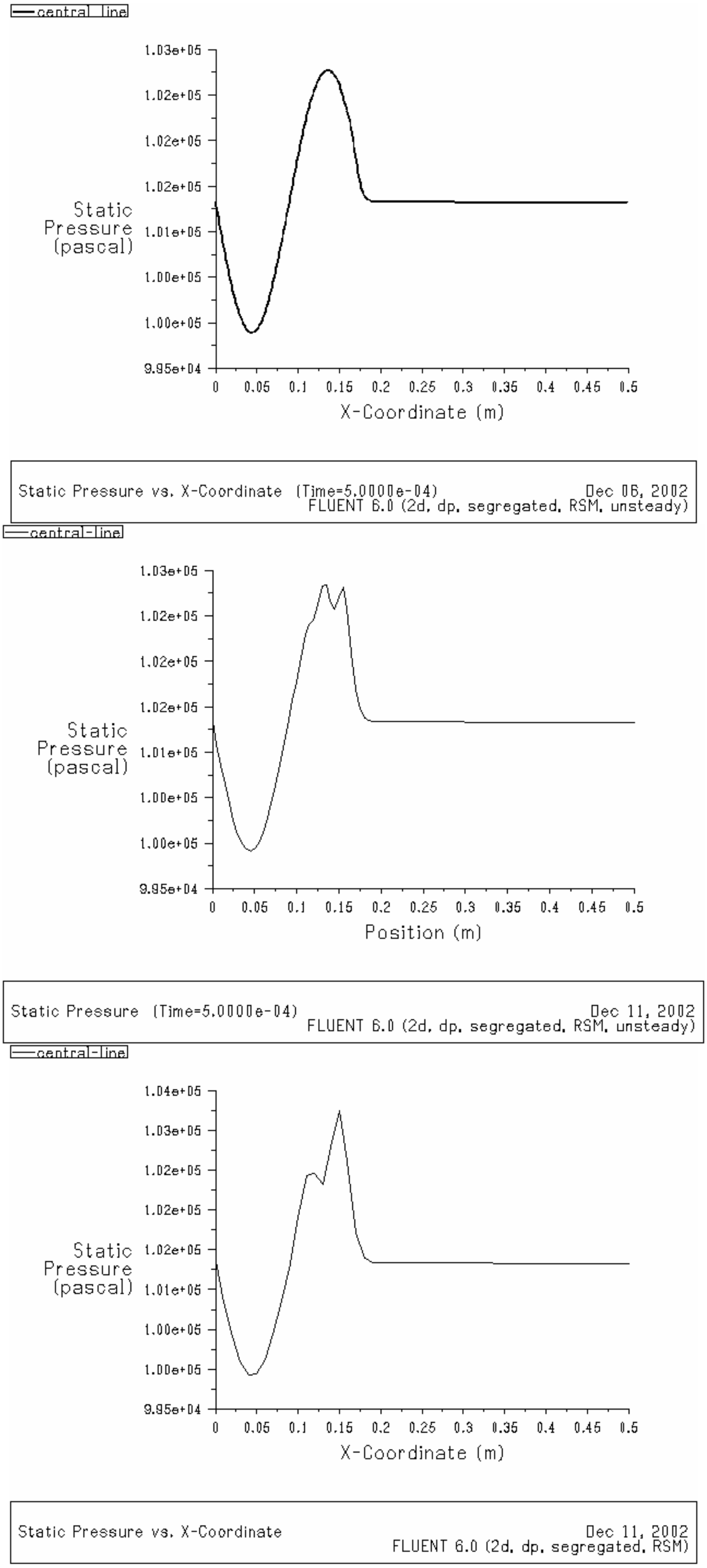


Figure 1: Effect of aliasing on pressure waves.

Effects of the spatial discretisation: aliasing-affected solutions.

The spatial discretisation affects the wave behaviour in different ways. An interesting wave-related problem is observed when the mesh refinement in the direction of waves propagation is not fine enough to describe their behaviour. This is obvious if one attempts to place on a mesh (as boundary or initial condition) a waveform with a spatial frequency content that cannot be resolved by the grid ($\lambda<2\Delta x$).

