

## Chapter 6, Linear Normal Models

### Properties:

- As GLM
- Maximum Likelihood Estimate (MLE)
- Least Square Estimate
- Deviance
- Hypothesis testing
- Outlier detection (delta-beta, Cook's distance and leverage)
- Colinearity

### Models:

- Multiple linear regression
  - ▶ Outlier detection / influential observation
  - ▶ Collinearity / multicollinearity
- Analysis of variance (ANOVA)
  - ▶ One factor ANOVA
  - ▶ Two factor ANOVA
- Analysis of covariance
- General linear model

# GLM formulation

- Response:  $Y_i \sim N(\mu_i, \sigma^2)$
- Expected value:  $E(Y) = \mu_i$
- Identity link function:  $\eta_i = \mu_i$
- Linear component:  $\eta_i = \mathbf{x}_i^T \boldsymbol{\beta} = \beta_0 + \beta_1 x_{i1} + \dots + \beta_{p-1} x_{ip-1}$

## Deviance

Let  $\beta_{max}$  be the parameter vector for the *saturated* modeled, and  $\beta$  for the model of our interest. Let  $l(\beta; y)$  be the log-likelihood function. The *deviance* of the model is

$$D = 2(l(b_{max}; y) - l(b; y))$$

where  $b$  and  $b_{max}$  are (ML) estimates.

## Saturated model

The richest possible model. Each combination of (all possible known) explanatory variables have their own  $\theta_i$ .  $b = b_{max}$

## Gaussian pdf

$$f(y; \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-0.5 \frac{(y - \mu)^2}{\sigma^2}\right)$$

### Definition central $F$ -distribution

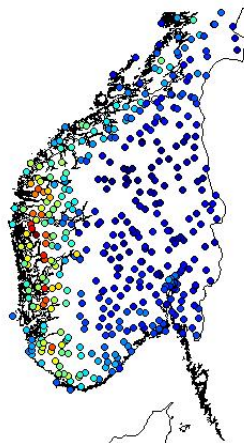
If  $X_1^2 \sim \chi^2(n)$ ,  $X_2^2 \sim \chi^2(m)$  and  $X_1^2$  and  $X_2^2$  are independent, then

$$F = \frac{X_1^2/n}{X_2^2/m} \sim F(n, m)$$

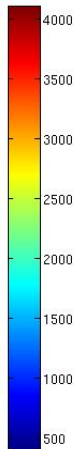
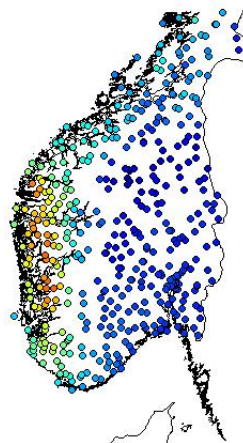
# Precipitation

5 years of daily precipitation observation and forecast for 450 locations  
⇒ 1.6 mill data.

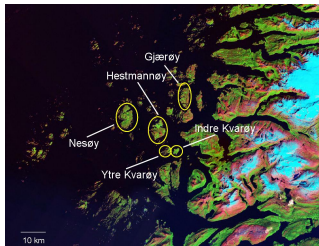
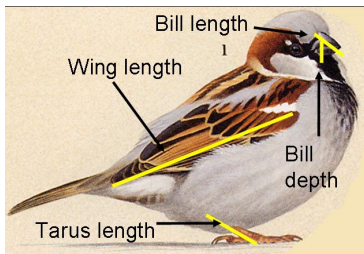
Yearly observed precipitation



Yearly forecasted precipitation



# House sparrows questions



- 1 Are birds heavier in summer than in winter?
- 2 Are birds relatively heavier on the outer islands in summer than on the inner islands?
- 3 Body mass modeled with tarsus length, wing length, bill length and bill depth.
- 4 Body mass modeled with tarsus length, wing length, bill length, bill depth and season.
- 5 Are birds heavier on the outer islands when we account for size (tarsus, wing length, etc.) ?