75032 Matrisemetoder 1996–08–13: Fasit

1a

$$A = LU = \begin{bmatrix} 1 & 0 & 0 & 0 \\ a & 1 & 0 & 0 \\ 0 & a & 1 & 0 \\ 1 & 0 & a & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & a \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

1b En basis for $\mathscr{N}(A)$ består av den ene vektoren

$$\begin{bmatrix} -a+2 \\ 2a-3 \\ -a \\ 1 \end{bmatrix}$$

mens $\mathscr{R}(A)$ har en basis bestående av de første 3 søylene i A.

1c

$$a = 0 \text{ eller } a = \frac{1}{2} \pm \frac{i}{2}\sqrt{3}.$$

2c

$$A = QR = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \\ -1 & -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 2u & 0 \\ 0 & 4 & 2v \\ 0 & 0 & 2 \end{bmatrix}.$$

2d

$$x = \begin{bmatrix} 2-u+uv \\ 1-v \\ 2 \end{bmatrix}.$$

3

$$A = Q\Lambda Q^{\mathrm{T}} = \begin{bmatrix} \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{2}} + \frac{\lambda}{6} & \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{2}} + \frac{\lambda}{6} & \frac{1}{\sqrt{3}} - \frac{\lambda}{3} \\ \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{2}} + \frac{\lambda}{6} & \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{2}} + \frac{\lambda}{6} & \frac{1}{\sqrt{3}} - \frac{\lambda}{3} \\ \frac{1}{\sqrt{3}} - \frac{\lambda}{3} & \frac{1}{\sqrt{3}} - \frac{\lambda}{3} & \frac{1}{\sqrt{3}} + \frac{2\lambda}{3} \end{bmatrix}$$