Gaussian Markov Random Fields: Theory and Applications

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Abstract

This note contains a list of errata and comments received from readers.

Errata

- page 49, second paragraph. "...and $n = 6$ as in Figure 2.7(b)..." [not $n = 7$]

  (Thanks to Ting-Li Su [`ting-li.su|lancaster|ac|uk`])

- page 49, second paragraph. "...that conditioning on node 6 in Figure 2.7(b)..." [not node 7]

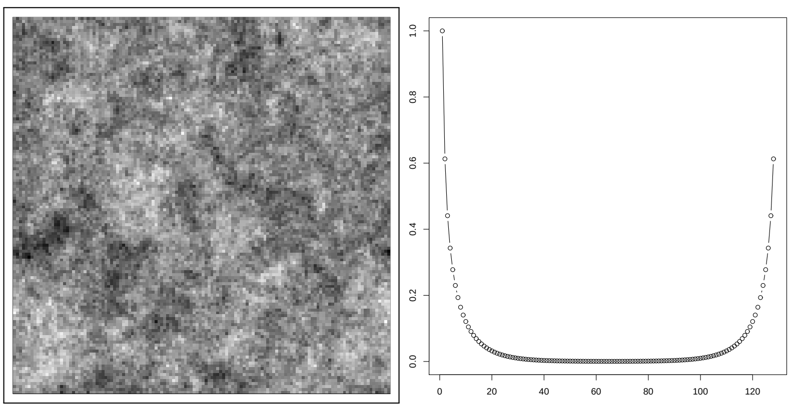
  (Thanks to Ting-Li Su [`ting-li.su|lancaster|ac|uk`])

- page 60, last paragraph. "The matrix $\boldsymbol{F}$ is (2.42) is well *known* as the..." [not *know*]

- page 66-67, the example. There was a bug in my (HR) R-code (sorry....), so the computed correlation matrix (i.e. the base) was wrong, it should be

```
> corr.matrix[1:5,1:5]
             [,1]         [,2]         [,3]         [,4]         [,5]
[1,] 1.0000000000 0.6128251941 0.4409615095 0.3430634723 0.2777151343
[2,] 0.6128251941 0.5082337593 0.4061834934 0.3285039400 0.2702886445
[3,] 0.4409615095 0.4061834934 0.3510965997 0.2977651891 0.2519989972
[4,] 0.3430634723 0.3285039400 0.2977651891 0.2624388713 0.2284247479
[5,] 0.2777151343 0.2702886445 0.2519989972 0.2284247479 0.2036164977
```

  and the sample and the plot in Fig.2.13 should be



  (Thanks to Guy Ruckebusch [`Guyruckebusch@aol.com`])

- page 68, second line of example. "order K" should be "order p". (Thanks to Reinhard Furrer [`reinhard.furrer|math|uzh|ch`].)
- page 90, example 3. The eigen vector e1 should be $-1/2(1,1,1,1)^T$ not $1/2(1,1,1,1)^T$, for the rest of the algebra to follow. (Thanks to Ting-Li Su [`ting-li.su|lancaster|ac|uk`].)
- page 97, second last paragraph. "...realization of an *integrated* Wiener process in continuous time...", should read as "...realization of a Wiener process in continuous time...".
- page 106, Figure caption. "...that the *variance* decreases if the..." [not *variances*].
- page 114, the expression after (3.42) should read

$$\left(\Delta^2_{(1,0)} + \Delta^2_{(0,1)}\right) x_{i,j}$$

and additionally, the Δ^2 operators should be *centered*, meaning that

$$\Delta^2_{(1,0)} x_{i,j} = x_{i+1,j} - 2x_{i,j} + x_{i-1,j}$$

(Thanks to Gang Liu [`gliu|mail|usf|edu`].)

- page 148, line -1: $\kappa_{\mathbf{x}}$ should read as $\kappa_{\mathbf{t}}$. (Thanks to Reinhard Furrer [`reinhard.furrer|math|uzh|ch`].)
- Page 157, middle of the page; In the expression for $g(p)$, it should read $\Phi^{-1}(p)$ instead of $\Phi(p)$. (Thanks to Maria Rogvin [`rogvin|stud|ntnu|no`].)
- Page 170, line -14: $\boldsymbol{x}_0$ should read as $\boldsymbol{\mu}_0$. (Thanks to Reinhard Furrer [`reinhard.furrer|math|uzh|ch`].)
- Page 174, line -5, in the equation: "$, \kappa_{\boldsymbol{v}} \boldsymbol{I})$" should read as "$, \kappa_{\boldsymbol{v}}^{-1} \boldsymbol{I})$". (Thanks to Reinhard Furrer [`reinhard.furrer|math|uzh|ch`].)
- page 176, Figure 4.17. The two figures are swapped; so (a) should be (b), and (b) should be (a). (Thanks to Sara Martino [`martino|math|ntnu|no`].)
- Page 184, "Bockner" should read "Bochner". (Thanks to Reinhard Furrer [`reinhard.furrer|math|uzh|ch`].)
- References. The reference *L. Fahrmeir and S. Lang, Bayesian inference for generalized additive mixed models based on Markov random field priors*, from 2001, was duplicated in the BIBTEX-file, hence appears as two distinct references. This is of course wrong; It's the same article.

