SIMULATION OF A TIGHT-MOORED AMPLITUDE-LIMITED HEAVING-BUOY WAVE-ENERGY CONVERTER WITH PHASE CONTROL

by

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ABSTRACT

A mathematical model is presented for a tight-moored heaving-buoy wave-energy converter, with a high-pressure hydraulic machinery for energy production and motion control. The buoy is cylindrical, has a diameter of 3.3 m, and the height is, at equilibrium, 3.1 m below mean water level and 2.0 m above. A valve in the machinery can be actively controlled, and it is used to obtain largest possible power production, and to limit the excursion of the buoy, in order to protect the hydraulic machinery. In addition, an end-stop device is provided as a safety measure, in case the control fails to limit the excursion. For comparison a quite similar hydraulic machinery, without active control, is also investigated.

A procedure is developed for control in irregular waves, and on the basis of a scatter table the year-average power production is estimated to be approximately 14.7 kW for the buoy with control and 5.3 kW for the buoy without control. Further, a duration curve is presented, which shows that for the buoy with control the mean power production is between 10 kW and 20 kW more than 70% of the year. For the buoy without control more than half of the annual energy output is obtained in less than 20% of the time. This shows that control will both increase the mean power production, and give a much more smooth output. The procedure is also reasonably successful in limiting the excursion, so that the end-stop device is not in regular operation.

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LIST OF SYMBOLS

The most frequently used symbols are presented in the following list. SI units are included in brackets. The derivative of a variable with respect to time is denoted by a dot above the variable.

A_o	[m^2]	Orifice area of valves
A_p	[m^2]	Net area of hydraulic piston
A_w	[m^2]	Water plane area of buoy ($A_w = \pi D_b^2/4$)
B_-	[kg/m]	Constant for damping term of the end-stop force
B_+	[kg/m]	Constant for damping term of the end-stop force
D_b	[m]	Diameter of buoy
f	[kg/s^3]	Excitation force kernel
F_c	[N]	Force from the end-stop device
F_e	[N]	Excitation force
F_f	[N]	Linear friction force
F_m	[N]	Net buoyancy force on buoy submerged to equilibrium position
$F_{max,1}$	[N]	Constant used in control strategy for operation in irregular wave
$F_{max,2}$	[N]	Constant used in control strategy for operation in irregular wave
$F_{min,1}$	[N]	Constant used in control strategy for operation in irregular wave
$F_{min,2}$	[N]	Constant used in control strategy for operation in irregular wave
F_r	[N]	Radiation force
F_u	[N]	Force from hydraulic system on buoy
F_w	[N]	Total wave force, $F_w = F_e + F_r$
g	[m/s^2]	Acceleration of gravity
h	[m]	Water depth
H	[m]	Wave height
H_s	[m]	Significant wave height
k_-	[N/m]	Constant for spring term of the end-stop force
k_+	[N/m]	Constant for spring term of the end-stop force
k	[kg/s^2]	Radiation force kernel
$m_r(\infty)$	[kg]	Added mass of buoy at infinite frequency
m_b	[kg]	Mass of buoy
p	[Pa]	Pressure
p_A	[Pa]	Pressure in gas accumulator *A*
p_B	[Pa]	Pressure in gas accumulator *B*, high pressure accumulator
p_C	[Pa]	Pressure in gas accumulator *C*, low pressure accumulator
p_D	[Pa]	Pressure in gas accumulator *D*, equal to cylinder pressure
P	[W]	Power
Q	[m^3/s]	Flow rate or flow per unit time
R_f	[kg/s]	Friction resistance
R_r	[kg/s]	Radiation resistance of buoy
s	[m]	Heave excursion of body, from equilibrium
s_-	[m]	Design limit for excursion in negative direction. Excursion at which the end-stop device is engaged.
s_+	[m]	Design limit for excursion in positive direction. Excursion at which

		the end-stop device is engaged.
S	[N/m]	Hydrostatic stiffness of buoy ($S = \rho g A_w$)
t	[s]	Time
T	[s]	Wave period
T_0	[s]	Natural period of buoy
T_{pred}	[s]	Prediction time for excitation force
T_z	[s]	Zero-upcross period
u	[m/s]	Heave velocity of buoy, equal to $\dot{s}$
V	[m^3]	Volume
V_b	[m^3]	Submerged volume of buoy in equilibrium position
V_A	[m^3]	Gas volume of accumulator *A*
V_B	[m^3]	Gas volume of accumulator *B*, high pressure accumulator
V_C	[m^3]	Gas volume of accumulator *C*, low pressure accumulator
V_D	[m^3]	Gas volume of accumulator *D*
γ		Ratio of the specific heat capacities
δ	[s^{-1}]	Damping coefficient
η	[m]	Wave elevation
μ		Discharge coefficient of orifice
ρ	[kg/m^3]	Density of water
ρ_o	[kg/m^3]	Density of oil
ω	[rad/s]	Angular frequency
ω_0	[rad/s]	Natural angular frequency without damping
ω_d	[rad/s]	Angular frequency with damping

1 INTRODUCTION

For wave-energy converters (WECs) operating in a wave climate where the WEC is small compared to predominant wavelengths, it is essential that means are provided for optimum control of the oscillatory motion, in order to achieve maximum power conversion. The first problem to resolve is then the conditions for optimum. Secondly, we need to discuss the general principles on how to approach optimum. Thirdly, designs have to be proposed and components developed, in order to implement the control in practice. So far, mainly the two first-mentioned problems have been addressed. In initial studies, optimum control was considered with sinusoidal waves,[1,2,3,4,5,6,7] but also irregular waves have been considered.[7,8,9,10,11] The purpose of the control is then to obtain optimum phase and optimum amplitude of the oscillation in order to maximise the converted power. Linear theory will, in general, give simple frequency-domain expressions for the optimum condition. However, it is inherent with the design of a WEC that there is an upper bound on the oscillation amplitude. Moreover, the energy-converting device has a limited power capacity. The optimum conditions will then depend on whether or not the oscillation amplitude is constrained. When the equation of motion becomes non-linear, as it does when the amplitude is constrained, the frequency-domain description becomes less suitable, and the optimisation must be carried out in the time domain, for instance by optimum control theory.[7,12,13,14,15,16] It is then more difficult to give simple expressions for the optimum conditions. The result of the optimisation will also depend on whether it is optimised for the power absorbed from the wave, the power input to the conversion machinery or the power output from the machinery. Moreover, when the incident wave is very large, we might want to control the WEC so that the power is within the capacity of the machinery, and so that the loads on the WEC are as small as possible.

In the present work it is focused on the second and third problem. A mathematical model is presented for a WEC, consisting of a floating body moving relative to a fixed reference, and the body is, in general, exposed to an irregular wave. The body is interconnected to the fixed reference by a piston-and-cylinder, exerting a force on the body. The cylinder is connected to a high-pressure hydraulic system, which is used for energy storage and production of useful energy. The hydraulic system has some components which can be actively controlled. This makes it possible to control the pressure in the cylinder and thereby the motion of the body. For comparison a hydraulic system without active control is also investigated. In the present text oscillation in heave only is considered. It should be noted that, if the system is axisymmetric, it is possible for this system to absorb power equal to the incident power of a wave front of width equal to the wavelength divided by 2π, when the oscillation is unconstrained.

The aim of the present work is to investigate (real time) procedures which control the motion of the WEC so that as much energy as possible is produced by the conversion machinery, while they at the same time protect the hydraulic piston-and-cylinder by limiting the excursion of the buoy. To do this the general principles on how to obtain optimum must be addressed, and the control strategy must be implemented through the control of the hydraulic system. Further, the hydraulic system must be designed so that it, in a best possible way, can realise the control strategy. These problems are investigated by time-domain simulations of the WEC.

