TMA4195 Mathematical modelling 2005

Exercise set 10

Advice and suggestions: 2005–11–23

Problem 1: (This is just a warm-up for the next exercise.) The *logistic growth equation* for a population $N = N(t)$ is sometimes written as

$$\frac{\dot{N}}{N} = r\frac{S-N}{S}.$$

S is called the *sustainable* population is this simple model. Why? Discuss. What is a reasonable scaling? (Hint: The scaled equation should contain no parameters at all.)

Problem 2: Around year 1980, the following model was suggested for the population of krill (N and whales H) in the Antarctic ocean:

$$\frac{\dot{N}}{N} = r\Big(1-\frac{N}{K}\Big) - aH - F_N$$
$$\frac{\dot{H}}{H} = q\Big(1-\frac{H}{\alpha N}\Big) - F_H$$

Here F_N and F_H are related to (human) harvesting of the two species. Give a reasonable interpretation of these and the other variables.

Assume that krill reproduce much faster than whales ($\varepsilon = q/r \ll 1$). Use the time scale of whale reproduction to rescale the equations, and note the singular nature of the result.

What equilibrium points do you find, and what are their stability properties?

What become of the equations if you set $\varepsilon = 0$? Can you make any sense of this from the point of view of singular perturbation theory?

Discuss the likely consequences of different harvest rates on the two species, as well as on the total amount harvested. Keep on discussing it until you're tired of it or discover nothing new, and then stop.