Comments

- **Finn Lindgren** [`finn|maths|lth|se`] (Mon, 31 Jan 2005), comment that (3.53) can be constructed using two sets of independent increments,

$$\begin{bmatrix} & a_2 & \\ a_2 & a_1 & a_2 \\ & a_2 & \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} a_4 & & a_4 \\ & a_3 & \\ a_4 & & a_4 \end{bmatrix}$$

where $(a_1, a_2) = (-16/\sqrt{2}, 2\sqrt{2})$ or $(16/\sqrt{2}, -2\sqrt{2})$ and $(a_3, a_4) = (-4, 1)$ or $(4, -1)$. Apart from the scaling factor of 12 this yield the same $\boldsymbol{Q}$ as (3.53). Using two sets of increments, it is not necessary to use a large neighborhood to define the increments for (3.53).
Reply: This is a clever observation!

- **Thomas Kneib** `[Thomas.Kneib|stat|uni-muenchen|de]` (Fri, 14 Jan 2005), comment that the IGMRF following from (3.46) does not give a second-order polynomial IGMRF according to the definition: the rank deficiency is to large.
  Reply: Thomas is right. We should had made this clear in the text and also commented that (3.46) is often called a restriction of an infinite IGMRF to the lattice of interest. The rank deficiency issue can be fixed considering special stencils along the boundary, but this a a quite involved and therefore not included in the book.
  Thomas Kneib (Mon, 14 Feb 2005) made us aware of the work *The Computation of Visible-Surface Representations* by Demetri Terzopoulos, which appeared in *IEEE Transactions on Pattern Analysis and Machine Intelligence*, Volume 10 , Issue 4, July 1988, pp 417 - 438, for which such boundary corrections can be found. A copy is available at book's www-page; see *References and further reading.*
  A function implementing the second order RW2 on lattice with proper boundary corrections, is available in `GMRFLib` from version 2.3; See the page 60 in Thomas Kneibs PhD-thesis A copy is available at book's www-page; see *References and further reading.*

- The subgraph $\mathcal{G}_A$ is often called the *induced subgraph.*

- The "conjecture" stated at the end of Section 2.7.2, is proved in the one-dimensional case by J. Makhoul, *IEEE Transactions on Acoustics, Speech and Signal Processing*, Vol 38, No 3, 1990. A copy is available at book's www-page; see *References and further reading.*

- The coefficients for the second order random walk model for irregular locations in Section 3.4.1, can be computed solving by approximating the numerical solution of the integrated Wiener process. These new coefficients replace those computed with an ad-hoc argument in Section 3.4.1. The coefficients and the derivation of them, are available in the report *A note on the second order random walk model for irregular locations*, by Finn Lindgren and Håvard Rue. A copy is available at book's www-page; see *References and further reading.*

- Efficient computation of marginal variances (and some covariances) for GMRFs is not discussed in the book. This issue is discussed in detail in either the technical report *Marginal variances for Gaussian Markov Random Fields* (Rue, 2005) and will be published as section 2 of *Approximate Inference for Hierarchical GMRF models* (Rue and Martino, 2006). Copies are available at book's www-page; see *References and further reading.*

- The (speculative) discussion on page 214–5 regarding *approximate inference not using simulation* is further discussed in *Approximate Inference for Hierarchical GMRF models* (Rue and Martino, 2005), and the follow up article *Approximate Bayesian Inference for Latent Gaussian Models Using Integrated Nested Laplace Approximations.* Copies are available at book's www-page; see *References and further reading.*

- **Reinhard Furrer**. Page 148, second last paragraph: In my opinion the description of Q_{ss} and Q_{tt} is not optimal. The second term of each matrix "and additional term on the diagonal due to (4.12)" or later "plus a diagonal matrix with $\kappa_{\mathbf{y}}$ on the diagonal" gives the impression that this second matrix can be written as $\kappa_{\mathbf{y}} I_{n+m}$. But the diagonal matrix should have zeros beyond n, because in (4.12) only t_i and s_i, $i = 1, \ldots, n$ are used.

- **Reinhard Furrer** Figure 4.15 (and similarly with Figures 4.17, 4.19): The legend has an inverted color scale.