When optimum control is considered in irregular waves it is necessary to predict the incident wave. This means that the control strategy is non-causal. How long time it is necessary to predict the incident wave will depend on the control strategy, and on the size of the device. If the prediction of the incident wave is imperfect, the optimum motion can only be realised approximately.

2 MATHEMATICAL MODEL

2.1 The forces

We consider a WEC in the form of a heaving body, oscillating relative to a fixed reference. In general, the geometry of the device and of the surrounding submerged solid boundaries is arbitrary, and influences the problem only through the hydrodynamic parameters of the device. The waves incident to the device are, in general, irregular. The total wave force on the body can be written as

$$F_w(t) = F_e(t) + F_r(t) \tag{1}$$

The excitation force is given by

$$F_e(t) = f(t) * \eta(t) = \int_{-\infty}^{\infty} \eta(\tau) f(t-\tau)\, d\tau \tag{2}$$

which is a convolution product. Here $\eta(t)$ is the surface elevation due to the incident wave at the origin and $f(t)$ is the excitation force kernel. An example of an excitation force kernel, for the geometry shown in figure 1, is given in figure 2. The radiation force on the body is given by [17]

$$F_r(t) = -m_r(\infty)\dot{u}(t) - k(t) * u(t) = -m_r(\infty)\dot{u}(t) - \int_{-\infty}^{t} k(t-\tau) u(\tau)\, d\tau \tag{3}$$

where $m_r(\infty)$ is the added mass of the body at infinite frequency, $u(t)$ is the vertical velocity of the body, $\dot{u}(t)$ is the vertical acceleration and $k(t)$ is the radiation force kernel. An example of a radiation force kernel, for the geometry shown in figure 1, is given in figure 3. Note that, in equation (3), the upper integration limit is t because the radiation force kernel, contrary to the excitation force kernel, is a causal impulse response function, that is $k(t) = 0$ for $t < 0$. The integration kernels have been obtained from the frequency domain expressions for the hydrodynamic parameters of the body, which have been computed by a method previously described by the author, using linear hydrodynamic theory and assuming an ideal incompressible fluid.[18] Note that, since the kernels are computed by linear hydrodynamic theory, these expressions are valid only for small excursions. How large the error becomes when the excursion is large depends on the geometry of the device and of the steepness of the incident wave.

A linear friction loss force is also included, and we choose to write

$$F_f(t) = -R_f u(t) \tag{4}$$

The friction resistance consists of contributions from viscous friction, mechanical friction, and conversion losses in the machinery. The friction resistance, R_f, is for simplicity assumed to be independent of the oscillation amplitude and of the frequency.

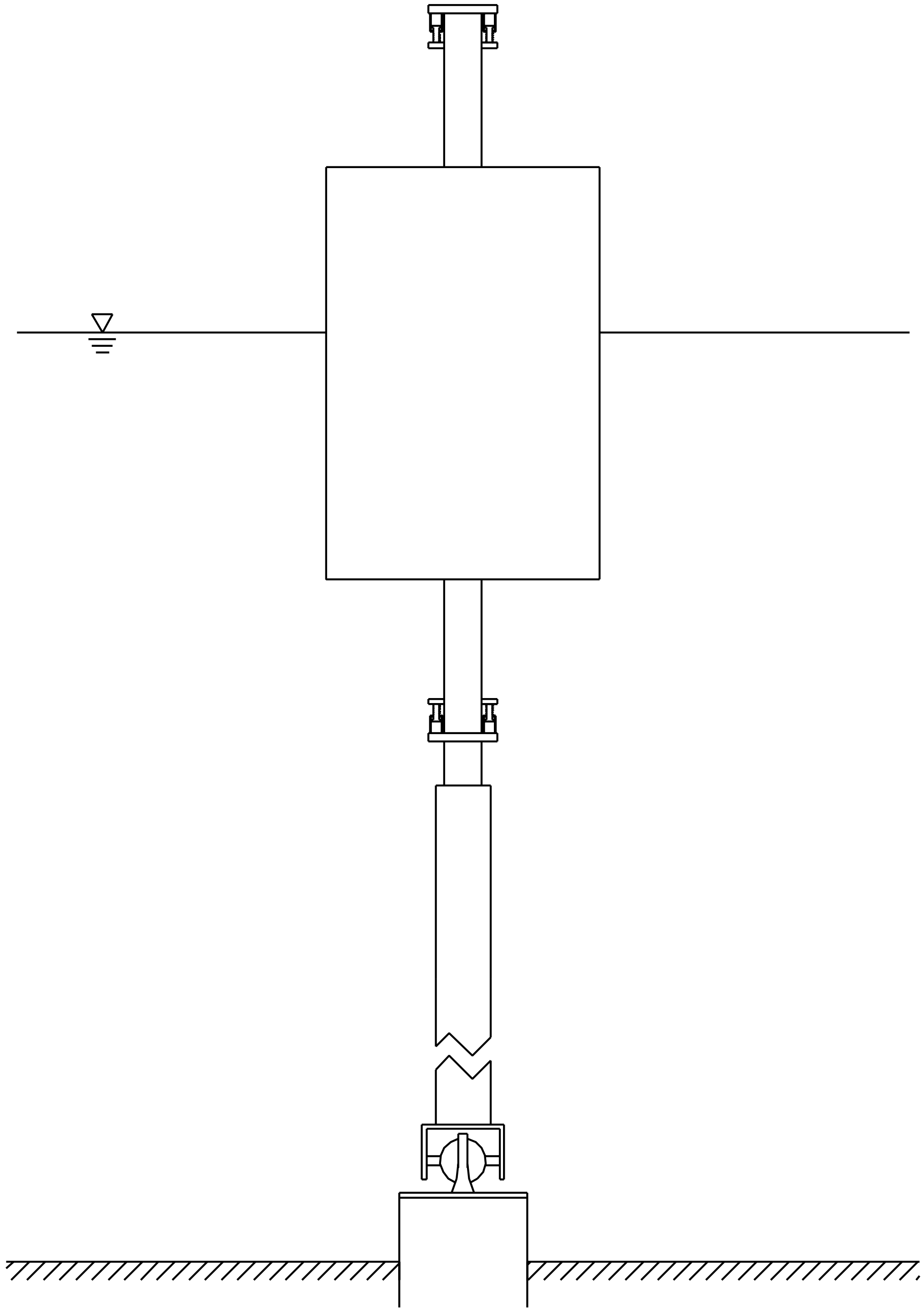

Figure 1. *Sketch of the geometry of the WEC. The diameter of the buoy is 3.3 m, the height is 5.1 m, of which 3.1 m is submerged at equilibrium, and the water depth is 25 m.*

