TMA4195 Mathematical modelling 2004

Exercise set 8

Advice and suggestions: 2004–10–12

Exercise 1: Show that the system

$$\dot{x} = x + 2y - x(x^4 + y^4),$$
$$\dot{y} = -2x + y - y(x^4 + y^4)$$

has at least one periodic solution.

Exercise 2: (Exam December 1994; somewhat modified.)
The following model has been suggested for the population (P) of moose in Trøndelag:

$$\dot{P} = kP\Big(1 - \frac{P}{M}\Big)\Big(\frac{P}{m} - 1\Big), \qquad 0 < m < M.$$

(a) Which properties of the population does the model attempt to describe, and what are its equilibrium points? Show how linear stability analysis can be used to analyze the stability and to make a qualitative sketch of the solutions.

(b) A simplified model which also includes hunters (J) has the form (after scaling)

$$\dot{P} = P(1 - P) - J,$$
$$\dot{J} = -\tfrac{1}{2}J + JP.$$

What are the (qualitative) properties of this model?
(*Hint*: The system is unchanged by the transformation $P \curvearrowright 1 - P$, $t \curvearrowright -t$. Try to understand what this means for the orbits of the system.)