Reproof of Fermat´s Last Theorem and Beal´s Conjecture: A Re-replication

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It has now been a month without disapproval of our Fermat's Last Theorem (FLT) proof replication on this site (www.math.ntnu.no/~hanche/blog/trell/). However, there has also been no retraction of the rather devastating denouncement offered by Harald Hanche-Olsen (HHO) (idem) on the deliberately somewhat idiomatic paper (Trell [1998]) under attack: "It is a disjointed mishmash of simple formulas, trivial manipulations of these, remarks of general philosophical flavour, references to the history of mathematics, and wild leaps of logic"….."The rest of the paper deals with Beal's conjecture"…"The author claims that FLT implies Beal's conjecture"…."A brief look reveals nothing of possible value in this part either".

Big words from a big M. A poor veteran scholar of panepistemic inclination, well, even a member of the British Association for the Philosophy of Science should shiver under the truly cosmic censorship - and the guilt by purest encyclopaedic association must be equally abysmal. Surely, we have reason to realise the concluding lines in our previous communication: "We feel compelled, therefore, to return in a forthcoming chapter by a demonstration that in effect we use the most original representatives and methods of genuine Diophantine whole-numbers and their operations well up to the time of Pierre de Fermat - and still going strong. And since intelligible words failed HHO altogether at the third-line, probably most akin to Fermat, reproof stage but yet brought defamation into it, we will have reason to come back to that also, for which the present channel has been officially designated by HHO.s extraordinary inauguration; the more the poorer his conduct regrettably is."

So; now and then. "Remarks of general philosophical flavour, references to the history of mathematics". Fine, that's the scope. Back to the future, Deus ex Machina, Eratosthenes' sieve…what's the problem? For it is all here again. Davies, amongst others, has reported “in some detail how to build an infinite machine within a continuous Newtonian Universe” [2001]. This essentially advances the Euclidean Universe and it is therefore of relevance to complement the latter-day “Platonist-Intuitionist debate about the nature of mathematics” (Davies [2001]) with a ‘Platonist-Institutionist’ description of how then the natural progenitors truly conceived and used numbers as veritable bricks in a productive synthesis by virtual self-assembly (Ikkala and ten Brinke, Kato, Whitesides and Grzybowski [all 2002]) of direct mathematical and physical space alike. Why differential calculus so entirely entered the Zeno paradox retreat - or parenthesis - of the last few centuries poses a fascinating epistemology per se (Trell [2003]), but will be omitted here since it has little to do with the prototype number practice (rather than number theory) which comprises the genuine heritage of the subject matter.

And since Davies and the type of infinite machine he promotes appear under Platonist label, it must be important as the true epistemology in kind to reintroduce what Platon and his contemporaries and disciples in fact thought and taught on the subject. In other words, Davie's seemingly provisional "Platonist intuitionist" position may need to be complemented by a "Scientific Realist" (Kukla[1998]) institution of the protagonists' actual posits on "the nature of mathematics" (Davies[2001]) as well as real realised space. Because they did not see substantial difference between matter and mathematics, between numbers and things. As Noel [1985] has expressed it: “the old Greek are famous for a completely brilliant idea, namely, to use spatial images to represent numbers”, where, notably, “Euclid’s mathematics was closely associated with his concept of the world, which in accordance with Aristotle was that the Universe was enclosed in a sphere, in the interior of which space and the bodies full-filled the properties of Euclidean Geometry".

In the current nanotechnological era there is a strong Renaissance in that direction (Winterberg [2000]) but partially incoherent with its sources. When it is concluded that "Plato would have insisted that God created triangles, out of which the Universe is made" while "Platonists of the early 21st century may insist that what God created were mathematical objects, called superstrings, out of which the world is made" (Fraser [2001]) this is in vital respects a deviation from both the archetypes and the prototypes at hand. Strings are curved but Plato (in *Timaios*) primarily reserved the spherical harmonies for the celestial rather than the terrestrial symmetries. For the latter he employed the (per se already well known) regular polyhedra "developed from the unit sphere" (Sutton [2001]) and in consequence lines up more with those today who again argue that in the dualistic interplay "between the curved and the straight" which is at "the heart of Greek geometry and indeed of geometry in general" (Netz [2002]), it is the rectilinear 'canvas' (Kamionkowski [2002]) that provides the flat screen (Bachall et al. [1999], Rees [2000]) of our physical realisation.

And the Platonic solid originally designated for this equally mathematical as material matrix was the cube, "completely filling the space with copies of itself" (Sutton [2002]). Triangles were engaged at many levels, but when it comes to their role as elementary constituents, the involved "triangular part is a diagonally divided quadrate, four of which recreate the whole square, which then form cubes" (Sutton [2002]).

What Plato really insisted is therefore that what God created, or actually "folded from planar substrates" (Whitesides and Grzybowski [2002]), were uniform cubes, out of the atomic clone of which geometric Earth and Ether are made. And this was the general idea of the age since time immemorial, including the consequential numerical bearings. For instance, the geometry that Euclid learnt from his Ionian teachers "was originally based on watching how people built" , and "the measurement of volume by the number of cubes with sides of standard length required to fill a solid space was probably first used by the Sumerians, who built with bricks" (Hogben [1937]).

How did the building proceed? There are at least two main continuous alternatives, one of which has been brought to the fore again both theoretically by e.g. Roger Penrose [1995] and in the recent nanotechnological "layer-by-layer" material self-aggregation and self-organization (Velikov et al [2002]). It can be described as a stepwise eccentric winding over the surface of the expanding box and was used in the previous replication to verbatim underpin a proof of FLT (www.math.ntnu.no/~hanche/blog/trell/).

