Linear Model

1. Response: $Y_i \sim N(\mu_i, \sigma^2)$, Y_is independent
2. Linear relationship: $E(Y_i) = \mu_i = x_i^T \beta_i$

We extend this class of models by:

1. Response from exponential family of distributions.
2. Non-linear link: $g(\mu_i) = x_i^T \beta$

Exponential family

Definition

$f(y;\theta)$ belongs to the exponential family if

$$f(y;\theta) = \exp[a(y)b(\theta)) + c(\theta) + d(y)]$$

Examples:

- Normal
- Binomial
- Poisson
- Chi-square
- Gamma
- Beta

- Inspired by Ch 3.5.2.
- Interested in languages that descend from the same historical languages.
  - Norwegian and Swedish from Norse.
  - Modern French and Spanish from Latin.
- Languages that are separated by time t.
- Probability that a particular meaning has cognate words, $\exp(-\lambda t)$.
- Data: A linguist (*Clue*) judges if N different meanings are cognate:

Meaning	Norwegian	Swedish	Cognate
Laugh	Le	Skratta	No
House	Hus	Hus	Yes

Similar data for Spanish and French.

Model

1. Probability function, $Y_i \sim f(y, \theta_i)$
2. Link function, $g(\mu_i) = x_i^T \beta$
3. Explanatory variables and parameters

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   - $Y_i \sim bin(1, p_i)$ with $p_i = \exp(-\lambda t_i) = \theta_i$
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3. Explanatory variables and parameters

   Parameter: $\beta = \lambda$

   Explanatory variable: t_i, time since separation.
   $X = [t_{ns}, t_{ns}, \ldots t_{ns}, t_{sf}, \ldots, t_{sp}]$. t-s are assumed known.

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Ideas extended model:

- Categories of meanings:
  - Feelings
  - Body parts
  - Mathematical terms
- Number of syllable.