

Hot tearing and thermally induced deformation in the mushy zone

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Abstract

A two-phase continuum model for an isotropic mushy zone has recently been presented by the authors. While the solid for temperatures below coherency is considered to be rigid in most other two-phase mushy zone models, this new model takes into account volume averaged velocities and stresses in both the liquid and solid phases. More specifically, both feeding of interdendritic melt due to solidification shrinkage as well as the thermally induced deformation of the solid phase are taken into account. This means that the main phenomena commonly associated with hot tearing are addressed. With reference to this recently proposed model, the purpose of the present paper is to discuss the constitutive behaviour of the volume averaged solid phase. Under the assumption of no elastic deformation, the volume averaged strain rate is sub-divided into a volumetric thermal part, a viscoplastic volumetric part, and a viscoplastic deviatoric part. It is suggested to model the thermal strain rate exactly as for a single-phase continuum provided that the solid fraction is high enough for coalescence (solid bridging) to occur. It is furthermore pointed out how liquid films between the solid grains, i.e., lack of coalescence, might decrease the possibility for thermal straining and thereby the driving force for thermally induced deformation. In spite of the incompressibility of the individual solid grains, incompressibility of the volume averaged solid can not necessarily be assumed due to the presence of liquid in the coherent mushy zone. However, when the solid fraction is close to one, it is suggested to impose incompressibility and to model the viscoplastic deformation by a creep law. Finally, some ideas are presented on how a possible hot tearing criterion can be based on the combined effects of an overcritical pressure drop representing pore formation and severe tensile deformation taking place just before the liquid fraction reaches one.

Introduction

This paper is directed towards some of the underlying mechanisms associated with hot tearing observed in direct chill (DC) cast aluminium ingots. During solidification of the alloys, a coherent mushy zone exist below a so-called coherency temperature at which the material starts to develop strength and above the eutectic or solidus temperature. It is commonly believed that hot tears are initiated and start to develop in this mushy regime at a stage where the fraction of solid is close to one [1, 2].

Hot tearing is associated with two different mechanisms [3], namely the lack of interdendritic melt flow to feed the solidification shrinkage, and a too large thermally induced deformation caused by the non-uniform cooling contraction of the casting. The two mechanisms were recently considered simultaneously in a model developed by Rappaz *et al.* [4]. They argued as Feurer [5] that a hot tear will nucleate if the liquid is no longer able to fill the intergranular openings caused by the solidification shrinkage. Considering columnar dendritic growth, this approach was then extended by taking into account the feeding associated with tensile deformation of the solidified material in the direction transversal to the dendrites.

Based upon the volume averaged conservation equations as formulated by Ni and Beckermann [6], Farup and Mo [7] recently presented a more general mathematical framework for an isotropic, coherent mushy zone with equiaxed grains. In this model, both the shrinkage driven interdendritic melt flow and the thermally induced deformation of the solid part of the mush are taken into account. Although not intended to be a new "hot-tearing theory", this model can be applied to compute and compare some key quantities generally believed to be associated with hot tearing.

For temperatures below coherency the solid is considered to be rigid in most two-phase mushy zone models, and volume averaged velocities and stresses in the liquid phase are quantified. In the model presented in Reference [7], however, volume averaged velocities and stresses in the solid phase are also taken into account. This leads to new challenges related to the constitutive modelling and experimental quantification of the mushy zone rheology. The purpose of the present paper is to address and discuss some of these challenges.

Modelling equations

The assumptions on which the new model is based are thoroughly outlined in Reference [7] and other references cited there, and will not be repeated here. Only a few remarks will be made concerning the differences between the present model and two-phase models applied for other transport phenomena in the mushy zone.

Above the coherency temperature it can be assumed that the solidified grains move freely in the liquid. The pressures in the two phases are then equal [6], which reflects that the solid phase does not pose any restriction against a "densification" (e.g., sedimentation of crystals). Below the coherency temperature, an additional pressure can be transmitted through the solid structure due to mechanical contact between the grains. The pressures in the two phases can then be different. Neglecting the surface tension, and assuming instantaneous pressure equilibration locally in the liquid and mechanical equilibrium at the solid-liquid interface, the interfacial liquid and solid pressures can both be set equal to the bulk liquid pressure [8]. In both phases of a coherent mushy zone, momentum transport due to acceleration is negligible, and due to the relatively large solid fractions under consideration, the momentum transport in the liquid phase is assumed to yield Darcy's law. It should also be pointed out that the liquid density is considered constant since free convection is not taken into account. The solid density is, on the other hand, not considered as constant due to the thermally induced deformation in the solid phase.