But such a wave may also be generated by the numerical procedure (typically via a product, such as *u Grad(T)*, of two short but resolvable waves), and the grid will misinterpret the too short wave as a longer wave that it can resolve. In this case aliasing is observed. If aliasing occurs and if the resulting time-integration becomes unstable (with a stable time-marching scheme, or reducing the time step), then the result is called aliasing instability [5]. The instabilities due to aliasing can be linear if they are due to a product such *u Grad(T)*, or non-linear if they are generated by a product such *u Grad(u)*.

An example of aliasing is provided in Figure 1. The pictures refer to the same problem studied before but with three different meshes respectively 500×50, 100×50 and 50×50, corresponding to elements in the x-direction with length *1mm*, *5mm* and *10mm*. The frequency of the incoming pressure wave is *2000 Hz*.

By reducing the number of elements the effect of the aliasing is to reallocate the energy of the wave introduced as boundary condition at a different frequency. Thereof the sine wave looses its initial shape and looks scattered. The effect is more pronounced for longer elements.

The literature on aliasing in CFD is rather sparse, but it is accepted that FDM are affected by aliasing [9,11], while Galerkin FEM are not. In general, schemes without quadratic semi conservation properties can be affected by aliasing. The speed of the waves depends on the spatial discretisation and increases by solving on coarser grids.

The effect of pressure waves on turbulence and in the EDC.

The EDC concept is a widely used approach used to link the dynamics of the small scales where combustion occurs with the mean features of the flow. It provides a connection based on the turbulence parameters of the flow. Tests were performed to clarify if and how acoustic disturbances affect the model. It turned out that the turbulence parameters are not strongly influenced by pressure waves. The indicator used to perform such tests was the mass fraction of fine structure. The concept defines for each location the a mass fraction of fines structures. Here, the species are molecularly mixed and thus able to burn, provided that an adequate temperature is reached. It is defined [2,7] as:

$$\gamma^* = 9.8\left(\frac{\upsilon\varepsilon}{k^2}\right)^{3/4}$$

Pressure waves were introduced as boundary conditions, and the behaviour of pressure, k and ε was analysed. Results are plotted (Figure 2) along a line at mid-channel, in the direction of the mean flow.

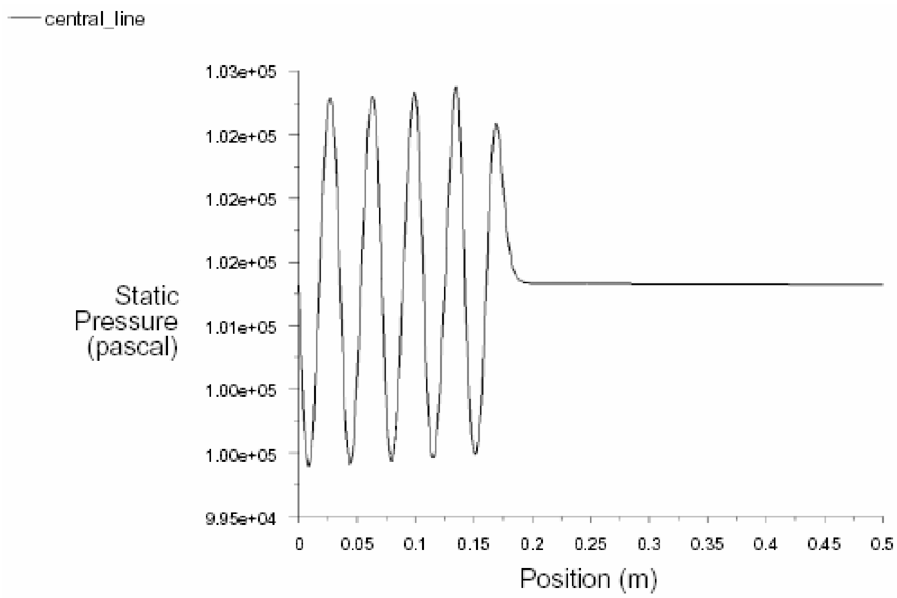
The pressure shows the expected behaviour, oscillation with a frequency of *10000 Hz* imposed as inlet boundary condition: the wave travels at the local sound speed. As expected, the effect on the mass fraction was approximately negligible, proving that the EDC concept did not feel the effect of the acoustic disturbances. The reason is that k and ε themselves are only weakly affected by acoustic disturbances. In this respect it is useful to refer to the k-equation, which is the trace of the