However, the other, and most straightforward and practically manageable, is to first pave the floor, starting by a row from a corner along the side, after that turning for the next row, and so on till the ground square or rectangle is filled. Then, with unbroken succession in reverse order in the next tier, and so on, till the box is filled in a hence really analytical way, i.e. continuous, spacefilling and non-overcrossing. This mode would probably be closest at hand for Diophantos as well as for Pierre de Fermat, and will be focused upon in the continuation.

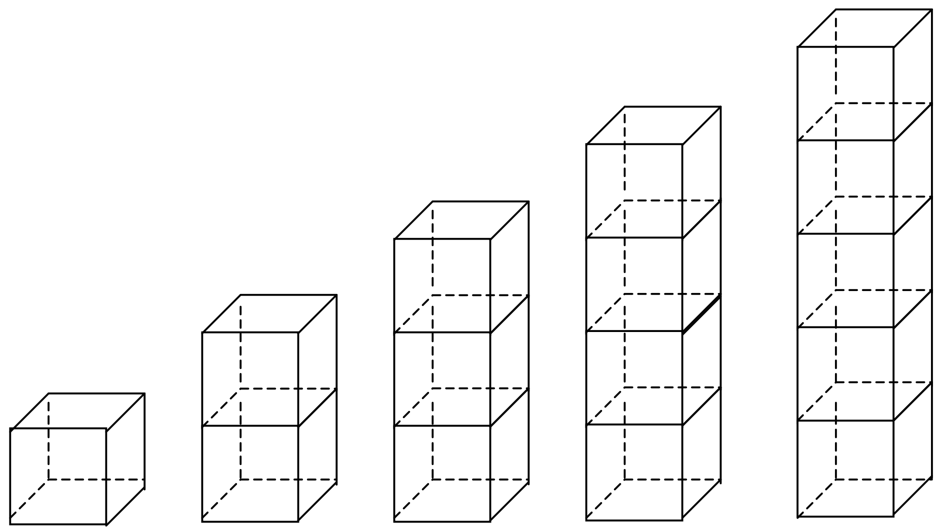
For it is important, that the comparative late Diophantos himself "stated the traditional definition of numbers to be a collection of units" when in his equations they "were simply put down without the use of a symbol" (Heath [1964], www [1997]). The effective quantum leap in relation to modern linear functions is of course the integer instead of point nature of the numerical unit. And pointless, too, would be to make this a heuristic controversy since it is all about reality: reality for the founders, reality of means and ends; reality of the very facts and findings of the case, i.e., that when ancient mathematicians well up to Cardano calibrated numerical and physical space alike they used what during thousands of years between the Sumerian bricks and Roman *tessellas**

* Oxford Concise Etymological Dictionary of the English Language: *Tessella* is Latin for little cube, diminutive of *tessera* = a die (to play with), a small cube. *Tile, tiling* are derived from another Latin word, *tegula*.

was the most refined of manufactured self-assembling forms: the cube, the irreducible whole-number bit, **One**, a cubicle, cubus, kaba, of arbitrary unit side, providing the atomic set of a myriad literal dice not alone for God to throw but for themselves to stow by cumulative fulfilment of their own properties (Noel [1985], Sutton [2002]).

In order to reconstruct the original procedure, it may be reminded that gauging and calculations in those days were much like surveying (Noel [1985]). For the first degree, *positio* alignment, the unit number cells then automatically deliver the measuring-rod by longitudinal plus or minus stacking like in the contemporary *abacus* over a single axis, here illustrated as the vertical (Fig. 1).

xyz = 1 xyz = 2 xyz = 3 xyz = 4 xyz = 5 ....



<u>Fig. 1.</u> Three-dimensional Diophantine whole-number cells (or, after Penrose [1995], polyominoes), one-dimensionally joined together in the arbitrary vertical direction to infinite series of integers of the first degree by the same discrete amount of the ground unit cubicle.

However, the added, in a double sense manifold value of the direct spatial realisation of whole numbers does not become apparent until with Diophantos formalising their exponentiations and subsequent equations. The natural procedure that offers for a serial power expansion is a sideways instead of length-wise multiplication of the digit by itself, producing at the second degree stage a square tile, step-by-step like the Sumerians did till the quadrate or rectangle is continuously and non-overcrossingly tessellated (Fig. 2). Then, in the same fashion, next layer is filled, and next, and next, till the resulting first-order third degree 'hypercube' is also analytically completed (Fig. 2).

