Gaussian Markov Random Fields: Theory and Applications.

Harvard RUE and Leonhard HELD. Boca Raton, FL: Chapman & Hall/CRC, 2005. ISBN 1-58488-432-0. 263 pp. $79.95 (H).

Rue and Held have constructed an impressive reference devoted to the theory and applications of Gaussian Markov random fields (GMRFs), with emphasis on recent computational development, in an accessible format. A GMRF is a multivariate normal distribution specified through its precision matrix, which is often sparse as a result of the Markov assumptions. GMRFs have applications in a wide range of fields such as spatial statistics, image analysis, structural time series analysis, longitudinal studies, and others. Many of these areas evolve from the seminal contribution of Besag (1974). In the last decade, intrinsic GMRFs (IGMRFs), a special type of GMRF with a degenerate precision matrix, have been widely used as improper priors for random effects in hierarchical spatial modeling (e.g., Banerjee, Carlin, and Gelfand 2004). Statistical inference for these models are primarily done with Bayesian methods via Markov chain Monte Carlo (MCMC) simulations, where fast sampling issues are crucial. In the statistics literature, Rue (2001) comprehensively discusses fast unconditional and conditional sampling from GMRFs and approximation techniques for more complicated GMRF applications, taking advantage of sparse matrix operations developed in the computer science literature. After a sequence of contributions by the authors and their colleagues covering a wide range of applications, a reference on the basic theory of GMRFs and state-of-art of the computational development and applications was needed. *Gaussian Markov Random Fields: Theory and Applications* is exactly such a book.

After a short introductory chapter, the book has four main chapters. Chapter 2 presents the most important results of GMRFs and algorithms for fast and exact sampling from GMRFs. The theoretical results of GMRFs are discussed using a canonical parameterization under which the density is proportional to $\exp(-\frac{1}{x}x^T Qx + b^T x)$ (with parameters b and precision matrix Q). This parameterization, in contrast to the commonly seen mean-covariance parameterization, is very useful for successive conditioning. The key component of an algorithm for sampling from a GMRF is the Cholesky factorization of the precision matrix, which can be efficiently constructed from reordering schemes recently developed in numerical mathematics. The authors give a nice exposition of the algorithms, pulling all the necessary components together and making the work self-contained. Chapter 3 presents detailed properties and sampling results for IGMRFs, whose wide applications makes it a highlight. Chapter 4 presents applications of GMRFs in hierarchical modeling. Blocking strategies for simultaneous updating GMRFs and their parameter estimators are discussed for models with both normal and nonnormal responses. Chapter 5 presents approximating techniques for GMRFs in more complicated models, for example, Gaussian fields parameterized by a covariance matrix and models with hidden GMRF components.

The book requires minimal background from the reader. Only very basic probability theory and linear algebra are needed to read the theoretical chapters (Chaps. 2 and 3). The application chapters (Chaps. 4 and 5) assume familiarity with Bayesian hierarchical models and their inference techniques via MCMC. Researchers who are familiar with the authors' recent work may find this book a mere summary. However, for researchers who want to step into GMRFs and graduate students who want to explore research opportunities, this book makes a perfect desktop reference. Enticing graduate students into hierarchical models should be easier with the help of this book. It can be used in a graduate level topics course following some preparation in Bayesian methods or spatial statistics. Students can implement the algorithms in their favorite computing environment and learn how to use the companion software. There are no exercises in this book. The proofs of some theorems can be left as homework problems.

The book is well organized. The four major chapters are sorted in a logical order, from basic theory, computational algorithms, to advanced applications. Some starred sections are advanced and may be skipped. The book is compact, making it easy to extract a feeling of accomplishment and progress. In particular, I appreciated that two topics are *not* covered. One involves details of the reordering techniques from the computer science literature. The authors "recommend leaving the issue of constructing and implementing algorithms for factorizing sparse matrices to the numerical and computer science experts" (p. 52). The other involves details on MCMC, which can be found in many existing books. The compactness distinguishes the book from those which try to be too complete and end up being intimidatingly thick.

The book has a dedicated website (*http://www.math.ntnu.no/~hrue/GMRF-book/*) which contains datasets, software, and other useful materials. The software companion of the book, a C library GMRFlib, is in the public-domain. This library provides functions implementing the algorithms described in the book: unconditional simulation of a GMRF, various types of conditional simulation from a GMRF, evaluation of the corresponding log-density, and generation of block updates in MCMC-algorithms. Following the instructions in the document, I installed the library without difficulty and ran the example code successfully. It would be nice if these facilities were available in a high-level computing environment, such as R, but concerns about speed in typical MCMC applications with GMRF components would likely drive one back to implementing in lower-level compiled codes. I think it is worth investing the time to learn how to use the library, which is made easier by the online document.

In summary, I can recommend this book as a very good graduate level textbook. I also believe that readers will learn much from the GMRFlib software.

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