Table 1: Nomenclature for quantities in Equations (1) to (6).

x_i	position vector	M_i	interfacial momentum transfer
t	time	$-(g\delta_{2i})$	acceleration due to gravity
g_k	volume fraction	$K(g_l)$	permeability (function of g_l)
ρ_k	mass density	μ	viscosity
v_{ki}	velocity vector	λ_k	heat conductivity
p_k	pressure	C_k	heat capacity
τ_{kij}	deviatoric part of stress tensor	L	latent heat
T	temperature	index $k = l$	liquid
Γ	interfacial mass transfer	index $k = s$	solid

With reference to the Nomenclature given in Table 1, the equations expressing the conservation in mass, momentum, and energy are

$$\frac{\partial(g_s \rho_s)}{\partial t} + \frac{\partial}{\partial x_i} (g_s \rho_s v_{si}) = \Gamma \quad (1)$$

$$\frac{\partial(g_l \rho_l)}{\partial t} + \frac{\partial}{\partial x_i} (g_l \rho_l v_{li}) = -\Gamma \quad (2)$$

$$p_l \frac{\partial g_s}{\partial x_i} - \frac{\partial}{\partial x_i} (g_s p_s) + \frac{\partial}{\partial x_j} (g_s \tau_{sij}) + M_i - g_s \rho_s (g\delta_{2i}) = 0 \quad (3)$$

$$-g_l \frac{\partial p_l}{\partial x_i} - M_i - g_l \rho_l (g\delta_{2i}) = 0 \quad (4)$$

$$(g_s \rho_s C_s + g_l \rho_l C_l) \frac{\partial T}{\partial t} + (g_s \rho_s C_s v_{si} + g_l \rho_l C_l v_{li}) \frac{\partial T}{\partial x_i} = \frac{\partial}{\partial x_i} \left[(g_s \lambda_s + g_l \lambda_l) \frac{\partial T}{\partial x_i} \right] + L\Gamma \quad (5)$$

where the energy equations for the liquid and solid phases are added due to the underlying assumption of local thermal equilibrium, and Darcy's law yields

$$M_i = \frac{g_l^2 \mu (v_{li} - v_{si})}{K(g_l)} \quad (6)$$

It is finally mentioned that the solidification was in Reference [7] taken into account by simply assuming a unique relation between g_s and T .

Considerations on the constitutive behaviour of the solid phase

Due to the quite low flow stress in metallic alloys at temperatures close to or above the solidus (or eutectic), elastic deformation is neglected. Only viscoplastic and thermal straining will be defined and considered.

Tensor components for the volume averaged solid strain rate can be defined by

$$\epsilon_{sij} = \epsilon'_{sij} + \frac{1}{3} \delta_{ij} \epsilon_{skk} = \frac{1}{2} \left(\frac{\partial v_{si}}{\partial x_j} + \frac{\partial v_{sj}}{\partial x_i} \right) \quad (7)$$

where the deviatoric part, ϵ'_{sij} , represents the incompressible part of the deformation, δ_{ij} denotes the Kronecker delta, and the sum convention applies. It is furthermore convenient to sub-divide the volumetric strain rate into two parts

$$\epsilon_{sii} = \epsilon_{sii}^T + \epsilon_{sii}^P \quad (8)$$

where ϵ_{sii}^T and ϵ_{sii}^P are related to the expansion/contraction of the solid by temperature and pressure, respectively. (Please note that $\epsilon_{sij}^T = 0$ and $\epsilon_{sij}^P = 0$ if $i \neq j$). Introducing $\frac{\partial v_i}{\partial x_i} = \epsilon_{sii} = \epsilon_{sii}^T + \epsilon_{sii}^P$ into Equation (1) then yields

$$\frac{1}{\rho_s} \left(\frac{\partial \rho_s}{\partial t} + v_{si} \frac{\partial \rho_s}{\partial x_i} \right) + \epsilon_{sii}^T + \epsilon_{sii}^P = \frac{1}{g_s \rho_s} \left[\Gamma - \rho_s \left(\frac{\partial g_s}{\partial t} + v_{si} \frac{\partial g_s}{\partial x_i} \right) \right] \quad (9)$$

where additional constitutive equations relating ϵ_{sij}^T and ϵ_{sij}^P to other quantities of the problem are required. In the proceeding, ϵ_{sij}^T will be referred to as the thermal strain rate. It should be noted that Equation (9) has a zero right hand side in the limit $g_s \rightarrow 1$. The single-phase continuity equation is then recognised.

