Analysis of a purported proof of Fermat's last theorem

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Abstract

Many amateurs believe they have an elementary proof of Fermat's last theorem (FLT). Finding such proofs has apparently not been made less attractive by the appearance of Wiles' proof. This note is about one such "proof". The paper [1] has a couple of unusual features for such a proof, perhaps the least interesting of which is the fact that its author is a professor of medicine at the University of Linköping. More interesting is the journal itself. If the reader is left wondering about the quality of a journal capable of accepting the paper [1], rest assured that I wonder as well. The journal in question is one of several journals published by Hadronic Press International, part of the mini empire built up by the physicist R. M. Santilli. Santilli is the originator of a series of theories and grand claims in mathematics and physics, including the so-called iso-, geno- and hypermathematics and (so far) culminating in the discovery of a new form of matter – the so-called magnecules – and a promised solution to the world's energy problems. To the extent that the present analysis throws any light on Santilli's brand of science, it is not exactly flattering.

Introduction. The paper under review is unlike any other mathematics paper I have ever read. It is a disjointed mishmash of simple formulas, trivial manipulations of these, remarks of a general philosophical flavour, references to the history of mathematics, and wild leaps of logic. The author rarely works with identities, preferring instead to work with expressions. Thus it is only by careful attention to the text that the reader is able to keep track of what these expressions signify, or what is claimed to be equal to what. The general vagueness of the text also makes it hard to pin down the exact location of any faulty reasoning.

In this analysis, I have gone out of my way to interpret the paper in the most positive light possible. This has not been easy, and I have of necessity been forced to make some guesses as to exactly what the author may have intended. In this way, I have managed to arrive at a precise fault in the chain of reasoning. However, this should not be interpreted as saying everything is OK if that point is repaired: I firmly believe that no repair of this "proof" is possible, for it really contains nothing of substance no matter how you look at it.

The paper begins by introducing the "representation space of FLT proof". This appears to mean representing the power X^n geometrically by the cube $[0, X]^n$ in n-dimensional space and its natural decomposition into X^n unit n-cubes (called *iso-units*).

As far as I am able to tell, this representation is only used to deduce the identity

$$Z^n - (Z-1)^n = Z^{n-1} + Z^{n-2}(Z-1) + \cdots + Z(Z-1)^{n-2} + (Z-1)^{n-1}$$

by counting unit cubes in the extra layer that needs to be added to the $(Z-1)^n$ cube in order to get the Z^n cube. (Of course, this is just a simple geometric series, easily summable by a standard formula (or by the equally standard remedy of multiplying by $Z-(Z-1)$, expanding, and cancelling terms).

The "proof" of FLT begins with the case $n = 3$, by noting the above expression for $Z^3-(Z-1)^3$ (which the author calls Δ) expands to

$$3Z^2 - 3Z + 1.$$

Next (and now I work harder to express this in a more conventional style) the author adds together the expressions for differences from $Z^3 - (Z-1)^3$ down to $(Z-\lambda+1)^3 - (Z-\lambda)^3$ to get

$$Z^3 - (Z-\lambda)^3 = 3\lambda Z^2 - 3\lambda^2 Z + \lambda^3.$$

The author never states this as an equation, but merely lists the righthand side, calls it Δ, and more or less leaves it to the reader to understand that this is $Z^3-(Z-\lambda)^3$. At this point I must quote the paper: “In order for Δ to rearrange into a cube (here identified as Y^3), fulfilling the $X^3+Y^3=Z^3$ relation, it must have a whole-number cubic root.” My interpretation of this is: Assuming $X^3+Y^3=Z^3$, write $X=Z-\lambda$ and arrive at the equivalent form

$$Z^3-(Z-\lambda)^3=Y^3.$$

The statement, then, is that $\Delta=Z^3-(Z-\lambda)^3$ must have a whole-number cubic root for a counterexample to FLT with $n=3$ to exist. (Clearly a true statement.)

Since Z is fixed and λ is variable at this point, let me improve on the author's notation to make what follows clear, and define the polynomial $\Delta(\lambda)$ by

$$\Delta(\lambda)=Z^3-(Z-\lambda)^3=3\lambda Z^2-3\lambda^2 Z+\lambda^3.$$

The author now notes that $\Delta(Z)=Z^3$ has a whole number cubic root, namely Z itself. I quote again: “Furthermore, standard cube root rules (two irrational and one rational root alone) confirm that these are the only, sequential (and infinitely many) whole-number cube roots of $3\lambda Z^2-3\lambda^2 Z+\lambda^3$. Hence Δ is always only the same as the whole Z^3, from which follows that the relation $X^3+Y^3=Z^3$ reduces to $X^3+Z^3=Z^3$, meaning that X^3 is zero, which proves FLT for $n=3$.”

As far as I can tell, the argument is this: Because $\Delta(Z)$ has only the rational cube root Z, and because no number has more than one rational cube root, $\Delta(\lambda)$ never has a rational cube root other than Z, no matter the value of λ.

This is, of course, completely fallacious, and nowhere near a proof of FLT for $n=3$. The fallacy is that of passing from the special ($\Delta(Z)$ has only the whole-number cubic root Z) to the general ($\Delta(\lambda)$ has only the whole-number cubic root Z, for any λ). Also, as far as I can tell, if the argument did hold water it should work equally well for $n=2$, thus disproving such identities as $3^2+4^2=5^2$.

Essentially the same argument is then repeated with $n=4$, 5, 6, 7 and 8 – with the only difference being that in the even cases, there are two root $\pm Z$.

I must also admit that I cannot see where the author uses the expansions of the polynomial $\Delta(\lambda)=Z^n-(Z-\lambda)^n$, that he so laboriously computes for $n=3,\dots,8$. In all cases, the essential argument boils down to what I describe above, which does not in any way depend on the exact expression for $\Delta(\lambda)$.

The rest of the paper deals with Beal's conjecture, which states that whenever a, b, c, r, s, t are positive integers with $r,s,t\geq 3$, and $a^r+b^s=c^t$, then a, b and c must have a common prime factor. It is not hard to show that this implies FLT, but the author of [1] claims that FLT implies Beal's conjecture.

A brief look reveals nothing of possible value in this part either. I do not have the stomach to try a detailed analysis at present.

Note: The only difference between this version and the one originally posted on 2003–01–22 is the correction of a few misprints.

References

[1] E. Trell. Isotopic proof and re-proof of Fermat's last theorem verifying Beal's conjecture. *Algebras Groups and Geometries* **15**, 299–318 (1998).