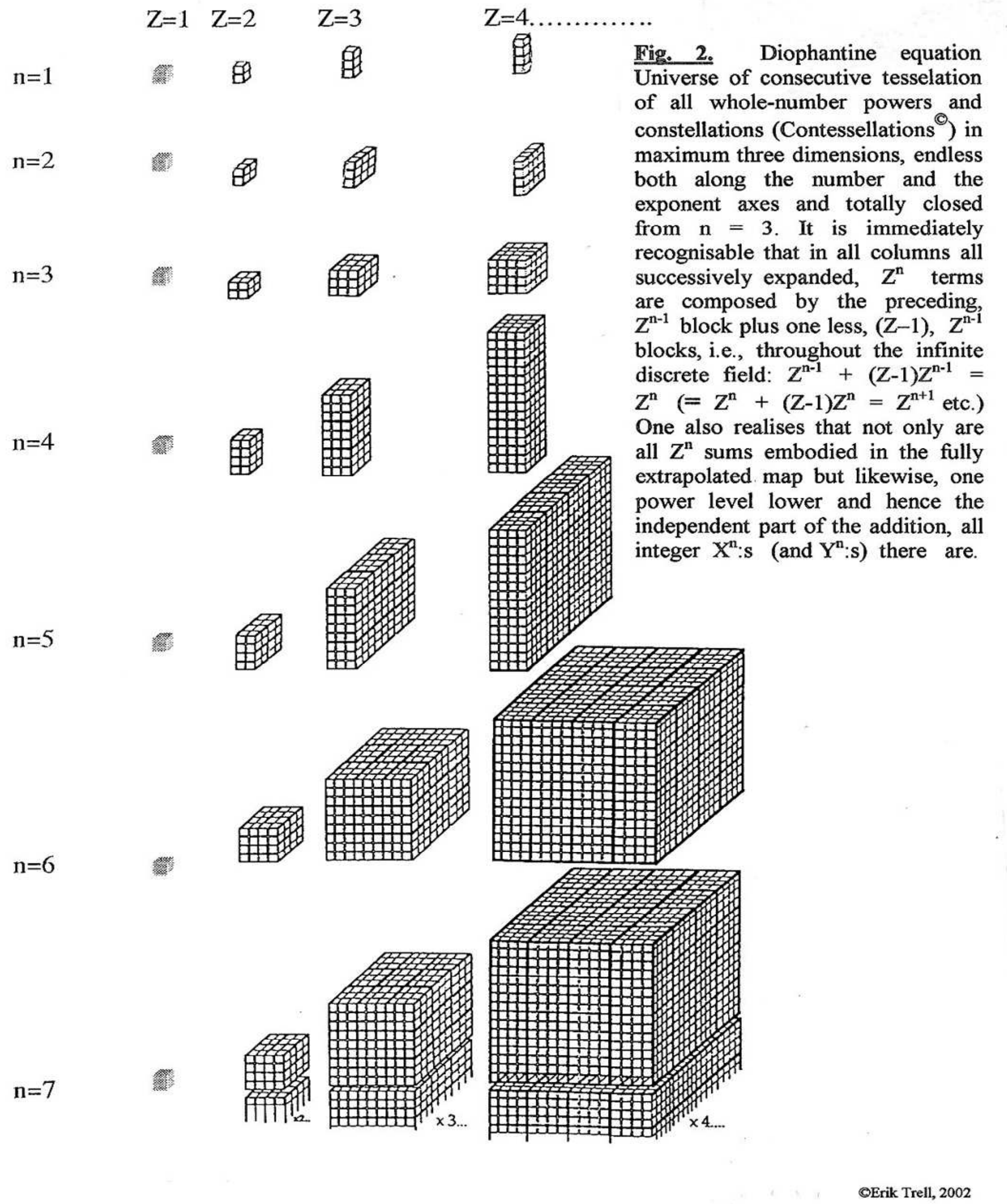


Fig. 2. Diophantine equation Universe of consecutive tesselation of all whole-number powers and constellations (Contessellations©) in maximum three dimensions, endless both along the number and the exponent axes and totally closed from n = 3. It is immediately recognisable that in all columns all successively expanded, Z^n terms are composed by the preceding, Z^{n-1} block plus one less, (Z–1), Z^{n-1} blocks, i.e., throughout the infinite discrete field: $Z^{n-1} + (Z-1)Z^{n-1} = Z^n$ ($= Z^n + (Z-1)Z^n = Z^{n+1}$ etc.) One also realises that not only are all Z^n sums embodied in the fully extrapolated map but likewise, one power level lower and hence the independent part of the addition, all integer X^n:s (and Y^n:s) there are.

In turn, that 'hypercube of the first order' in same periodic progression re-multiplied by the base number yields a 4^{th} power in the shape of a quasi-one-dimensional 'hyper-rod of the second order', which in forthcoming multiplications generates a 5^{th} degree second order hypersquare, then 6^{th} degree hypercube, then 7^{th} degree hyperrod, 8^{th} degree hypersquare etc. in an endless cyclical "self-assembly at all scales" (Whitesides and Grzybowski [2002]) that eventually contains all whole-number (and fractional) powers that there at all are (Fig. 2). It is important to re-emphasise that the build is successive also within each sheet by the zigzag lining up of the individual tessellas so that they never clash.

The entire Diophantine equation Block Universe is thus generated by a recursive, perpendicularly revolving algorithm in a maximum of three dimensions, thereby reproducing the hierarchically retarded, non-overcrossing, i.e. analytical space-filling of consecutively larger constellations, imaginable up to the size and of twist of galaxies, no matter if taking place during actual time or an instantaneous phase transition in the sufficient ordinary Cartesian co-ordinate frame. A continuous “rod-coil-rod…..self-assembly of phase-segregated crystal structures” (Kato [2002]) - which “in turn form assemblies or self-organize, possibly even forming hierarchies” (Ikkala and ten Brinke [2002]) - precipitates in a completely saturating, consecutively substrate-consuming way, displacing other stepwise cumulative syntheses (Fig. 2). This is of utmost relevance, since, with bearing to and like Fermat´s Last Theorem (FLT), “far from being some unimportant curiosity in number theory it is in fact related to fundamental properties of space” (www [1997]). And the geometrical uniformity, that all whole-number powers from n = 3 and infinitely onwards are realised in sufficiently three dimensions as saturated regular parallelepipeds which per primordial definition are composed by integer blocks alone, is of equal cardinal importance for the demonstrations *ad modum* Cardano to be exposed in the continuation.

