

## **H. Rue, L. Held: Gaussian Markov random fields. Theory and applications**

**Chapman & Hall/CRC, Boca Raton–London–New  
York–Singapore 2005, pp. 263, EUR 44.99. ISBN 1-58488-432-0**

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Gaussian Markov random fields (GMRFs) are mainly applied in hierarchical models as they represent a convenient form to model stochastic dependence between observed data. The dependence may be spatial, temporal or spatiotemporal. All these dependency-structures are covered by the book in given examples, case studies and lots of provided algorithms.

In the introduction the main ideas in the context of Markov random fields, such as autoregressive processes, conditional independence and their implications for precision and covariance matrix are presented. These are extended to the ideas of conditional autoregression and neighbouring structures. The authors give a broad overview on the applications of GMRFs such as time series analysis, analysis of longitudinal and survival data, graphical models, semiparametric regression and splines, image analysis and spatial statistics. Key references are provided for each area.

The second chapter deals with the general theory of GMRFs and starts by recalling all essential basics on topics such as graph theory and Markov properties. Linear algebra techniques are briefly revised at appropriate places, such as the Cholesky decomposition of the precision matrix which is essential for efficient simulation from a GMRF. The authors introduce useful simulation algorithms starting with algebraic basics which are adopted and extended in various places of the book. As the number of neighbours is typically small in real applications, numerical methods for sparse matrices play a central role in this chapter. The authors impressively show how to reduce computational time by clever factorizations, bandwidth reduction and nested dissection. A numerical case study is performed comparing computational time of different methods presented for temporal, spatial and spatiotemporal models.

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Furthermore this chapter contains more advanced methods for stationary GMRFs and the parameterization of GMRFs.

The third chapter introduces intrinsic GMRFs which are intensively used as prior distributions. Intrinsic GMRFs are improper, i.e., their precision matrix is not of full rank. The relation between intrinsic GMRFs and GMRFs conditioned on linear constraints is cleverly derived and allows to transfer algorithms introduced in the second chapter to the new situation. This chapter focuses on intrinsic GMRFs of first order on the regular and nonregular spaced line, including only locally constant means, and on regular and irregular lattices. The discussions are extended to higher to order intrinsic GMRFs for all applications.

The fourth chapter applies the methods and algorithms introduced to a number of case studies. It includes a comparison of GMRFs to alternative methods such as the Kalman filter and the forward-filtering-backward-sampling algorithm as well as the reformulation of logit and probit models for categorical data by auxiliary variables in order to retrieve GMRF full conditionals. Nonnormal responses, where such an augmentation of the parameter space by auxiliary variables is not possible, require an approximation of the log-likelihood by a second-order Taylor expansion. This is demonstrated for the univariate as well as for the multivariate case. Some of the examples from the first part of this chapter are revisited using the approximations discussed in the second part.

The last chapter deals with recent developments extending the range of applications of GMRFs. The first part links GMRFs to Gaussian fields frequently used in geostatistics leading to computational benefit. The second part discusses the construction of non-Gaussian approximations by hidden GMRFs in order to improve the model fit of non-Gaussian applications as described in the second part of the fourth chapter.

Each chapter ends with additional bibliographic notes for further reading.

The Appendices provide information on common distributions and the GMRFLib library written in C and Fortran which includes useful program code and its documentation. The source code is certainly a further plus of the book, although a better implementation of the program code in a widely used software package such as R would be appreciated.

The book provides an excellent fundament for the work with GMRFs for students and researchers at a reasonable price. Knowledge on Bayesian techniques for hierarchical models and statistical inference using Markov chain Monte Carlo methods is useful precondition, though the methods are briefly reviewed.

The authors provide examples from a wide range of applications, such as rental prices, rainfall data, and joint disease mapping of oral cavity and lung cancer mortality. These data as well as the GMRFLib library is available from the book's website <http://www.math.ntnu.no/~hrue/GMRF-book/>.