Reynolds-stress equations (i.e. the sum of the normal-Reynolds-stress equations). The mass-weighted Reynolds-stress equations can be written

$$\frac{\partial}{\partial t}\left(\overline{\rho}\overline{u_i''u_j''}\right)+\frac{\partial}{\partial x_k}\left(\overline{\rho}\overline{u}_k\overline{u_i''u_j''}\right)=D_{T,ij}+D_{L,ij}$$

$$-\overline{\rho}\left(\overline{u_i''u_k''}\frac{\partial\overline{u}_j}{\partial x_k}+\overline{u_j''u_j''}\frac{\partial\overline{u}_i}{\partial x_k}\right)+\overline{p'\left(\frac{\partial u_i''}{\partial x_j}+\frac{\partial u_j''}{\partial x_i}\right)}-\overline{u_j''}\frac{\partial\overline{p}}{\partial x_i}-\overline{u_i''}\frac{\partial\overline{p}}{\partial x_j}-\varepsilon_{ij}$$

Pressure oscillations have two main effects on the production of turbulent kinetic energy. The first one is the influence on velocity gradients, which modifies the production term (3rd term on the right-hand side). This term does not need any modelling. The effect of the pressure variation goes through density variation, which change the dilatation of the mean flow ($\partial\overline{u}_i/\partial x_i$). The effect also goes through the pressure-strain correlation (fourth rhs term), which requires modelling. In RSE



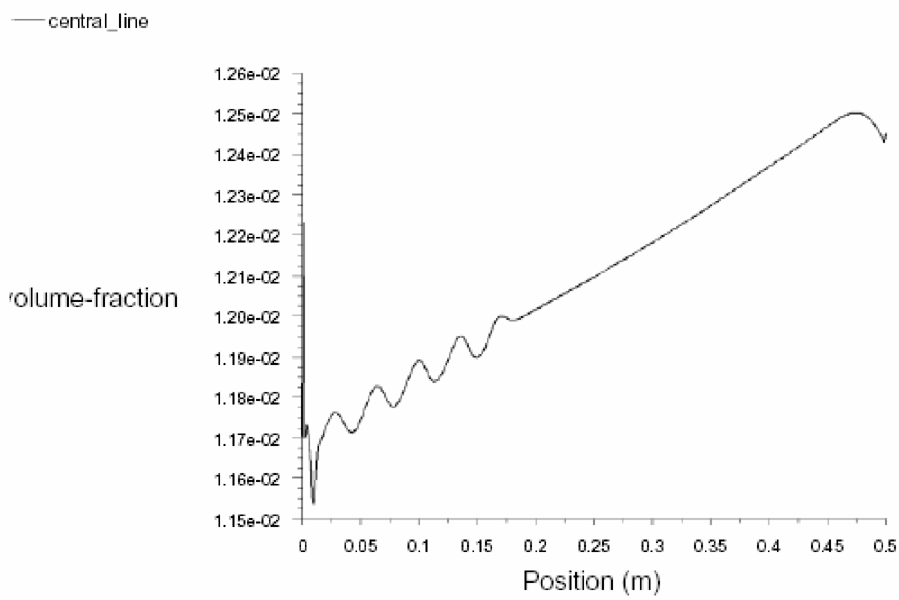


Figure 2: Pressure and EDC fine-structure mass fraction.

models from early 1980s and before (e.g. [3]), the model of the pressure-strain correlation is often purely redistributive and the trace is zero (i.e. zero term in the *k*-equation). This is also correct when the density is constant. Finally, the effect of the two last pressure terms, actually the mean of the mass-weighted velocity fluctuation, $\overline{u_i''}$, remains to be modelled.

When the density fluctuates or varies – and consequently, when pressure fluctuates or oscillates – the pressure-strain correlation in the *k*-equation should be non-zero. Thus, if the RSE model include a purely (or mainly) redistributive model of the pressure-strain correlation, no (or weak) effects from the pressure waves can be expected on *k* and *ε*, and consequently on the quantities of the EDC. Newer and more advanced pressure-strain models include some effects of varying density. In this study the, a quadratic pressure-strain model by Speziale et al. [12] was used. This model has been demonstrated to give superior performance in a range of basic shear flows, including plane strain and axisymmetric expansion and contraction [12]. However, the effect of varying density is modelled through the mean-flow dilatation and proportional to the similar terms in the production term. Hence, the effect of the pressure waves on the production of turbulent kinetic energy is small (Figure 6).