That the (Western) situation was essentially the same up to the days of Cardano and hence also current for Fermat is namely another undeniable mathematical and philosophical fact, as most clearly demonstrated by the former in his Ars Magna [1545]. Quoted from Karen Hunger Parshall [1988]; "For quadratic equations, Cardano, like his ancestors, built squares, but for third degree equations, he constructed cubes". He concluded "that only those problems which described some aspect of three-dimensional space were real and true. In his words: *"For as positio [the first power of the unknown] refers to a line, guadratum [the square of the unknown] to a surface, and cubum [the unknown cubed] to a solid body it would be very foolish for us to go beyond this point. Nature does not permit it"*" (Ib.).

That indeed Nature does not allow a truly analytic (that is, continuous, space-filling and non-overcrossing) simultaneous physical distribution over more than three linearly independent dimensions had been shown already by Aristotle, and so was the state of the art also for Fermat, when in the exclaimed (but unexplained) *demonstrationem mirabilem* in 1637 of his last theorem he manipulated plain *"cubos"* in equal *en bloc* manner without the use of algebraic symbols (www [1997]).

But whereas Cardano "was unable to conceive of….a four-dimensional figure" geometrically (Hunger Parshall [1988]), this, and its continuation may well have been that instant flash of insight for the one century younger Fermat mind: just perpetuating the identified row-rectangle-octagon cycle to ensuing powers by the same undulating iteration and reiteration of the ground unit cube which comprised the genuine whole-number atom of the still prevailing protagonist era. The consequences would have been immediately recognised, too, for Fermat, but why he did not pass on the veritable blockbuster remains as an enigma. Perhaps he did not want to destroy future number theory fun, or it was just an act of that cryptic jeopardy game which seems to have been going on in the esoteric circles when mathematics was often a jealously protected secrecy.

While the previously replicated proof of FLT follows the horizontal axis of Fig. 2, the reproof engages the vertical. Thus considering the stepwise growth of each number for every new power, it is clearly an ascending differential function, too, and as such exhaustive, that is, filling and so occupying the whole space by its continuous iteration. As demonstrated in Fig. 2, the second degree corresponds to a two-dimensional square in the arbitrary z direction by adding to the one-dimensional number column, X^1 (= X), one less further such columns: $X + (X-1)X = X^2$. The ensuing stage is equally straightforward – and straight-angular. It is a periodical twisting, or unwinding of the space, where the third degree in like manner is entered along the x axis by the continued zigzag addition of (X-1) X^2 planes: $X^2 + (X-1)X^2 = X^3$.

And so it continues. Focusing on the stepwise growth of the exponents of all separate integers, FLT and the latter-day progeny called Beal's Conjecture (BC) can be proved, too, by this complementary "dynamical evolution of our toy model universe" (Penrose [1995]), which will here be performed in algebraic notation. Expressed in the forefather FLT designation, BC states that all possible whole-number power, $X^n + Y^m = Z^p$, additions must share an irreducible prime factor in all its terms (Mauldin [1997-], Mackenzie [1997]).

From what has been said earlier and by extrapolation from Fig. 2, it can be observed that all manifold blocks grow from the preceding one in the same column by adding upon this one less of the same than its base number:

$$X^n + (X-1)X^n = X^{n+1}$$

This borders to trivial but has profound bearings and consequences, notably in regard of the prevailing X = integer requisite. First, it is a universal relation; All X^n.s are represented, both by the first summand term and by the sum one step up (or successively higher by the relations $X^n + (X^2-1)X^n = X^{n+2}$ and, with non-integer roots of the multiplicative coefficient, $X^n + (X^3-1)X^n = X^{n+3}$, $X^n + (X^4-1)X^n = X^{n+4}$ etc. ad infinitum, according to the general formula, $X^n + (X^p-1)X^n = X^{n+p}$, where the specific case, p = n or multiples thereof, is excluded from integer solutions since when by definition X^n has a whole-number n:th root, (X^{n-1}) cannot have one).

It strikingly reminds of the actual world where three dimensions likewise are the most in which a continuous physical realisation can be simultaneously distributed in a non-overcrossing and space-filling, that is, analytical order. Already Aristotle deduced that with additional extensions the geodesics will get entangled by their equally higher-dimensional co-ordinate points no longer being able to avoid colliding with each other within one and the same static compartment.

Also by observations on the own free mobility in experienced space but fixed transport in time he reached conclusions akin to modern expressions like that ”invariant...orthogonal transformation of co-ordinates” can lastingly keep clear of obliterating themselves in a given neighbourhood over at the most three linearly independent axes so that when ”in the theory of relativity, space and time co-ordinates appear on the same footing”, the corresponding Lie algebra, or 4x4 matrix ”inhomogeneous Lorentz transformations” must contain a ”translational part” (Carmeli [1977]). The latter is here offered, too, as the perpetual way out from the final cubicle recess in a filled power box to the next.

The principal condition is that all X^n.s are regenerated in the Z sum one power higher whereas the Y term is a full member only when its (X-1) or (X^p-1) multiplicator has an integer n:th root - and when not can still be retrieved and mobilised as a discrete factor subset within the sum block. Then, one starts to realise that $X^n + (X-1)X^n = X^{n+1}$ (etc.) is also the unique, i.e., the only possible non-overlapping or non-gapping binary $n \geq 3$ manifold tessellation in the entire whole-number $n > 2$ exponential space, which naturally verifies FLT by exclusion and the secondary BC by the inclusion in all terms of the common irreducible prime factor in X.