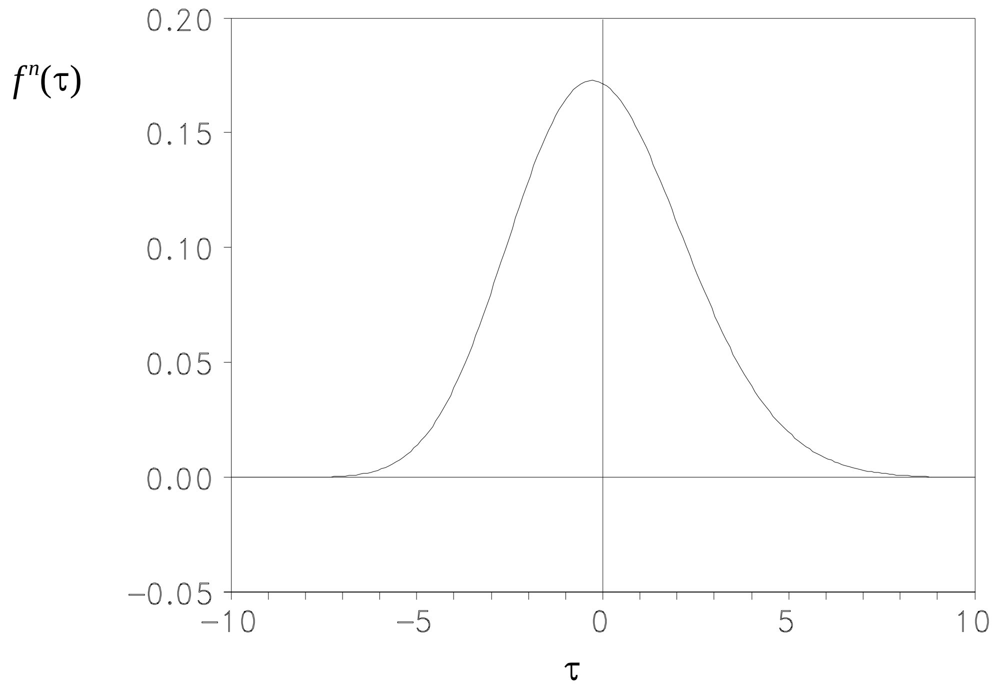

Figure 2. *The non-dimensional heave excitation force kernel $f^n(\tau) = f(t)(D_b/(2g))^{1/2}/S$ versus the non-dimensional time, $\tau = (2g/D_b)^{1/2}t$, for a vertical cylinder with draft/diameter = 0.94 and depth/diameter = 7.58.*

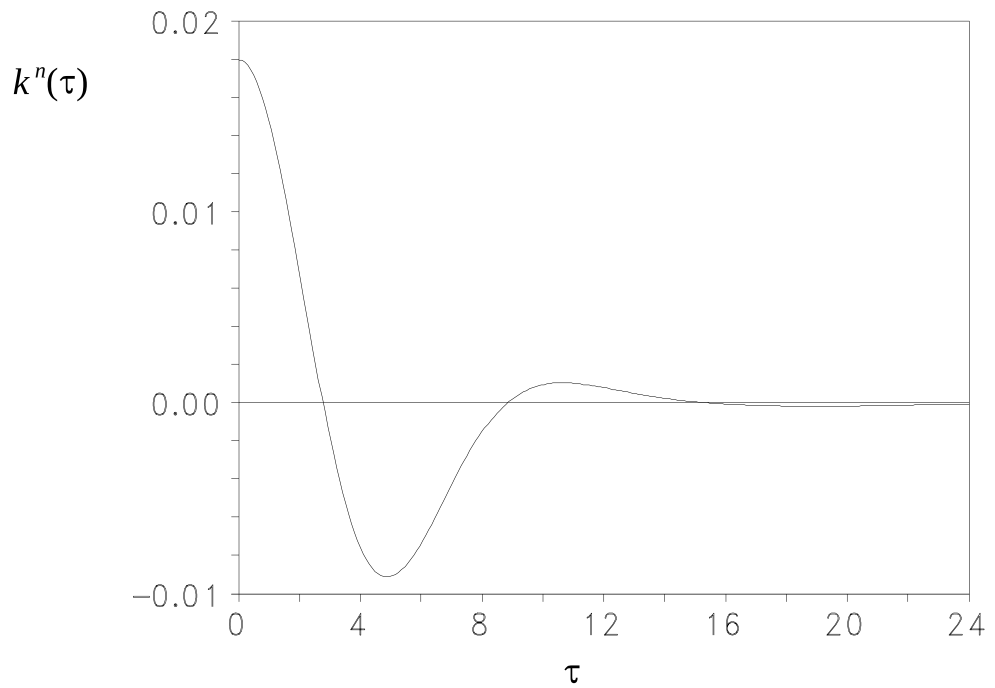

Figure 3. *The non-dimensional heave radiation force kernel $k^n(\tau) = k(t)/S$ versus the non-dimensional time, $\tau = (2g/D_b)^{1/2}t$, for a vertical cylinder with draft/diameter = 0.94 and depth/diameter = 7.58. Note that $k(t) = 0$ for $t < 0$.*

The WEC is equipped with a hydraulic machinery which produces a load force $F_u(t)$, which is given by the pressure difference across the piston multiplied by the net piston area. The load force (which also includes a pretension force to balance the force F_m described below) is working between the body and the fixed reference, and is taken to be positive when it is acting on the body in the positive z-direction. Thus, $F_u(t)$ is negative when there is a tension force in the piston rod and the force reacting anchor. The two hydraulic systems investigated, which are also used to produce useful energy, are described in Appendix A, and shown in figure 4 and 5.

The net buoyancy of the body, that is the difference between the buoyancy and the weight of the body at the equilibrium position, is given by

$$F_m = g(\rho V_b - m_b) \tag{5}$$

where V_b is the submerged volume of the body at equilibrium and m_b the mass of the body. This force equals the load force from the hydraulic system in the equilibrium position, but it is acting in the opposite direction.

For a real WEC the excursion has to be limited, for instance because of the finite length of hydraulic rams. It is often necessary to include a deceleration cushion at the end of the stroke, and this function is carried out by the end-stop device. This device dissipates kinetic energy of the load gently, and reduces the possibility of mechanical damage to the cylinder. The force from the end-stop device is named $F_c(t)$, and might include both damping terms and spring terms. The excursion at which the end-stop is engaged is termed the design limit of the excursion. A more detailed description of this force is given in Appendix B.

2.2 The equation of motion

The equation of motion for the body may now be written as follows, when the forces described in the previous section are included,

$$m_b \ddot{s}(t) + Ss(t) = F_w(t) + F_f(t) + F_u(t) + F_c(t) + F_m \tag{6}$$

where s is the vertical distance of displacement from equilibrium for the body and $S = \rho g A_w$ is the hydrostatic stiffness, A_w being the water plane area of the body. We have chosen to consider S as constant, not depending on the excursion, which is correct for a vertical cylinder. We note that when the WEC in the equilibrium position ($s(t) \equiv 0$) and with no incident wave, we must have $F_u(t) \equiv -F_m$, as mentioned in the previous section. The equation of motion can be reorganised as follows

$$\frac{1}{m_b + m_r(\infty)} \left(F_e(t) - \int_{-\infty}^{t} k(t-\tau)\, u(\tau)\, d\tau - R_f\, u(t) - Ss(t) + F_u(t) + F_c(\right. \tag{7}$$

which, together with $\dot{s}(t) = u(t)$, constitutes a (second-order) set of state equations.