Thermal shock in a duct flow

The first part of this work was intended to show techniques and related limits that arises in the treatment of pressure waves with finite-volume methods. Nevertheless it has provided quantitative measures of the effect of acoustic disturbances on the combustion model. In real combustors the mechanisms at the core of the amplification phenomenon is the heat release unsteadiness, which provides the energy to the system. This leads to self-excited acoustic modes growing until they reach a non-linear limit loop.
The way how combustion unsteadiness affects the acoustic modes is part of the closed loop outlined above. Since the combustion in gas turbine is essentially turbulent, changes in the mixing rate will modify the turbulent velocity of the flame (S_T) leading to unsteady heat release within the flow field. The local temperature suddenly increases as the flame front is moving and hot spots induce pressure and density variations. The effect of a hot spot has been simulated in order to understand how the pressure field is influenced.

Heat release as acoustic source

Unsteady heat release is known to be a source of sound, and a simple model has been derived for plane waves.
Provided that the reactants and products behave as perfect gases and there is no change in the number of molecules during the chemical reaction, by combining the energy equation with the linearised momentum equation one obtains the inhomogeneous wave equation, the term on the RHS describing how unsteady heat generates pressure waves.

$$\frac{1}{c^2}\frac{\partial^2 p'}{\partial t^2}-\frac{\partial^2 p'}{\partial x^2}=\frac{(\gamma-1)}{c^2}\frac{\partial q}{\partial t}$$

The effect of the mean flow has not been modelled, however its *Mach* number is normally small and it is convenient to neglect it.
The mean flow has two main consequences: it affects the speed of the acoustic waves, which then travel downstream with speed *c+u* and upstream *c-u*. Furthermore the equations of mass,

momentum and energy are coupled leading to the convection of linear waves of entropy and vorticity. This can be easily seen looking at the entropy and vorticity equations for a constant density case [1]:

$$T\left(\frac{\partial S}{\partial t}+\vec{u}\frac{\partial s}{\partial x}\right)=\Phi+\frac{1}{\rho}\nabla^2 T$$
$$\frac{\partial \vec{\omega}}{\partial t}+\vec{u}\frac{\partial \vec{\omega}}{\partial x}=\vec{\omega}\cdot\nabla\vec{u}+\upsilon\nabla^2\vec{\omega}$$

The two linear convection terms on the LHS describe the linear waves travelling at the flow speed which is normally small compared with the speed of sound, and this explains why for small Mach number they are normally neglected. Nevertheless, the strengths of the acoustic and entropy waves are coupled in the continuity equation by the density and the velocity.
There exist simplified heat release models that can be included into the one-dimensional waves equation to obtain analytical solutions, but they cannot be applied to solve more advanced problems where it is necessary to include accurate and case dependent time scales and chemistry. The problem is, these equations are all strongly coupled, and hence there is no practical way to obtain accurate results by simplifying them too much. On the other hand, methods like FVM require particular attention because the equation coupling is often implemented in ways that can lead to wrong results.

Thermal shock and acoustic modes

Combustion noise is generated by unsteady heat release, but our interest here is not directed to capturing the closed loop mechanism providing energy to acoustic divergence. Rather, we want to study how a pressure wave is originated by temperature disturbances. Combustion unsteadiness has been emulated by superimposing a high-temperature spot to the flow field and following the solution in time. The hot spot acts like a step in the flow domain, and it is convected downstream by the mean velocity. The result is pressure waves travelling in both directions and reflected by the boundary conditions.

Numerical Modelling

The flow field was the same rectangular 2D channel (*0.5×0.05m*) used above to study how pressure waves convect and influence turbulence. In this case, the temperature of a thick layer across the flow composed by 5 cell rows (*5 mm*) situated at middle of the channel had been patched in order to simulate the effect of a hot spot (Figure 3).