This is best mathematically expressed by the regular differential chain equation:

$$X^1 + (X-1)X^1 + (X-1)X^2 + (X-1)X^3 + (X-1)X^4 \dots + (X-1)X^{n-1} = X^n$$

Which can be further generalised to

$$X^p + (X\text{-}1)X^p + (X\text{-}1)X^{p+1} + (X\text{-}1)X^{p+2} + (X\text{-}1)X^{p+3} \ldots + (X\text{-}1)X^{p+(n-1)} = X^{p+n},$$

hence providing a formal mathematical proof of the uniqueness of the ascending differential function by its "layer-by-layer…complete close-packed" (Velikov et al. [2002]) continuous iteration gradually sweeping over and so covering the entire Diophantine equation space. FLT and BC are demonstrated in the passing since all $integer^n$ + $integer^n$ additions in the exhaustive set yield $integer^{(\geq)n+1}$ sums, and the mutual X term obviously shares irreducible prime with itself.

Yet it may be of interest to illustrate the situation more expressively. The *sine qua non* of FLT and BC is the pure integer requirement of all the X, Y, Z base numbers, i.e. that the simultaneous 'external' coefficient of their X^n, Y^m and Z^p terms = 1. It is a binary splicing already at the outset absorbing all ligands in the solution by their mutual double-bonds and thus works like a global Eratosthenes' sieve (Noel [1985]), filtering the space from ascending 1^n, 2^n, 3^n, 4^n …..$(\infty\text{-}1)^n$ X exponential series so that the horizon for lower base number power inclusions is gradually pushed up precisely out of reach.

This goes over all magnitudes of n, even n = 1, because, for instance, 2 can only be combined with 4 to form 6. However, in that power it is an unbound relation since 2 can be combined with endlessly many other integers to form endlessly other integer sums, all members of the X^1 subset. When the power of the sum is 2, the situation is the same because it is formed by two basically first-degree terms; $X^1 + (X\text{-}1)X^1 = X^2$, and X^2 can thus be added together by other first-degree terms which might even be squares. But from $X^2 + (X\text{-}1)X^2 = X^3$ and onwards the relation is locked in all its members; the first term X^2 piece exactly and exclusively determining also the unique missing second degree quantitative fraction delivered to the sum member of the common set which exactly and exclusively has to be filled by the missing puzzle piece of the addition.

By such homogeneity of its algorithm, the totality of binary Diophantine additions comprised by the universal $X^n + (X\text{-}1)X^n = X^{n+1}$ (etc.) equation technically forms a folded but wholly even and dense X^n membrane, or 'n-brane', which, at all its points, by a mathematically equally constant, fixed and unbroken gear of itself lifts itself to the next level of itself. The totality elevates to the totality, in just one and the shortest rise, between one floor and the next, all monolayer shafts in the single interstice filled to the last unit corner, doubly obstructing other manoeuvres. In consequence, FLT and BC are proved by the effective displacement of other, necessarily higher solutions, by the gradual occupation from the bottom of all lowest solutions with the universal X^n as first term.

From the whole-number condition of the second term it is possible to regenerate <u>all</u> FLT and BC additions, most transparently by reformulating the equation to:

$$(X^n+1)^n + X^n(X^n+1)^n = (X^n+1)^n + [(X^n)^{-n} (X^n +1)]^n = (X^n+1)^{n+1},$$

This is clearly in ground level exponential state as shown when posed as

$$1^{x} (X^n+1)^n + 1^{x} X^n(X^n +1)^n = 1^{x} (X^n+1)^{n+1},$$

and is accordingly unique already because of one rational solution alone to equations with all base terms of degree 2 and over. It is easy to exemplify for any X^n, e.g. 5^{13} = 1220703125, when the coupled equation becomes:

$$(1220703126)^{13} + (1220703125) \times (1220703126)^{13} = (1220703126)^{14},$$
that is, $(1220703126)^{13} + [5(1220703126)]^{13} = (1220703126)^{14}$,

and indeed for any magnitude , for instance when $X^n = 12345^{6789}$:

$$(12345^{6789}+1)^{6789} + (12345^{6789}) \times (12345^{6789} + 1)^{6789} = (12345^{6789}+1)^{6790} =$$
$$(12345^{6789}+1)^{6789} + [(12345)(12345^{6789} + 1)]^{6789} = (12345^{6789}+1)^{6790}.$$

This virtual extraction of all second terms can be systematised by the variety of Eratosthenes' sieve that Davies suggested [2001], viz. first let it (the infinite machine) "solve the problem for *n* = 1"; then it "passes to….*n* = 2" and so on "down the chain". So let us start doing it with, in our notation,
X = 1:

for 1^1 : $(1+1)^1 + [(1^1)^{-1}(1^1+1)]^1 = (2)^1 + (1 \times 2)^1 = (2)^2$;
for 1^2 : $(1+1)^2 + [(1^2)^{-2}(1^2+1)]^2 = (2)^2 + (1 \times 2)^2 = (2)^3$;
for 1^3 : $(1+1)^3 + [(1^3)^{-3}(1^3+1)]^3 = (2)^3 + (1 \times 2)^3 = (2)^4$;
for 1^4 : $(1+1)^4 + [(1^4)^{-4}(1^4+1)]^4 = (2)^4 + (1 \times 2)^4 = (2)^5$;
for 1^5 : $(1+1)^5 + [(1^5)^{-5}(1^5+1)]^5 = (2)^5 + (1 \times 2)^5 = (2)^6$;
etc. till $n = (\infty - 1)$;