When the solid fraction is close to one, there is experimental evidence indicating coalescence of the solid skeleton, i.e., solid bridging exists between the different grains [9] as sketched in Figure 1(a). In this case, the thermal strain rate can be related to the volume change with temperature as in a single-phase continuum

$$\epsilon_{sij}^T = \frac{1}{3} \delta_{ij} \beta \left(\frac{\partial T}{\partial t} + v_{si} \frac{\partial T}{\partial x_i} \right) \approx \frac{1}{3} \delta_{ij} \beta \frac{\partial T}{\partial t} \quad (10)$$

where $\beta = -\frac{1}{\rho_s} \left(\frac{\partial \rho_s}{\partial T} \right)$ denotes the thermal expansion coefficient. The approximation indicates that the temperature change due to advective energy transport in the solid phase is negligible.

For somewhat smaller values of g_s , continuous films of liquid can exist between the individual grains as indicated in Figure 1(b). In this situation Equation (10) is not necessarily correct. Due to the liquid films, one could imagine that the grains contract (at constant g_s) without affecting the positions of the individual mass centres of the grains. Different from a single-phase continuum, this means that ϵ_{sij}^T would *not* be given by the volumetric change of the solid with temperature. In other words, one can imagine that the thermal strain rate be zero for the lower values of g_s in the coherent part of the mushy zone, meaning that no driving force for thermally induced deformations would be present.¹ Such an effect could be incorporated in the constitutive model by assuming that ϵ_{sii}^T be zero for all values of g_s below a given value. An elaboration on this would be to introduce a g_s -dependent factor on the right hand side of Equation (10) starting at zero for some value of g_s and then approaching one as g_s tends towards one.

Due to the presence of liquid in the coherent mushy zone (cf. Figure 1 (a) and (b)), it was pointed out in Reference [10] that a difference between the solid and liquid pressures can lead to a change in the solid volume, i.e., $p_s > p_l$ can lead to a "densification" of the solid. In spite of the incompressibility of the individual (microscopic) solid grains, this means that the *volume averaged* (macroscopic) be compressible. A constitutive equation for the volumetric strain rate caused by such a difference in the pressures could be on the form

$$\epsilon_{sij}^P = \delta_{ij} f(p_s, p_l; g_s) \quad (11)$$

such that $\epsilon_{sii}^P = 0$ when $p_s = p_l$, $\epsilon_{sii}^P < 0$ when $p_s > p_l$ and $\epsilon_{sii}^P > 0$ when $p_s < p_l$. Innovative experiments are required in order to determine the function f , keeping in mind the limiting case of the incompressible fully solid (single-phase) continuum at $g_s = 1$.

In addition to establishing a relation for the mechanically imposed volumetric strain rate, relations for the strain rate associated with the incompressible part of the viscoplastic deformation represented by ϵ'_{sij} must be incorporated in the model.

¹Please note that in a real solidification situation, there will always be a driving force for thermally induced deformations in adjacent parts of the mushy zone or fully solid material in which g_s is sufficiently close or equal to one.

A simplified approach

Experimental data, both qualitative in order to identify the different effects involved and quantitative in order to determine realistic values for the strain rates under given conditions, are limited [11] or even non-existent. In order to establish some constitutive relations that at least are not totally unrealistic, a very simplified approach was suggested in Reference [7]. For the entire coherent part of the mushy zone incompressibility was here imposed (i.e., $\epsilon_{sij}^P = 0$) along with the assumption that Equation (10) is valid even when liquid films exist.²

The assumption of incompressibility simplifies the description of the viscoplastic deformation considerably. In Reference [7] proportionality between corresponding tensor components ϵ'_{sij} and τ_{sij} was assumed (cf. Levy–Mises flow law). The effective viscoplastic strain rate $\bar{\epsilon}_s = \sqrt{\frac{2}{3} \epsilon'_{sij} \epsilon'_{sij}}$ was then related to the effective stress $\bar{\tau}_s = \sqrt{\frac{3}{2} \tau_{sij} \tau_{sij}}$ by a simple power law.