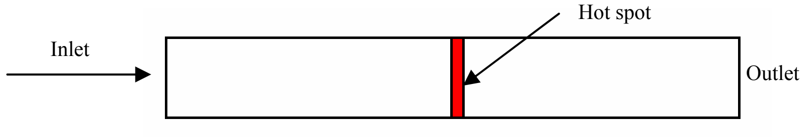


Figure 3: Computational domain.

Tests have been carried out at different conditions of flow rate, inlet temperature and pressure, and with different spot temperatures. The solution showed how the hot spot was convected approximately at the local flow speed, while pressure waves were convected at the speed of sound.
The ideal-gas flow was compressible and turbulent. The turbulence model adopted was the RSE with quadratic pressure-strain correlation [12]. The upstream stabilisation scheme was again the second-order upwind. The QUICK scheme has been tested as well.

The use of the boundary conditions was limited to the conditions available in FLUENT™ for compressible cases. At the inlet either the mass flow or the total pressure can be specified, while at the outlet, one can only fix the static pressure. Fixing the outlet static pressure introduces a node for the acoustic modes.

Solution Strategies

The flow field was first solved without the hot spot, leading to a steady solution. To study the time behaviour of the pressure waves, the temperature field of the converged solution was patched with a higher temperature in the central zone (the hot spot). The fluid-dynamics fields then obtained was then used as the initial condition of the unsteady computation.
The spatial discretisation adopted was the same which proved to be unaffected by aliasing in the previous tests, while the time step is 10^{-6} *s*. The solver was claimed to be implicit and therefore should be unconditionally stable. However, this was not completely convincing because segregated solvers (as well as the coupled one implemented in Fluent) do not solve all the equations at the same time step. As a matter of fact, at least the turbulence equations are solved decoupled. This might explain some solution divergences observed in performing the simulations.

Behaviour of the pressure waves from a hot spot.

The hot spot induces pressure waves on a wide spectrum, which travel at the local sound speed and are reflected at the boundaries. The imposed BC are both nodes for the acoustic modes. Therefore the channel, after a short transitory, is acting like an organ pipe, whose modes have a period defined as ω=π/(cL). In this case, the frequency of the *n-th* mode was $f_n = 330 \cdot n$ and it was a function of the sound speed equal to *330 m/s* at the local temperature.
By sampling the pressure in the middle of the channel (point A), it was possible to identify the modes and observe the error due to the phase speed (Figure 4).

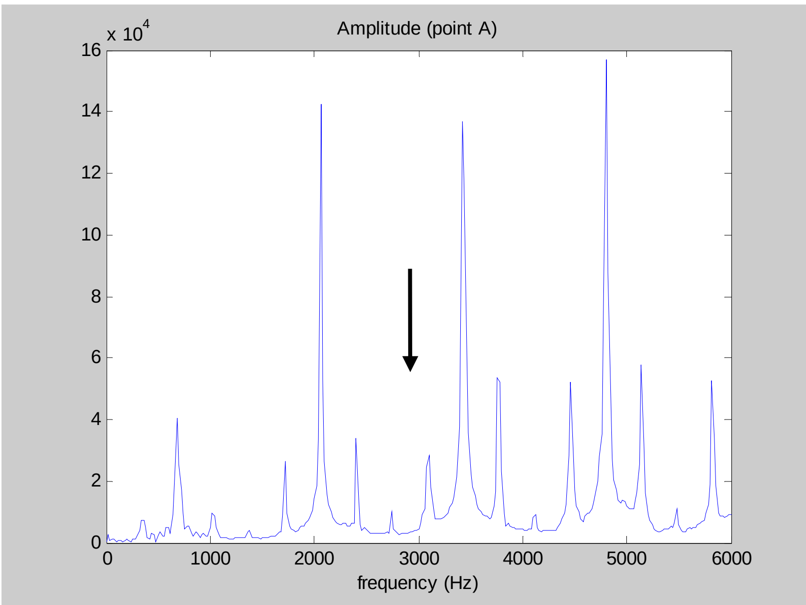


Figure 4: FFT analysis of pressure.

At high frequencies, the difference between numerical and theoretical frequency becomes appreciable. When *n=12*, corresponding to the *12th* mode of the pipe, the theoretical frequency was given by $f_{12} = 12 \cdot 330 Hz = 3690 Hz$, while the physical frequency was approximately *4200 Hz*. The error was due to the incorrect evaluation of the sound speed and it is in accordance with the value reported in Table 1.