And X = 2

for 2^1 : $(2+1)^1 + [(2^1)^{-1}(2+1)]^1 = (3)^1 + (2 \times 3)^1 = (3)^2$;
for 2^2 : $(4+1)^2 + [(2^2)^{-2}(4+1)]^2 = (5)^2 + (2 \times 5)^2 = (5)^3$;
for 2^3 : $(8+1)^3 + [(2^3)^{-3}(8+1)]^3 = (9)^3 + (2 \times 9)^3 = (9)^4$;
for 2^4 : $(16+1)^4 + [(2^4)^{-4}(16+1)]^4 = (17)^4 + (2 \times 17)^4 = (17)^5$;
for 2^5 : $(32+1)^5 + [(2^5)^{-5}(32+1)]^5 = (33)^5 + (2 \times 33)^5 = (33)^6$;
etc. till $n = (\infty - 1)$;

and X = 3;

$$\text{for } 3^1: (3+1)^1 + [(3^1)^{-1}(3+1)]^1 = (4)^1 + (3 \times 4)^1 = (4)^2 ;$$
$$\text{for } 3^2: (9+1)^2 + [(3^2)^{-2}(9+1)]^2 = (10)^2 + (3 \times 10)^2 = (10)^3 ;$$
$$\text{for } 3^3: (27+1)^3 + [(3^3)^{-3}(27+1)]^3 = (28)^3 + (3 \times 28)^3 = (28)^4 ;$$
$$\text{for } 3^4: (81+1)^4 + [(3^4)^{-4}(81+1)]^4 = (82)^4 + (3 \times 82)^4 = (82)^5 ;$$
$$\text{for } 3^5: (243+1)^5 + [(3^5)^{-5}(243+1)]^5 = (244)^5 + (3 \times 244)^5 = (244)^6 ;$$

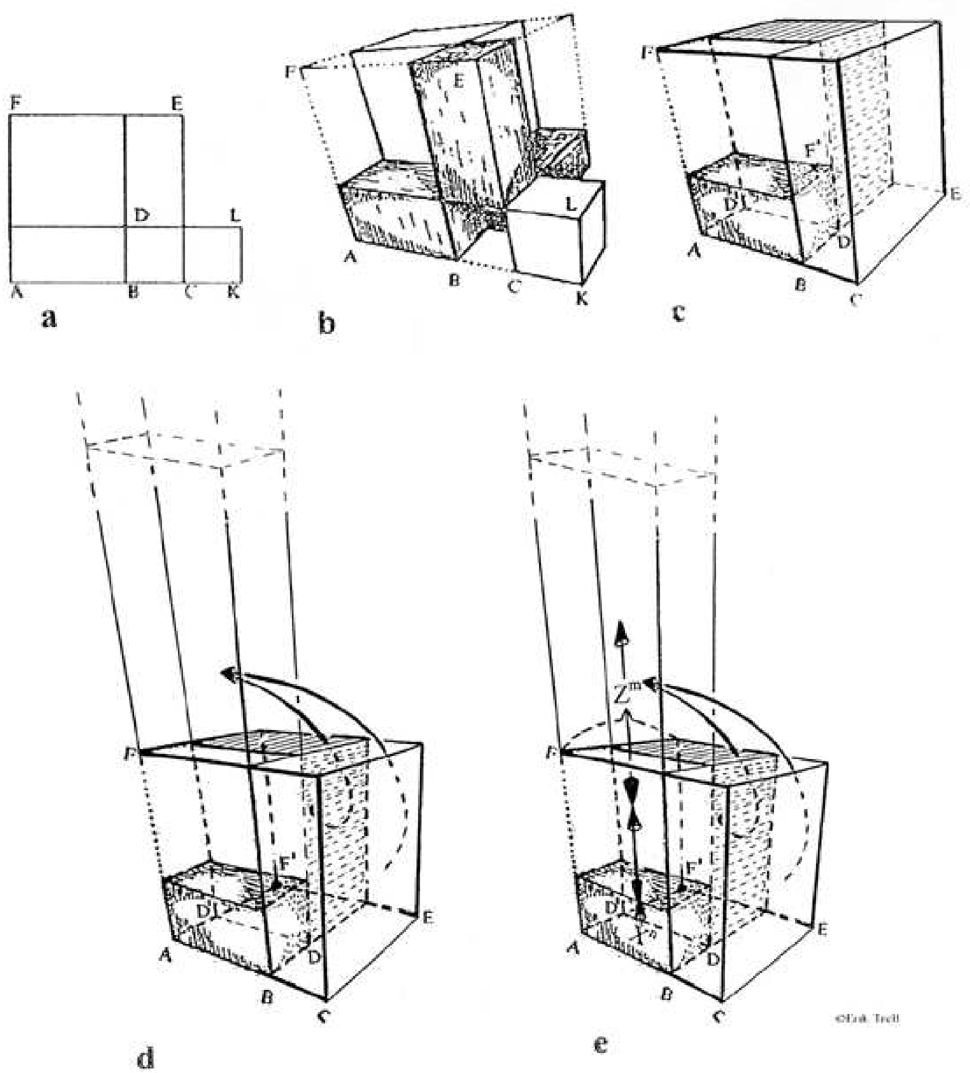
etc. till n = (∞ - 1);

And so it goes on, for every consecutive X and every consecutive n, till both X = (∞ - 1) and n = (∞ - 1) , and hence, for every whole-number X^n introjected in the second term there is but one pure FLT/BC equation where all terms are ground whole-number powers, i.e., in the irreducible form with all external coefficients = 1 (that, for instance $(82)^4 + (3 \times 82)^4 = (82)^5$ can be expressed as e.g. $(6724)^2 + (60516)^2 = (37073984321)^1$ does not alter that), screening off other solutions. Because the equation thus drains the whole space of binary additions of whole-number powers it also proves both FLT and BC since (here stated in most general form) $(X^n+1)^n + [(X^n)^{-n} (X^n+1)]^n = (X^n+1)^{n+1}$ excludes n.th power sums (FLT), and the mutual (X^n+1) shares least prime factor (BC).