The limited number of quantitative measurements of the viscoplastic deformation in partially solidified aluminium alloys indicates that such experiments are considerably more complicated than similar mechanical testing of a fully solidified alloy (see e.g., References [12] and [13]). When such mushy zone measurements are to be applied in a two phase model, it should furthermore be kept in mind that the mush in most of the experiments in the literature is considered as a single-phase continuum for which the effective strain rate is related to the effective stress by a creep law. In order to incorporate such a creep law into the present two-phase model, the single-phase variables associated with the experiments must be interpreted in terms of two-phase quantities. It is referred to Farup and Mo [7] and Martin *et al.* [10] for further details.

In spite of the simplifications on which the new model is based, it should be taken into consideration that hot tears develops when g_s is very close to one, i.e., near the limit at which the simple approach to the constitutive relations is fairly reasonable. This might explain the fact that several well-known effects can be qualitatively predicted by the new approach. Examples are the so-called lambda curve (the variation of liquid pressure drop with alloy composition) and the stress build-up in the mushy zone [7]. For lower values of g_s , the results from the new model are questionable, even from a qualitative point of view. However in this regime, hot tears are healed by melt feeding anyway. A detailed quantitative model for this "healing regime" is therefore of less interest.

Ideas on a possible hot tearing criterion

Because pores may act as nucleation sites for hot tears, it has been suggested that the formation of hot tearing is closely related to the formation of microporosity [4]. It should be emphasised that models for pore formation during solidification could be incorporated in the present modelling framework by introducing a pore fraction as a third phase in addition to the solid and liquid phases. A possible hot tearing criterion could then be based on the combined effects of pore formation and severe tensile deformation taking place just before the liquid fraction reaches one. In order to simplify further, the pore formation, or more precisely, the lack of melt feeding could be represented by an overcritical pressure drop.

²In Reference [7], Equation (10) was actually imposed from a slightly different point of view. Here it was assumed that the solid fraction in the entire coherent part of the solidification interval could change only due to interfacial mass transfer. This is mathematically expressed by a zero right hand side of Equation (9), which in turn can be considered as imposing the single-phase continuity equation for the solid phase

$$\frac{1}{\rho_s} \left(\frac{\partial \rho_s}{\partial t} + v_{si} \frac{\partial \rho_s}{\partial x_i} \right) + \epsilon_{sii} = 0$$

as a "closure equation" for the model. Introducing $\rho_s = \rho_s(T)$ and $\beta = -\frac{1}{\rho_s} \left(\frac{\partial \rho_s}{\partial T} \right)$ into this equation, however, yields Equation (10) when $\epsilon_{sii} = \epsilon_{sii}^T$.

Conclusion and final remarks

With reference to a recently proposed two-phase model for an isotropic mushy zone in which both shrinkage driven interdendritic melt flow and thermally induced deformation of the solid phase are taken into account, the constitutive behaviour for the volume averaged solid phase have been addressed.

- Under the assumption of no elastic deformation, the volume averaged strain rate has been sub-divided into a volumetric thermal part, a viscoplastic volumetric part, and a viscoplastic deviatoric part.
- Assuming coalescence (solid bridging) for solid fractions close to one, it has been suggested to model the thermal strain rate exactly as for a single-phase continuum.
- It has been pointed out how liquid films between the solid grains, i.e., lack of coalescence might decrease the possibility for thermal straining and thereby the driving force for thermally induced deformation.
- Due to the presence of liquid in the coherent mushy zone, it has been pointed out that the volume averaged solid could be compressible in spite of the incompressibility of the individual solid grains.

In the previously mentioned two-phase modelling framework, a very simplified approach for the constitutive behaviour of the solid was suggested. For the entire coherent part of the mushy zone, incompressibility was imposed along with a constitutive relation for the thermal strain rate similar to that applying for a single-phase continuum. The viscoplastic deformation was furthermore modelled by a simple creep law. The two-phase mushy zone model could then be applied to compute and compare some key quantities generally believed to be associated with hot tearing.