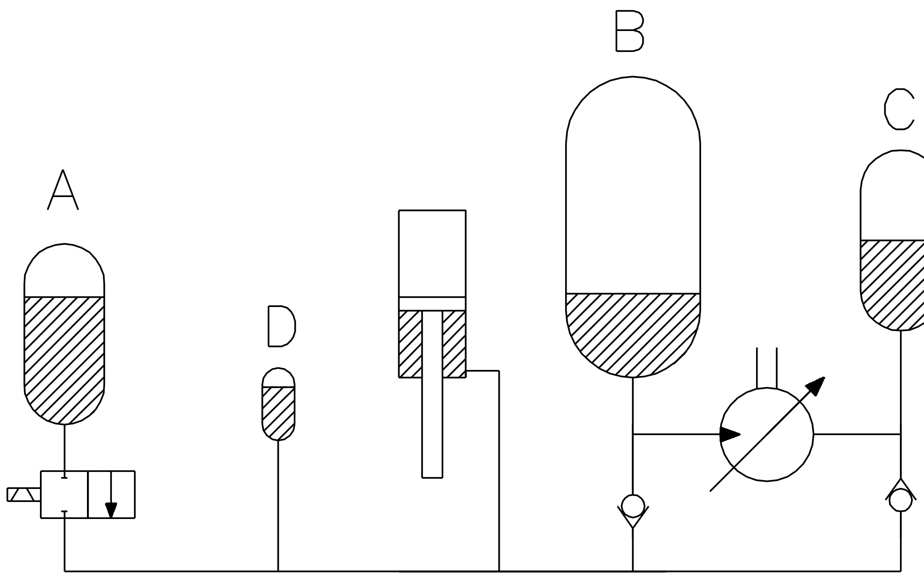

Figure 4. *Sketch of the hydraulic system proposed for the buoy with control.*

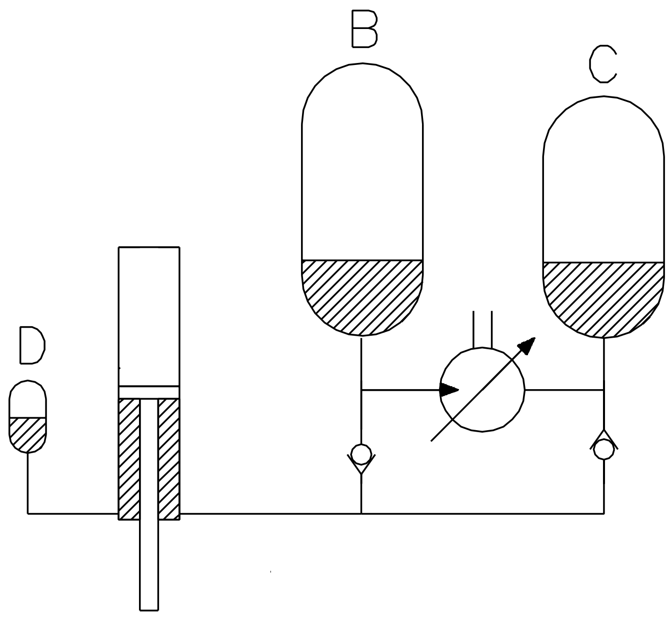

Figure 5. *Sketch of the hydraulic system proposed for the buoy without control.*

3 THE COMPUTER PROGRAM

The main purpose of the computer program, which has been developed, is to carry out an integration of the equation of motion (7). This is done by a fourth order Runge-Kutta procedure[19] with variable step length. The procedure advances the solution through a time interval of predetermined length, first by one step and afterwards the interval is divided into two time steps. The two solutions obtained for the excursion at the end of the interval are compared, and if the difference is below a given value the solution is accepted. If the discrepancy is too large, the number of time steps is doubled and the integration is repeated. This procedure is repeated until the discrepancy between the two solutions with the largest number of time steps is below the given value. The procedure then moves on to advance the solution through the next time interval.

The convolution integrals for the excitation force (2) and the radiation force (3) are evaluated by a trapezoidal approximation, where the time step equals the length of the predetermined intervals with which the equation of motion is advanced, and the values are determined in both ends of each interval. This is done to reduce the computing time and the requirement for storage of previous values of the solution. Interpolation by splines is used to obtain values inside the intervals. Further, the integrations are truncated in order to reduce the computing time. From figure 2 and 3 we see that this is an assumption which is easy to justify, since the integration kernels tend fast to zero as $|t| \to \infty$.

The surface elevation of the incident wave is read from a file with a certain sampling time, and spline interpolation is used to determine the wave elevation between the samples, if necessary.

The pressure in the cylinder, and thereby the force from the piston-and-cylinder on the buoy, is determined as follows. A gas accumulator (labelled *D*) is connected directly to the cylinder, as shown in figure 4 and 5, and the pressure in this accumulator is assumed to be the same as in the cylinder. The flow between the accumulators, and thereby the oil volume and pressure in the accumulators, are determined by the flow through the valves and by the motion of the piston. The system of pipes connecting the accumulators is assumed to be of minor importance. The equations used to relate the pressures, volumes and flows are described in Appendix A.

For each time step in the solution of the equation of motion, a procedure, that perhaps can best be termed as a Euler-algorithm, is used to advance the solution for the hydraulic system, using several shorter time steps. The flow through each of the valves is determined at the beginning of each time step, and assumed to be constant during the time step, not taking into account the pressure change during this time interval. However, in some cases analytical solutions are used within the procedure. From the flow through the valves and the motion of the piston, the gas volumes and pressures of the accumulators are determined at the end of the time step. The number of time steps used by the procedure has a minimum value, and the number is increased if, during one particular time step, the pressure change in the cylinder is above a given value. This is done to reduce the computing time, and at the same time get an acceptable accuracy for the solution.

4 SUMMARY OF RESULTS

Calculations have been carried out for the cylindrical buoy shown in figure 1, with the two alternative hydraulic systems described in Appendix A, and shown in figure 4 and 5. The diameter of the buoy is 3.3 m and the height is, at equilibrium, 3.1 m below mean water level and 2.0 m above. The shape of the buoy is chosen for mathematical convenience, and a real buoy should not have sharp edges. The mass of the buoy is $9.7 \cdot 10^3$ kg. This means that the force from the hydraulic system, acting on the buoy, in the equilibrium position, is 173 kN downwards, and the net buoyancy force is 173 kN upwards. These forces are the same for both hydraulic systems investigated, and the piston in the hydraulic system is connected to the ocean floor and the cylinder is connected to the buoy. During operation there should always be tension in the piston rod. The friction resistance is set to R_f = 200 Ns/m, which is approximately 10% of the maximum radiation resistance. The excitation force kernel and the radiation force kernel for this geometry are given in figure 2 and 3, for water depth $h = 25$ m. It has further been found that the added mass at infinite frequency is approximately $m_r(\infty) = 8.7 \cdot 10^3$ kg. The frequency domain hydrodynamic parameters of the device have been computed by a method previously described by the author.[18] The hydraulic cylinder is envisaged to be 5.0 m long, 2.5 m in each direction from the equilibrium position of the piston. However, the length of the cylinder does not enter into the mathematical model. The piston has a stroke of 2 m in each direction, from the equilibrium position, before the end-stop device comes into operation. This is the design limit of the excursion, which is the same for both hydraulic systems. The maximum time step used for the integration of the equation of motion is 0.04 s, and the integration kernels are assumed to be zero for $|t| > 6.2$ s (corresponding to non-dimensional time $\tau = 15$ in figure 2 and 3). A summary of values of constants and initial values of variables used in the calculations, is given in table 1 and table 2.

4.1 Regular waves

Calculations have been carried out for a number of combinations of wave period and wave height, with sinusoidal incident waves. To avoid problems with transient motions when starting the calculation, the wave height is gradually increased from zero to the desired value, and then held at this value for the rest of the wave series. Further, the calculation is not stopped before a steady periodic solution has been obtained for five to ten periods. For large waves the buoy will become fully submerged during parts of the wave cycle, and the simulation is not expected to give correct results. This has determined the maximum wave height used for the calculations.