In conclusion, what has been quite extensively done here is a "brute force" exposition that every discrete X, Y and Z power can be explicitly retrieved by a simple, but universal numerical formula. As so much brute force it is actually superfluous because contemplating that the entire whole-number Diophantine Equation Block Universe chart of Fig. 2 and its infinite extrapolation really comprises each and every separate element in its total space and that each and every of them likewise has a specific numeric formula of its solitary constitution, it is indeed almost a truism that there cannot be more formulas either so that both FLT and BC instantly follow.

However, as "in the natural selection of ideas, the standard of rigorous, geometrical demonstration applied to algebraic fact which Cardano adopted represented a favourable variation in the theory of algebra" (Parshall [1988]), the proof can be given a more distinct infinite machine execution in sole geometrical terms. In fact, it is then possible to build directly upon Cardanos solution of the third degree equation, which he performed by cubes, in which category the rectangular parallelepipeds were also included (Fig 3 a-e).

There, like "Cardano asked his readers to complete the cubes formed on AB + BC, in just the same way the Arabs or Leonardo completed the square" (Parshall [1988]), individual parallelepipeds will be accordingly manufactured by a virtual 'manifold extruder', recreating what is here regarded as the at least fully plausible prior art.



<u>Fig. 3.</u> **a)** "Bottom plate" of Cardano's cubic solution of the general third-degree equation. **b)** Cardanos graph of cubical completion of same. **c)** Present extraction of cornered X^n inclusion in Z^m = ACEF' cube with identification of side, back and roof difference slabs amounting to Y^p in FLT/BC addition. **d)** Injection of side and back slabs on top of roof slab of same irreducible crossection area as X^n. **e)** This means that Y^p is a multiple of X^n, hence sharing prime factor. And since $X^n + Y^p$ is whole Z^m, this must be a multiple of X^n, too, hence sharing prime factor as well, but cannot be of n:th degree when $Y^{p(=n)}$ is, because the Y number is occupied at the n:th degree where X^n is too small to fill the gap to next higher, Z number to be lifted to n:th power.

Recalling the vertical growth of power levels in Fig. 2, i.e. the geometrical "shafts" of manifold increment, what singular engine is it that Fig. 3 by direct filling out the shaded form from the living past outlines? In principle its infinite machine action is akin to the way Archimedes measured volume by displacement from the outset basin, and it could accordingly be called an one-stroke parallelepiped re-assembler, which beyond this slightly peculiar designation is precisely what it perpetually does. When an arbitrary whole-number parallelepiped is cornered from the origin within a larger arbitrary whole-number parallelepiped, the difference between the two forms one side and one back and one roof slab eccentrically enveloping the inclusion body. Only if they can be combined to a full whole-number parallelepiped, too, the basic requirements of a FLT/BC equation are fulfilled, the further, severely restricting provision of which is that all the parallelepipeds are whole-number powers.

However, we don´t need to worry about this at the moment, but analyse the general Archimedean communicating vessel premises established. It is obvious that any continuous permutation of net marginal volume, be it of the side or back slab alone or of both, around the combined inclusion body and its roof slab can be transferred as a likewise continuous multiple of and over the cross-section of their mutual column (Fig. 3c,d), where the occasional constellation that all tributaries thereby are whole numbers, let alone whole-number powers (Fig. 3e), are but rare and extremely rare special cases, respectively. Since the specific whole-number coupling is established by the inclusion body fully defined as X^n and its cross-section therefore is a multiple of X, this applies to all sections of the Archimedean channel above it, so that already at this stage it can be determined that they must share least common prime denominator as well. Hence the proper genealogy of the BC baby comes out in the preliminaries as drained in the FLT bath-water.

In the further filtering of this also FLT follows from a mere consideration of its hypothetical stipulations, explicitly stating that the second term, Y, must likewise be of n:th degree. In practice, the potential second term candidates are therefore all n:th root whole-number multiples of X^n. Combined with X^n, this gives rise to the Z^m sum (Fig. 3e), which obviously is a multiple of X, too. However, since all successive whole-number multiples of X^n are already occupied by the $Y^{p(=n)}$ term, the Z^m sum is always pushed up to the nearest higher dignity if at all a whole-number power itself, thus proving FLT. From this, the unique formula, $X^n + (X-1)X^n = X^{n+1}$, and a whole lot more, can be derived the other way around, but this goes beyond the scope of the present communication, the main objective of which is to show that the proper spelling-out of the FLT acronym should righteously be Fermat's Last Triumph. Contrary to the assertion that "the problem may require a brand-new approach that would not only re-prove the Fermat theorem but a whole lot more" (Mackenzie [1997]), the old-brand directions may still yield even better.

Indeed, the roots are branching out again. Although HHO had difficulties in digesting literary flavour as well as philosophical essence, perhaps now the many quotations here might convince of their belonging to visionary Mathematics from when this was a mother discipline of all Science. There can be no fault in drawing from the well-springs nor in attempting a certain eloquence in the presentation. And the original compass of Philosophy is pan-epistemology, that is, profoundly including pragmatic Natural History as well. In order to preserve this ideal condition (and to curb any perennial cycle of ecclesiastic retreat to self-referential ivory towers), some refreshed life-blood injection from the left side of the heart should be warmly received. It is of course even more healthy and revitalising when bringing, the simpler the better, a verifiable portion of Science's finest essence, namely, Truth, also as reproducible objective data, in the authoritative Wittgenstein understanding that “the world is the totality of facts” (Hossack [2000]). What better provenance of universal philosophy and mathematics can moreover be recruited than their very founding fathers which are here verbatim represented?