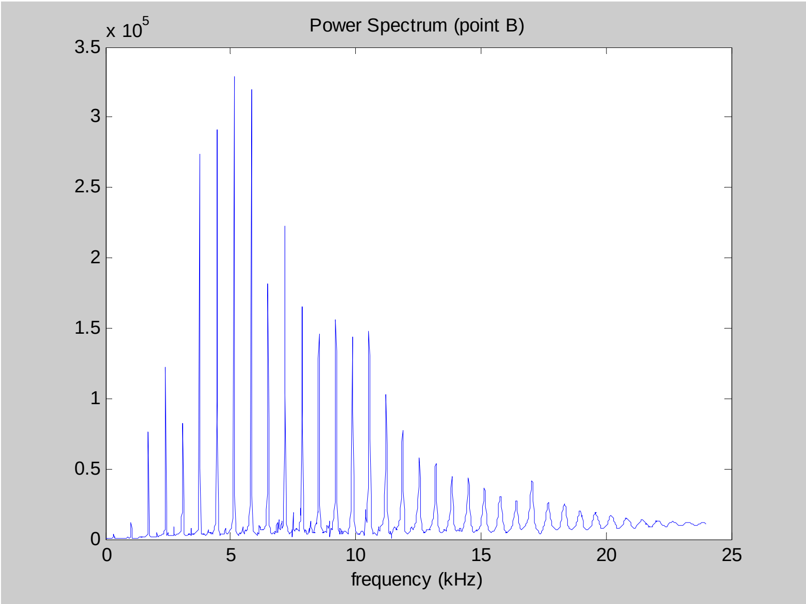


Figure 5: Power spectrum of pressure.

By observing the pressure spectrum on a wide range, it is also possible to observe how the dissipation acts. Figure 5 refers to the sampling in a point at ¼ of the channel (point B).
The damping became appreciable at a frequency which was very close to the one corresponding to the Taylor scales, while the modes seemed to be completely damped at a frequency close to the Kolmogorov scale (Figure 5). The Kolmogorov time scale is defined as $\tau_\eta = (\upsilon/\varepsilon)^{1/2}$, corresponding in this case to a frequency of *24 kHz*. The Taylor time scale is $\tau_t = 3.87\tau_\eta$, corresponding to *6.2 kHz*. At the low frequency, the modes have very small amplitude. One reason was that the hot spot perturbed the modes with longer amplitude, while the shorter amplitudes modes were less disturbed.

Effect on turbulence and the EDC

The analysis showed that it was possible to simulate the acoustic modes with good accuracy and to capture their attenuation.
It remains to investigate how the EDC takes them into account. In this case, the turbulent kinetic energy k and the dissipation ε are used as indicators. In particular, the FFT analysis showed that the pressure waves had only a marginal effect on both quantities.
This indicated that the fine-structure mass fraction was not influenced by acoustics. A Fourier analysis of the turbulence parameters (Figure 6) showed narrow spectra compared with the power spectra of pressure and velocity, that is, k and ε are stronger at low frequency. From a physical point of view, the dissipation spectrum should be wider and have its main area at higher frequencies. However, in turbulence models, the dissipation ε is a function of large-scale quantities such as Reynolds-stresses, mean-velocity components and their gradients. Thus, the "model ε" is a large-scale quantity, and accordingly, the power spectrum shown in Figure 6 was not unexpected.
Figure 5 also shows the energy spectrum of pressure waves to be discontinuous, unlike the turbulent spectrum, and characterised by a redistribution of energy between discrete frequencies which are defined by the acoustic modes of the flow field.