Calculations have been carried out for sinusoidal waves to determine how the buoy should be controlled to obtain maximum power production, when the excursion is limited to ±2.0 m. That is, the end-stop device should not come into operation. This is a rather severe restriction on the piston-stroke. When the excursion is not constrained, maximum power production is obtained by opening the controllable valve approximately a quarter of the natural period of the buoy ($T_0/4$) before the extremum of the excitation force; in this case 0.55 s has been used. This value was determined by running simulations with different wave periods and different opening instants, while the natural period of the buoy has been determined from a free oscillation to be approximately 2.5 s (cf. section 4.4). With this choice of opening instant, the extremum of the buoy velocity will approximately coincide

Constants			
A_o	0.0079 m^2	R_f	200 kg/s
A_p	0.0173 m^2	s_-	-2.0 m
$B_- = B_+$	9200 kg/m	s_+	2.0 m
D_b	3.3 m	S	86.4 kN/m
F_m	173 kN	T_0	2.2 s
g	9.81 m/s^2	T_{pred}	4.4 s
h	25 m	V_b	26.5 m^3
$k_- = k_+$	50 kN/m	γ	1.4
$m_r(\infty)$	8700 kg	μ	0.611
m_b	9700 kg	ρ	1030 kg/m^3
		ρ_o	850 kg/m^3

Table 1. *Values of constants used in the calculations, for both systems.*

Initial values for system with control			
p_A	10 MPa	V_A	0.16 m^3
p_B	15 MPa	V_B	0.52 m^3
p_C	5 MPa	V_C	0.092 m^3
p_D	10 MPa	V_D	0.0005 m^3
Initial values for system without control			
p_B	10 MPa	V_B	0.8 m^3
p_C	10 MPa	V_C	0.55 m^3
p_D	10 MPa	V_D	0.0005 m^3

Table 2. *Initial values of variables used in the calculations.*

with the extremum of the excitation force. The valve is closed when the buoy has excursion extremum, that is when the velocity is zero. A sketch of how the buoy will typically move is shown in figure 6. When the wave height increases, the valve is opened later, which means delaying the velocity phase relative to the excitation force, so that the excursion of the buoy does not exceed the design limit.

Figure 7 shows the mean power production, neglecting energy losses in the hydraulic motor, for the buoy with control, and figure 8 shows the corresponding results for the buoy without control. For the buoy without control the power production is very small for small wave heights, but it increases rapidly as the wave height increases. The power production is approximately proportional to the square of the wave height. For short wave periods the increase is somewhat slower, and for the long wave periods it increases more rapidly with wave height.

For the buoy with control the oscillation is unconstrained for all wave periods examined for wave heights 0.5 m and 1.0 m, and the power production increases rapidly with the wave height. The controllable valve is opened 0.55 s before the extremum of the excitation force, for all wave periods. This means that we have approximately coinciding extremum of buoy velocity and excitation force. For short wave periods the power production also increases with the wave period, while for longer wave periods it is almost independent of the wave period. For wave height 1.5 m it is necessary to open the valve later, in order to limit the excursion for wave periods 7 s and 8 s, for wave height 2.0 m for all wave periods except 4 s and for wave heights 2.5 m and 3.0 m for all wave periods examined. The time between the opening of the controllable valve and the extremum of the excitation force, as fraction of the wave period, is shown in table 3. This fraction is negative when the valve is opened after the extremum of the excitation force. For the largest wave heights the phase control was used in a way which kept the buoy from becoming fully submerged. This meant constraining the excursion more than what would have been necessary if only the stroke of the piston had been considered, resulting in an excursion significantly smaller than the design limit and a small power production, especially for waves with short period. For the longest wave periods the power production is almost constant for wave heights from 1.5 m to 3.0 m. In practice the buoy may well become fully submerged, and the only reason for using this control is to obtain correct simulation results.

The power production in small waves could probably have been increased if the power take-off device could work with a diminished pressure difference across the hydraulic motor, that is a diminished pressure difference between the high pressure accumulator and the low pressure accumulator. The check valves would then open more easily, and power production could have been obtained with a reduced oscillation amplitude, and thereby less reradiated power due to the heave motion of the buoy.

Table 4 shows the mean power lost in the hydraulic system, for the buoy with control. The power absorbed from the wave is the sum of the power production, the losses in the hydraulic system and the friction losses. The losses in the hydraulic system are, in this model, associated with the turbulent flow through the valves. When the controllable valve is opened, there is usually a pressure difference across it, and some oil will flow through the valve to equalise the pressure. The associated energy loss seems to be much larger than the loss associated with the flow after the pressures have become almost equal, and there is a more continuous flow through the valve. This latter part of the loss is present for all valves, and will depend on the diameters of the valves, which in this case have been

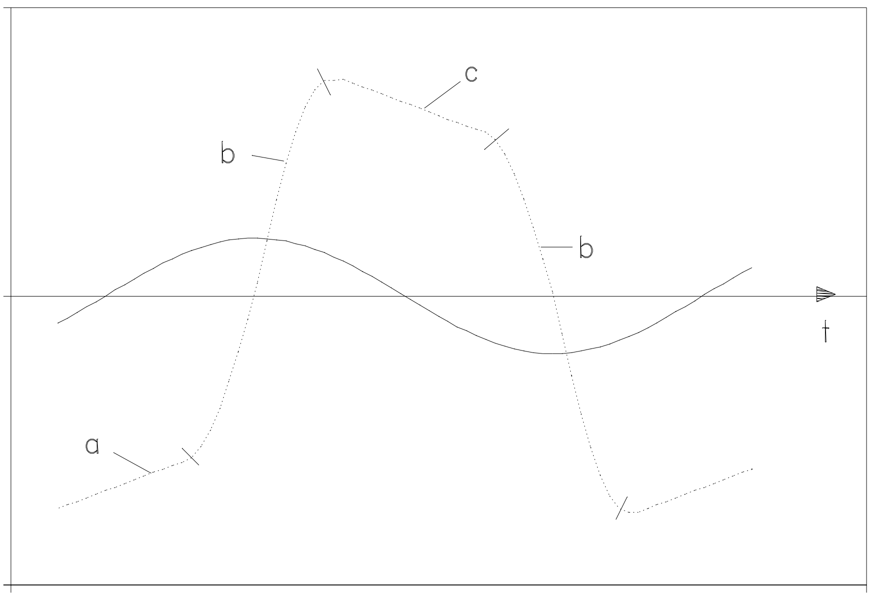

Figure 6. *Typical buoy motion for unconstrained oscillation (dotted line) relative to wave elevation (solid line) for the buoy with control. In the part of the curve marked* a (c) *the check valve to the hight (low) pressure accumulator is open, and in the parts marked* b *the controllable valve is open.*

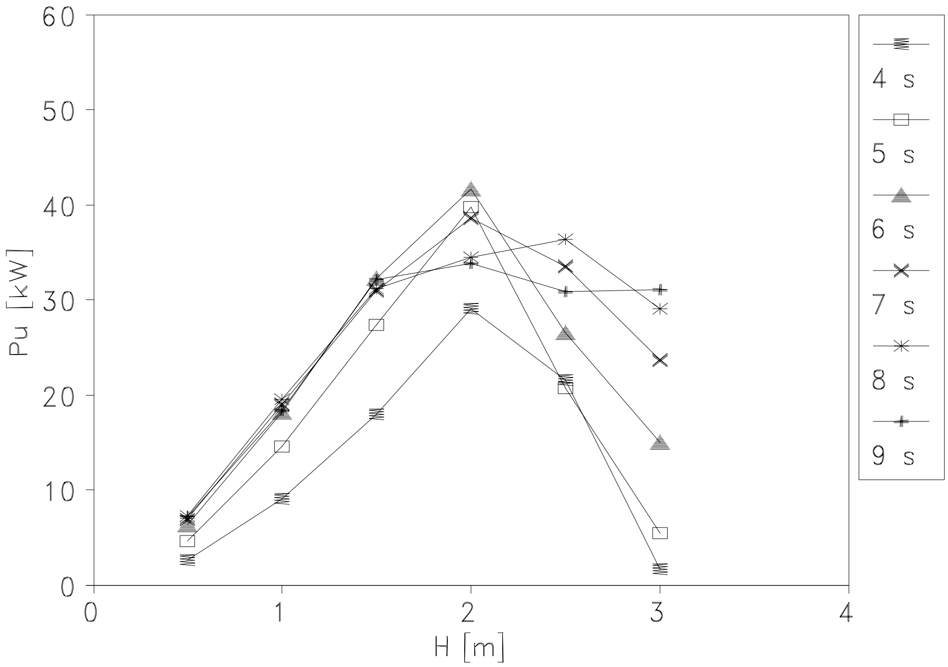

Figure 7. *Mean power production in sinusoidal waves, as function of the wave height, for different wave periods, for buoy with control.*