And there is ample additional legacy, one of the more famous of which is Hilbert's formalism, i.e., that "mathematics should be regarded as being, at heart, nothing other than a collection of formal games, each one played according to completely specified rules" (Devlin [2002]). Another, equally much stimulating also the informed 'paramaths' to "playing the Euclidean geometry game in terms of those objects" (idem) is the Scientific Realism stance which holds that "theoretical posits are as real as the tables and chairs" (Kukla [1998]). What then about reproducible Diophantine equation boxes and block Universe? It seems entirely compatible with what Davies perceives about the ideological confession of the Platonist “intuitionists….to what can actually be proved in the real world” [2001].

When he agrees with both Turing and Earman and Norton [1996] that an “infinity machine is defined to be a computer” he aligns with the recent informatics signals that “mathematical research as well as physics and many other fields would benefit from increased emphasis on development of deployable mathematical software and relatively less emphasis on abstract mathematical results” and that “such software can lower the barriers between those who think in ‘practical’ terms and those who think in ‘abstract’ terms” (Petti [1995]).

But he exclusively employs his “machine in a continuous Newtonian universe. By this we mean a universe obeying Newton´s laws in which matter may be subdivided more and more finely while retaining the same properties”. However, it is known that this “infinite descent” approach (which also Fermat fruitlessly entered instead of - perhaps for the suggested reasons - publishing his *mirabilem* demonstration) “would not be tractable in a fractal-like universe of clusters within clusters ad infinitum, such as Carl Charlier envisaged early in the 20th century" (Unruh [2002]). The same thus applies to any machine along that direction trying to "carry out an infinite number of computations within a finite time", either "by performing the individual computations faster and faster", or "design the computer in such a way that the memory can be doubled indefinitely" or/and that "the machine can indeed produce a scaled-down version of itself"; all "in the manner of the Zeno paradox" (Davies [2001]). However, he overlooks one quite relevant aspect of modern computer output which the alternative, ascending construction copes with, namely that of animation, i.e. that "the scientific content in a physical model might in the future be captured in simulation" (Petti [1995]). The omission is paradoxical also because in effect many operations and algorithms he suggests are in principle ascending, for instance the earlier mentioned version of Eratosthenes' sieve.

Similarly he aptly predicts that "our machine depends upon advanced nanotechnology". However, the needed re-consolidation of constitutionally omniscientific Philosophy by its otherwise dissociated Natural History strand is again indicated by the corresponding state of the art that "an essential part of nanotechnology is self-assembly" (Whitesides and Grzybowski [2002]), preferably by binary "layer-by layer growth" including "formation of…superstructure…as a result of the templating effect" of the primary deposition (Velikov et al. [2002]). The congenial scientist would positively respond to such collegial "learning from one another. Different fields of science take different roads to understanding, each bring something to self-assembly" (Whitesides and Grzybowski [2002]), too. When Davies summarises that "neither our machine nor Turing machines can actually be built, because of fundamental properties of the real universe", he is therefore right; and none-the-less the real infinity machine is the real universe, and can be reproduced.

And the present ascending animation, which with the appropriate proportionally factor is applicable to spherical geometry as well, certainly does not convey absurdities like that elementary particles be cubic (Trell [1983, 1990, 1991, 1992, 1998 b,c, 2000]). But in an updated cosmology where the world returns as the possible analytical substantiation, or "inflation", from some singular scintillation, or "fluctuation", of an elementary quantum against an already available, again essentially "flat universe" (Bachall et al. [1999], Rees [2000]), it provides the exponentially enlarging dispersion of the thereby re-instituted Cartesian co-ordinate frame, or 'canvas' (Kamionkowski [2002]), for a stratified realisation of the spark and current between contrasting, yet infinitesimally approximating logical categories, or "branes" (Seife [2002]), such as "between the curved and the straight" which is at "the heart of Greek geometry and indeed of geometry in general" (Netz [2002]).

Once more there is double endorsement since “during the past few decades, it is the canvas itself that has increasingly become the focus of study” (Kamionkowski [2002]). In conclusion, the ancient plan still holds the draft, but of the block rather than the piston of the motor. This is the straight-to-round reversal in the back to the future excursion which together with contemporary observations backs up, too, that the eventual GUT of the infinity machine is not a single trail but that its comprehensive organic formula must be a symbiosis of complementary topological forms; a system of more than one principal equation, just like the cell needs a wall as well as a nucleus to work.

Such terms and parables are no more philosophically profane than "tables and chairs", no more jargon and verbose than many an ancient or recent text, no more outlandish than the Platonic solids or Eternal-Universe branes; and they have equally profound background and implications, not the least as a reintroduction of Kant's heuristic *Teleologie als ob* when "the idea of the archetype is currently making something of a comeback" (Laubichler [2003]). With that we may depart from our treatise, eventually noting that it is only at this crucial vantage point that Marius Sophus Lie enters the arena (Trell and Santilli [1998]), not as an FLT/BC midwife as HHO should have realised had he read research articles as observant as lay newspapers, but as a guide when getting from the distributed Cartesian co-ordinate system to internal particulate symmetries (Trell [1983, 1990, 1991, 1992, 2000]); excitingly enough and to what significance it may have exactly against the cube's externally, intermediary and internally inscribed spheres. Reproducible descriptive data like these are just the kind of stuff that should deserve to be seriously picked up and refined by the nominal *cogniti* instead of such adverse reactions that may sometimes rise from an especially xenophobic sectarian entrenchment. So, HHO, wake up! Maybe, just maybe, it is not Pandora's box but Columbi egg that does not lie but lies before your eyes.

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