TMA4195 Mathematical modelling 2005

Exercise set 3

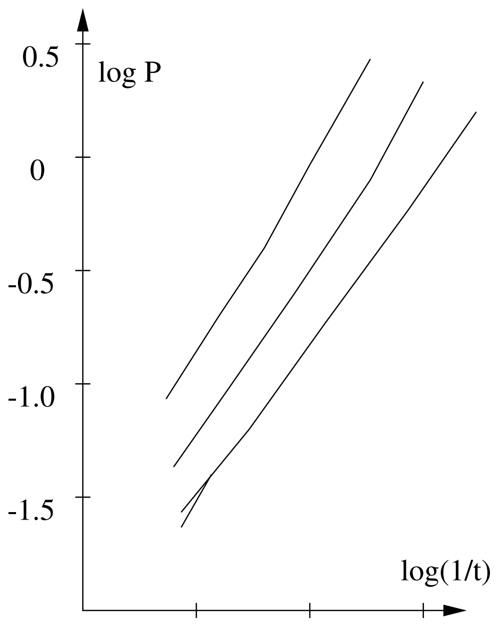
Advice and suggestions: 2005–09–15

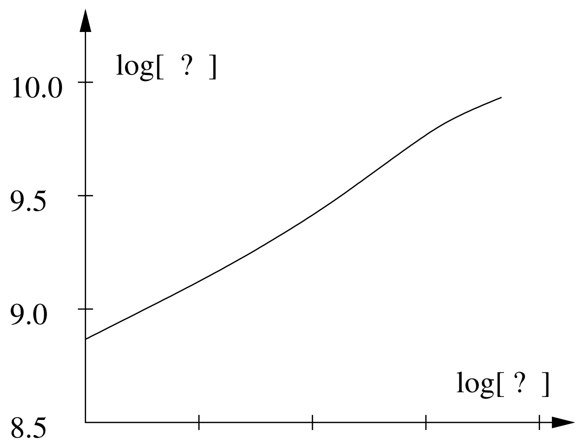
Exercise 1: An open, vertical, cylindrical tank with diameter D is filled with a liquid of density ρ to a height h. The bottom has thickness d and (Young's) modulus of elasticity[1] E. The weight of the liquid causes the bottom of the tank to sag a bit, most in the middle (we assume that the edges of the bottom stay put). Show that the amount of sagging δ in the middle can be expressed as

$$\frac{\delta}{D} = \Phi\Big(\frac{h}{D}, \frac{d}{D}, \frac{E}{Dg\rho}\Big).$$

How can you improve on this result with a small bit of physical insight? (*Hint*: Does the liquid influence the bottom other than through its pressure?)

Exercise 2: By measuring the pressure drop p in the feed pipe against the time t taken to fill a vessel of volume V, BOSE, BOSE and RUERT (around 1910) plotted the relations shown in the lefthand figure for water, chloroform, boroform,[2] and mercury. Show by dimensional analysis (using the density ρ and the viscosity[3] μ) that there should be one common relation that turns these curves into one. That is, find the variables along the axes ofVON KÁRMAN's representation of the same data, as shown on the righthand figure.





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[1] Young's modulus of elasticity has the dimension of a force per area.

[2] No, I don't know what boroform is, and I cannot find it in any dictionary or encyclopaedia. But that is unimportant for our purposes here.

[3] Dynamic viscosity μ has dimensions which can be derived from its definition: A *stress* is a force per area (of which pressure is an example), and shear stresses in a fluid are proportional to the velocity gradient (units: velocity per length). The dynamic viscosity μ is the constant of proportionality.

Exercise 3: (Exam January 1999) Important parameters in the assessment of foods like meat and fish are water and fat content. For use in quality control there is a need for instruments which quickly and reliably can yield an approximate value for these parameters. In particular, it is an advantage for the test to be non-destructive, so that foods can be tested and then sold. One suggested instrument uses the fact that water, fat and proteins have different thermal properties. The idea is to add a known amount of energy in the form of heat to the surface of the food, and then to measure the resulting increase in the temperature. For this instrument to be useful, it should satisfy the following two criteria:

1. The measurement should not take much more than a half minute.
2. The measurement should give a representative average value for water and fat content not only near the surface, but down to a centimeter or deeper.

Approximate values for the specific capacity of heat and heat conductivity for the most important parts of food are:

			Water	Fat	Protein
Heat capacity	c	$\mathrm{J\,kg^{-1}\,K^{-1}}$	4180	2000	1300
Heat conductivity	k	$\mathrm{J\,m^{-1}\,K^{-1}\,s^{-1}}$	0.56	0.09	0.16

You can assume the density to be 1000 $\mathrm{kg/m^3}$ for all three. Employ a simple dimensional analysis technique to assess whether it would seem probable that such an instrument could be made to fulfill the requirements.