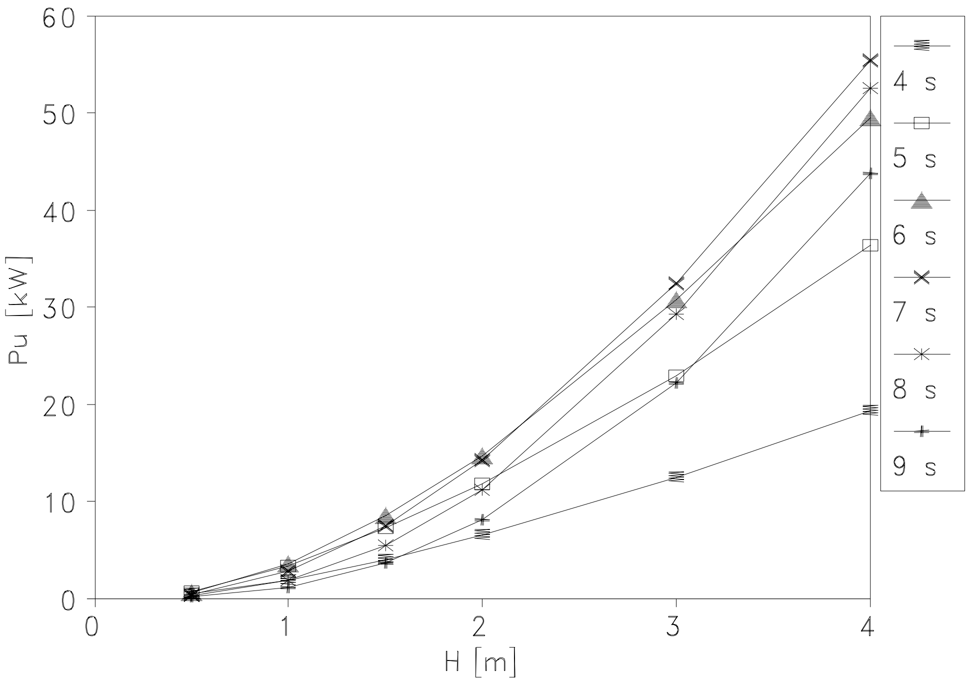

Figure 8. *Mean power production in sinusoidal waves, as function of the wave height, for different wave periods, for the buoy without control.*

T [s]	*H* [m]					
	0.5	1	1.5	2	2.5	3
4	0.14	0.14	0.14	0.14	**0**	**-0.1**
5	0.11	0.11	0.11	**0.06**	**-0.06**	**-0.12**
6	0.09	0.09	0.09	**0.02**	**-0.07**	**-0.12**
7	0.08	0.08	**0.03**	**-0.01**	**-0.07**	**-0.11**
8	0.07	0.07	**0.03**	**-0.04**	**-0.08**	**-0.11**
9	0.06	0.06	0.06	**-0.04**	**-0.09**	**-0.11**

Table 3. *Time interval between the opening instant of the controllable valve and the extremum of the excitation force in sinusoidal waves, in fractions of the wave period. Negative numbers mean that the valve is opened after the extremum. Results from calculations where the control was used to constrain the excursion are shown in bold.*

0.1 m for all the valves. If the valves are smaller, the pressure drop across the valves becomes larger, and the loss increases, provided the flow remains the same. Reducing the diameter of the two check valves does not increase the power loss significantly, since the flow through these valves is relatively small. The diameter of the operable valve is more significant, since the flow through this valve is much larger. If the diameter is reduced, the damping loss of the motion increases, and the excursion of the buoy is reduced. This leads to a considerable reduction in the absorbed energy for certain sea states. For the buoy without control the only losses included in the model of the hydraulic system are the losses associated with the flow through the check valves, and for the valve diameter used here these losses are negligible.

In Appendix C an exact analytical solution is presented for optimum oscillation of a buoy moving sinusoidally in sinusoidal waves, when the oscillation amplitude is constrained. In this case the excitation force and the buoy velocity are in phase for all the waves investigated. It is optimised for the power input to the hydraulic machinery, which can be compared to the sum of the power lost in the hydraulic machinery and the power production in the present simulations. The results obtained for the simulations are in the range from 70% to 90% of the analytical results, for wave heights from 0.5 m to 2.0 m. For the analytical solution the oscillation is constrained for all combinations of wave height and wave period examined, except for the wave with period 4 s and height 0.5 m. This indicates that the oscillation amplitude should ideally have been larger for wave heights 0.5 m and 1.0 m for the simulations, since the oscillation is not constrained for any wave periods for these wave heights. Moreover, since the motion of the buoy is not sinusoidal it should be possible to absorb somewhat more energy than when the motion is sinusoidal.[16] However, for the analytical solution it has been assumed that the control force necessary to establish the desired oscillation is available, while in the simulations a realistic system has been used to generate the control force. On this basis the simulation results must be assumed to be reasonably good. For the largest wave heights the simulation results for the power production drops rapidly compared to the analytical results. This is a result of the control being used in a way which kept the buoy from becoming fully submerged.

Maximum and minimum pressure in the cylinder over the wave cycle, are shown in table 5, and we observe that the pressure is well within the limits described in Appendix A. Maximum force in the piston rod is for these simulations given by the maximum force from the hydraulic piston-and-cylinder, since the end-stop device is never in operation. The maximum force has relatively little variation with wave period as well as with wave height, which can be observed from the maximum cylinder pressure given in table 5.

The extreme values of the excursion, in positive and negative direction, over the wave cycle, are shown in table 6. The excursion should not exceed ±2.0 m in these calculations, to avoid use of the end-stop device. When the controllable valve is opened approximately a quarter of the natural period of the buoy before the extremum of the excitation force, the results are not very sensitive to the opening instant of the valve. This is so because, when the wave period is long compared to the natural period of the buoy, the excitation force will not change rapidly in the time interval around the extremum of the buoy velocity. However, when the oscillation is constrained the results are more sensitive to the opening instant, since the excitation force changes more rapidly when the buoy velocity has its extremum. Note that, for the largest wave height, it is not the stroke of the piston which determines the opening instant of the valve, but the imposed condition that the buoy shall never be totally submerged. This leads to a relatively small excursion for the buoy.

