Shooting balloons

- N trail subjects, $i = 1, 2, \ldots, N$
- Each shot n_i times, trying to hit balloons.
- Count hits y_i.
- Explanatory variables:
  - Experienced / non-experienced gunman
  - Wind speed

Shooting balloons, data

Trail person	1	2	3	...
Experiences	1	0	0	...
Wind speed	2.13	0.59	1.03	...
n_i	6	3	5	...
y_i	2	1	1	...

Shooting balloons, model

- $Y_i \sim bin(n_i, \pi_i)$, $i = 1, 2, \ldots, N$
- $\eta_i = logit(\pi_i)$
- 1. $\eta_1 = \beta_0 \Rightarrow Y_i \sim bin(n_1, \pi)$
  2. $\eta_i = \beta_0 + \beta_1 x_1$
  3. $\eta_i = \beta_0 + \beta_2 x_2$
  4. $\eta_i = \beta_0 + \beta_1 x_1 + \beta_2 x_2$

Where $x_1 = 1$ for experienced gunman, otherwise $x_1 = 0$ and x_2 is wind speed.

Hight of male students

- In population: $Y \sim (179.8, 6.5^2)$
- Mean of 91 male students: 181.9
- NTNU students higher then Norwegians?

Score function and information

Score statistics

Let $l(\theta; y_i)$ be log-likelihood function. Then the score statistic is:

$$U(\theta; y) = \frac{\partial l(\theta; y_i)}{\partial \theta}$$

Information

Let $U = U(\theta; Y)$ be the score statistic. Then the information is

$$\Im = Var(U) = -\frac{d^2 l}{d\theta^2}$$

Taylor approximations for $l(\beta)$ near estimate b:

$$l(\beta) = l(b) + (\beta - b)U(b) + \frac{1}{2}(\beta - b)^2 U'(b)$$

Approximate $U'(b)$ with $E(U') = -\Im(b)$:

$$l(\beta) = l(b) + (\beta - b)U(b) - \frac{1}{2}(\beta - b)^2 \Im$$

For a vector b

$$l(\beta) = l(b) + (\beta - b)U(b) - \frac{1}{2}(\beta - b)^T \Im(\beta - b)$$

5.3 Taylor series approximations for log likelihood

$$U(\beta) = U(b) - (\beta - b)U'(b)$$

Approximate $U'(b)$ with $E(U') = -\Im(b)$:

$$U(\beta) = U(b) - (\beta - b)\Im(b)$$