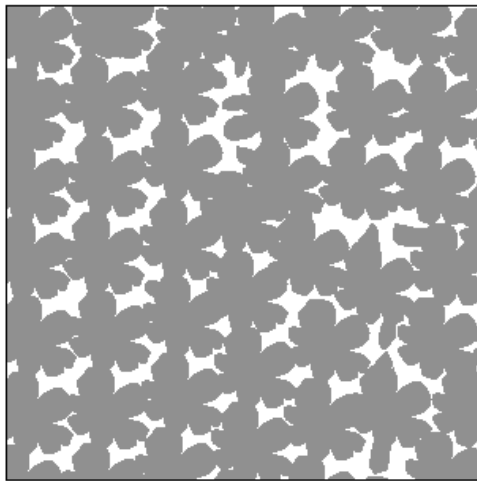
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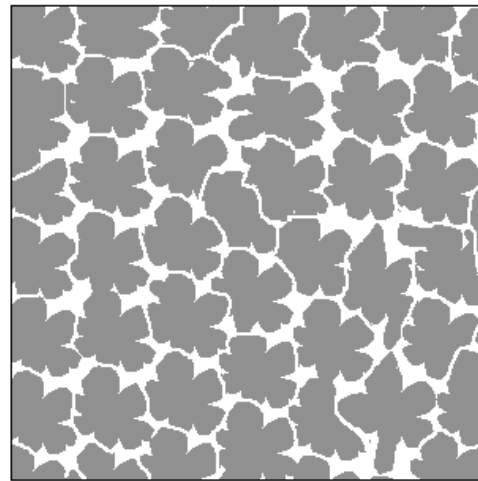
References

- [1] J. Campbell. *Castings*. Butterworth Heinemann, 1991.
- [2] M. L. Nedreberg. *Thermal stress and the hot tearing during the DC casting of AlMgSi billets*. PhD thesis, University of Oslo, Dep. of Physics, February 1991.
- [3] G. K. Sigworth. Hot tearing of metals. *AFS Transactions*, **106**:1053–1069, 1999.
- [4] M. Rappaz, J.-M. Drezet, and M. Gremaud. A new hot-tearing criterion. *Metallurgical and Materials Transactions A*, **30A**:449–455, 1999.
- [5] U. Feuerer. Matematisches Modell der Warmrissneigung von binären aluminum Legierungen. *Giesserei Forschung*, **28**:75, 1976.
- [6] J. Ni and C. Beckermann. A volume-averaged two-phase model for transport phenomena during solidification. *Metallurgical Transactions B*, **22B**:349–361, 1991.

- [7] I. Farup and A. Mo. Two-phase modelling of mushy zone parameters associated with hot tearing. Accepted for publication in the *Metallurgical and Materials Transactions*, November 1999.
- [8] C. Y. Wang and C. Beckermann. Equiaxed dendritic solidification with convection: Part I. Multi-scale/multiphase modeling. *Metallurgical and Materials Transactions A*, **27A**:2754–2764, 1996.
- [9] I. Farup, J.-M. Drezet, and M. Rappaz. In-situ observation of hot-tear formation in succinonitrile-acetone. Submitted to the *Materials Science and Engineering*, September 1999.
- [10] C. L. Martin, D. Favier, and M Suéry. Viscoplastic behaviour of porous metallic materials saturated with liquid. Part I: Constitutive equations. *International Journal of Plasticity*, **13**(3):215–235, 1997.
- [11] C. L. Martin, D. Favier, and M Suéry. Viscoplastic behaviour of porous metallic materials saturated with liquid. Part II: Experimental identification on a Sn-Pb model alloy. *International Journal of Plasticity*, **13**(3):237–259, 1997.
- [12] J. M. Drezet and G. Eggeler. High apparent creep activation energies in mushy zone microstructures. *Scripta Metallurgica*, **31**(6):757–762, September 1994.
- [13] P. Wisniewski and H. D. Brody. Tensile Behaviour of Solidifying Aluminum Alloys. In M. Rappaz, M. R. Özgü, and K. W. Mahin, editors, *Modeling and Control of Casting and Welding Processes V*, pages 273–278, Warrendale, PA, USA, 1991. TMS-AIME.



(a)



(b)

Figure 1: Two phase volume element for (a) a coherent mushy zone where bridging between the grains has occurred, and (b) a coherent mushy zone in which liquid films exist between the grains.