T [s]	*H* [m]					
	0.5	1	1.5	2	2.5	3
4	0.6	1.4	2.3	3.4	**2.1**	**0.4**
5	0.9	1.6	2.5	**3.3**	**1.4**	**0.3**
6	0.9	1.5	2.5	**2.7**	**1.3**	**0.5**
7	0.9	1.6	**2**	**2**	**1.1**	**0.1**
8	0.8	1.2	**1.6**	**1.5**	**1.3**	**0.6**
9	0.6	0.9	1.5	**0.7**	**0.6**	**0.1**

Table 4. *Mean power lost in the hydraulic machinery (valves) in sinusoidal waves, in kW, for the buoy with control. Results from calculations where the control was used to constrain the excursion are shown in bold.*

T [s]	*H* [m]					
	0.5	1	1.5	2	2.5	3
4	15.3 4.6	16.1 3.7	16.9 3.0	17.9 2.4	**17.2** **2.8**	**15.2** **4.6**
5	15.6 4.2	16.6 3.2	17.8 2.5	**18.8** **2.1**	**17.3** **2.8**	**15.7** **4.1**
6	15.8 4.0	16.9 3.0	18.2 2.3	**19.0** **2.0**	**17.7** **2.5**	**16.6** **3.2**
7	15.8 3.9	17.0 2.9	**18.1** **2.4**	**18.8** **2.1**	**18.4** **2.3**	**17.5** **2.6**
8	15.9 3.9	17.1 3.0	**18.2** **2.4**	**18.4** **2.2**	**18.7** **2.1**	**18.1** **2.4**
9	15.8 4.0	17.0 3.0	18.3 2.3	**18.4** **2.1**	**18.2** **2.3**	**18.3** **2.3**

Table 5. *Maximum and minimum cylinder pressure, in MPa, over the wave cycle, in sinusoidal waves, for the buoy with control. Results from calculations where the control was used to constrain the excursion are shown in bold.*

T [s]	H [m]					
	0.5	1	1.5	2	2.5	3
4	0.97 -0.92	1.25 -1.19	1.48 -1.44	1.72 -1.74	**1.30** **-1.26**	**0.54** **-0.52**
5	1.15 -1.09	1.52 -1.44	1.77 -1.76	**1.84** **-1.92**	**1.29** **-1.23**	**0.65** **-0.57**
6	1.29 -1.22	1.68 -1.59	1.95 -1.97	**1.82** **-1.89**	**1.27** **-1.21**	**0.77** **-0.66**
7	1.36 -1.29	1.78 -1.73	**1.92** **-1.91**	**1.86** **-1.92**	**1.36** **-1.31**	**0.90** **-0.83**
8	1.40 -1.30	1.72 1.59	**1.91** **-1.85**	**1.70** **-1.80**	**1.37** **-1.44**	**1.00** **-0.96**
9	1.36 -1.27	1.66 -1.55	1.93 -1.97	**1.73** **-1.94**	**1.30** **-1.29**	**1.09** **-1.04**

Table 6. *Extrema of excursion, in positive and negative direction, in m, over the wave cycle, in sinusoidal waves, for the buoy with control. Results from calculations where the control was used to constrain the excursion are shown in bold.*

It has also been tested what happens if the excursion is limited by opening the valve earlier relative to the extremum of the excitation force, instead of later, as it is done for the results presented here. This results in a significant reduction in the power production when the oscillation is constrained. This has the following explanation. When the extreme velocity of the buoy occurs before the extremum of the excitation force, the buoy follows the wave elevation more closely, and the pressure necessary to open the check valves is not established in the cylinder. This is a result of the design of the proposed hydraulic system, and it is not generally valid for all kinds of power take-off. However, for the present WEC this strategy can be used if one wishes to protect the machinery.

4.2 Irregular waves

For simulations in irregular waves a Pierson-Moskowitz (PM) spectrum[20] with wind speed as parameter is used. This spectrum is supposed to describe a fully developed sea-state. The zero-upcross period increases linearly with the wind speed and the significant wave height has a quadratic increase with the wind speed. Time series are generated for the wave elevation, which are composed of 100 components of regular waves with frequencies from 0.01 to 1.0 Hz, with $\Delta f = 0.01$ Hz. The amplitudes of the wave components are obtained from the spectrum, and the phases are random. The wave series repeats itself after 100 s, due to the choice of Δf.

The total length of the calculated time series have been 300 s, which is three times the repeating time of the wave series. The first 200 s were not used, as this time interval was expected to include transient motions. Separate calculations were carried out to confirm that the last 100 s represented a periodic solution. For wave series with significant wave height larger than 2 m the buoy becomes totally submerged during short periods of the series, for the largest wave crests. No results are presented for wave with significant wave height larger than 4.6 m, corresponding to wind speed larger than 15 m/s, since the buoy would then be totally submerged during a significant part of the series, and the simulated results are not expected to be correct.

For operation of a WEC with control in irregular waves, it is necessary to develop some sort of strategy on how to absorb as much energy as possible, and at the same time protect the machinery in large waves. In this case we need a strategy on how to control the operable valve. The strategy used here has the excitation force as input, and it is described in Appendix D. The basic idea of this strategy is to have excitation force and buoy velocity in phase when the waves are small, and gradually delaying the velocity phase as the wave height increases, thereby limiting the excursion of the device. The procedure searches for the extrema of the excitation force, and uses them to determine the opening instant of the valve. If an extreme value is within certain limits, the valve is opened a quarter of the natural period of the buoy before the extremum. This will give approximately coinciding extrema of buoy velocity and excitation force. Otherwise, the procedure searches for an interval after the extremum where the excitation force is such that the excursion of the buoy is likely to be within the design limits. The procedure contains some design-specific parameters, which have been determined by running the program several times, for different sea states. The parameters were changed to maximise the power production, and at the same time keep the excursion within the design limits.

Figure 9 shows the mean power production, as function of the significant wave height, for the buoy with control as well as for the buoy without control. Each entry in the figure is a mean value based on 30 simulations, and the height of the vertical lines are equal to twice the standard deviation. For the buoy without control the power production is approximately proportional to the square of the significant wave height. For the buoy with control the power production increases rapidly for small wave heights, and for significant wave heights larger than 1.5 m it continues to increase although slowly. The power production is almost constant for waves with significant wave height from 2 m to 3 m, and this is probably due to the design of the control strategy. The time delay in opening the controllable valve will always be $T_0/4$ or longer, relative to the unconstrained opening instant, when the oscillation is constrained. This is a result of the relative simplicity of the control strategy. This means that when the wave height reaches a value where it is necessary to start constraining the oscillation, the power production can decrease, because the delay in opening the operable valve is longer than necessary. The power production is significantly larger for the buoy with control than for the buoy without control, especially for waves with significant wave height below 3 m.

For the buoy with control the mean power production is rather small for very low wave heights, because the pressure necessary to open the check valves is not established in the cylinder when the excursion is small. The pressure in the cylinder must be higher than the pressure in the high pressure accumulator and lower than the pressure in the low pressure accumulator, during parts of the series, in order to obtain energy production. For small waves the necessary excursion is difficult to establish, since the energy absorbed is reradiated and lost in friction. This results in a relatively small energy production for waves with significant wave height below 1.0 m. As discussed for regular waves, if the power take-off device had been designed to work with a diminished pressure difference across the hydraulic motor, the power production could probably have been increased for small waves. The pressure difference across the motor would then have to be determined by the sea state, and it would have to be possible to adjust the pressure in the accumulators on the same time scale as the change in the sea state.

The absolute value of the most extreme excursion in each series is shown as function of the significant wave height in figure 10, for the buoy with control as well as for the buoy without control. Each entry in the figure is a mean value based on 30 simulations, and the height of the vertical lines are equal to twice the standard deviation. For the buoy with control the design limit for the excursion (in this case 2 m) is rapidly approached when the significant wave height increases, but the control strategy is reasonably successful in constraining the excursion. We note that the variation in the extreme excursion increases with the wave height. This indicates that there are certain large waves that the control procedure does not handle well. With a better control procedure it should be possible to reduce the variation, so that the device, on average, could operate closer to the design limit.

For the buoy without control the increase in the excursion is almost linear as function of the significant wave height, and for the largest wave heights the most extreme excursion exceeds the design limit in almost all the simulations. The cylinder was envisaged to have a total length of 5.0 m, meaning the extreme excursion could be 2.5 m in each direction from the equilibrium position. It is clear that for operation in a wave climate with significant wave heights larger than approximately 4 m it is necessary to have a longer cylinder. Further, we notice that when the cylinder is designed to handle the most extreme sea states, most of the time only a part of the stroke is in use. Designing a more efficient end-stop device, so that the motion of the buoy is more efficiently stopped when the piston approaches the end of the cylinder, could probably reduce the need for a longer cylinder. It could also be possible to mechanically lock the buoy in a fixed position for the largest waves.