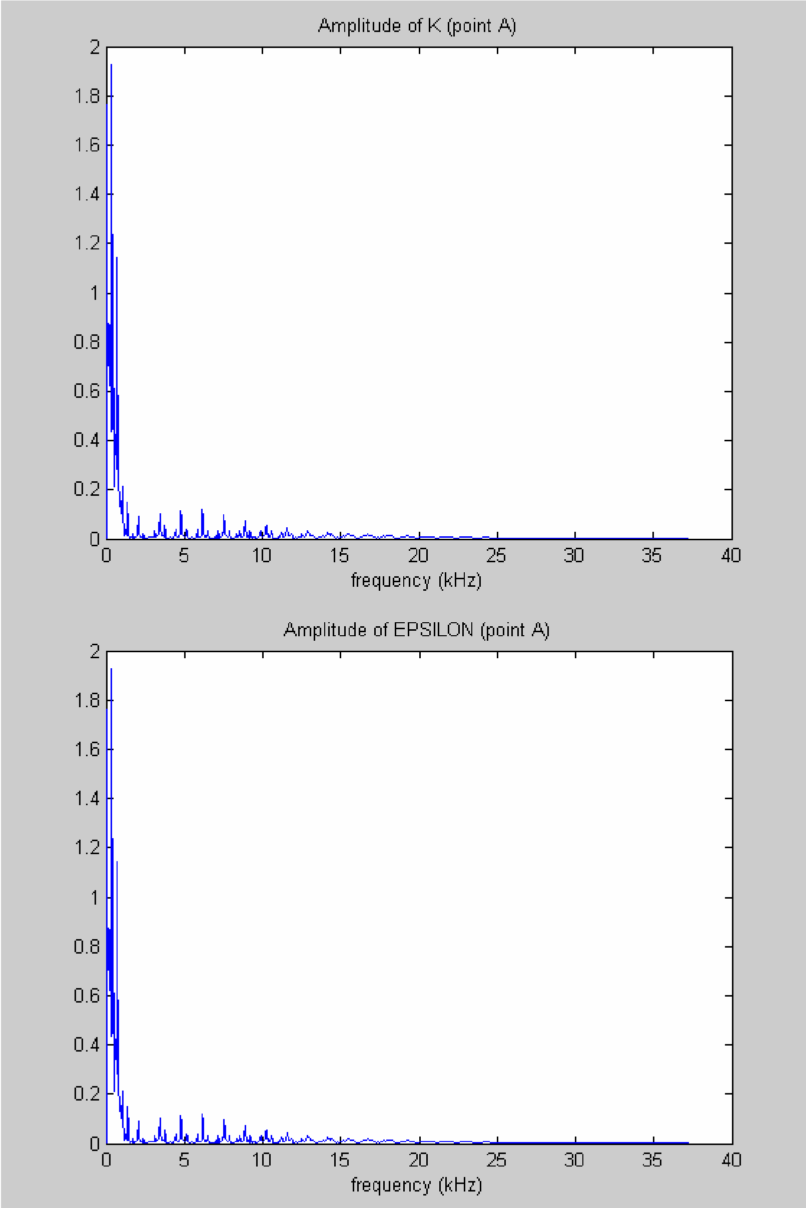


Figure 6: FFT analysis of k and epsilon.

Concluding remarks

Simulation of problems including pressure oscillation with fineite-volume methods requires care in order to limit error due to the phase speed and aliasing. Mesh dependency tests have to be performed in order to ensure good accuracy in the frequency spectrum of the problem. The use of RSE turbulence model with an advanced pressure strain correlation leads to reasonable results showing coherent energy spectra. However, the effect of pressure oscillation was only weakly captured by the turbulence model. Accordingly, also the turbulence quantities of the EDC, and hence, the reaction rate, was only marginally influenced by pressure oscillations.
The EDC concept was developed in order to catch the effect of turbulence on the mixing rate. When dealing with pressure waves the concept shows a weak correlation that needs to be modelled and studied in detail.

List of symbols

c	speed of sound
c^*	numerical speed of sound
C_p	specific heat at constant pressure
$D(\lambda)$	dissipation spectrum
$E(\lambda)$	energy spectrum
f	frequency
k	turbulence kinetic energy
L	pipe length
p'	pressure oscillation
S	entropy
t	time
T	temperature
U	velocity
u_n	velocity at n-th iteration
x	spatial coordinate
γ	specific heat ratio
γ^*	fine-structure mass fraction
Δx	spatial discretisation in the x-direction
Φ	dissipation function
ε	turbulence-energy dissipation rate
λ	wave number
η	strain viscosity
υ	viscosity
ρ	density
τ_t	Taylor time scale
τ_η	Kolmogorov time scale
ω	vorticity

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