The control procedure can probably be improved. In the present approach, only the excitation force is used for determining the opening instants of the valve, and for instance the position of the buoy is not considered. A better approach could be to determine an approximate opening instant first, based on the excitation force, and then afterwards carry out integrations of the equation of motion for different opening instants around the approximate opening instant, in order to determine the best possible choice. This should increase the power production, and at the same time reduce the use of the end-stop device in large waves. Further, the current control procedure seems to allow for excessively high oscillations in small waves. When the waves are small and the excursion amplitude is large, the absorbed power over the wave cycle is sometimes negative, due to the energy lost in friction, lost in the hydraulic system and reradiated due to the heave motion of the buoy. This is possible since the energy stored in the gas accumulator controlled by the controllable valve and as potential energy of the buoy is used to drive the motion of the buoy. The procedure should check that the energy absorbed over the next cycle is likely to be positive, before the oscillation cycle is started. It is also possible to use the displacement volume of the hydraulic motor for control, since the pressure in the high pressure accumulator and the low pressure accumulator will influence the motion of the buoy. However, in the present work it has not been investigated how the control of the pressures in these accumulators can be used to increase the power production and to

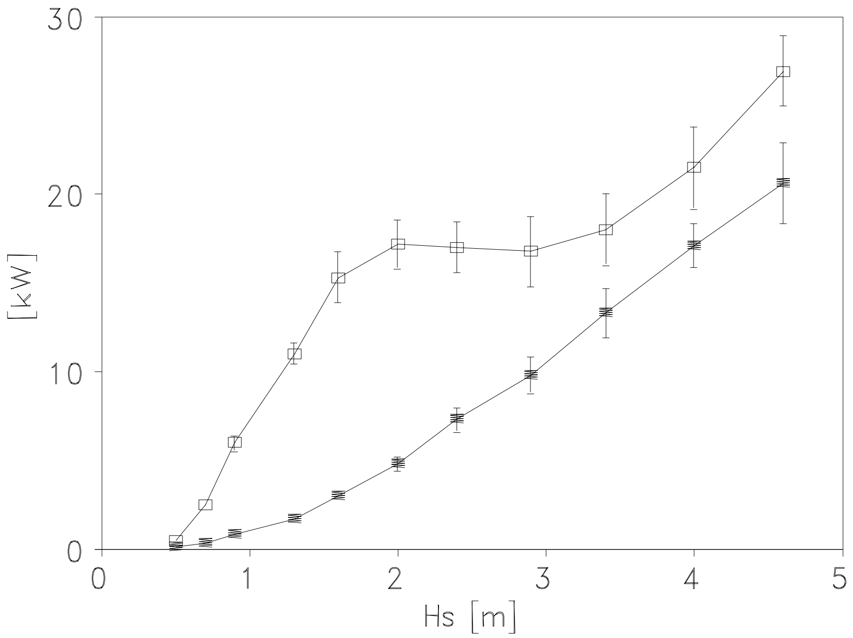

***Figure 9**. Mean power produced in irregular waves, as function of significant wave height. Empty (filled) squares are for the buoy with (without) control. The uncertainty is indicated by the vertical lines, which height is equal to twice the standard deviation.*

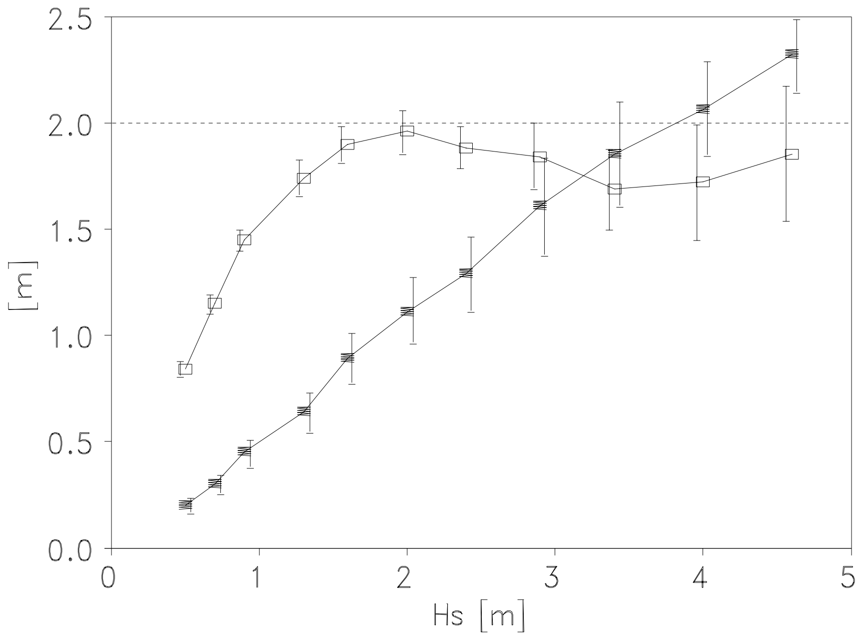

***Figure 10.** Absolute value of the most extreme excursion in irregular waves, as function of significant wave height. Empty (filled) squares are for the buoy with (without) control. The uncertainty is indicated by the vertical lines, which have height equal to twice the standard deviation.*

constrain the excursion of the buoy.

A control strategy where the excursion is limited by opening the valve earlier relative to the extremum of the excitation force, has also been tested, and compared to the present approach. As described for regular waves, this resulted in a significant reduction in the power production for large waves. However, this strategy could be used if one wishes to protect the hydraulic machinery in very large waves without stopping the machinery.

4.3 Year-average power production

In order to determine the year-average power production a JONSWAP-spectrum[21] is chosen as a basis for generating irregular waves. As input parameters to the spectrum are chosen the significant wave height, H_s, and the zero-upcross period, T_z.[10,22] In addition the peakedness of the spectrum is taken to be 3.3. Time series for the wave elevations are generated, which are composed of 250 components of regular waves with frequencies from 0.01 to 2.5 Hz, with $\Delta f = 0.01$ Hz. The amplitudes of the wave components are obtained from the spectrum, and the phases are random. The wave series repeats itself after 100 s, due to the choice of Δf.

The total length of the calculated time series, for each sea state, has been 400 s, which is four times the repeating time of the wave series. For computation of the converted power the first 300 s were not used, since we want to make sure that a stable periodic solution has been obtained.

Calculations have been carried out for most of the sea states in a scatter diagram (significant wave height versus zero-upcross period) from "Haltenbanken" (64° 10.5' N, 9° 10.0' E) off the Norwegian coast.[23] This scatter diagram is based on observations from the period 1974 to 1978, and the average incident wave power per unit width is approximately 37 kW/m.[24] In these calculations energy losses in the hydraulic motor have been neglected. No calculations have been performed for sea states with significant wave height larger than 6.5 m, since the buoy will then be totally submerged a significant part of the series, and the simulation results are not expected to be correct. However, this accounts for only approximately 1% of the observations. For these sea states the mean power production and the power lost in the hydraulic system have both been assumed to be zero.

To obtain an estimate for the year-average power production of the WEC a summation is carried out over all sea states of the probability of the sea state multiplied by the mean power production in the sea state. For the buoy with control four simulations (with different random phases) have been performed, and the results were from 14.5 kW to 14.9 kW. By doing the same for the power lost in the hydraulic system, year-average values from 0.71 kW to 0.74 kW were obtained. For the buoy without control five simulations have been performed, and the results for the year-average power production were from 5.2 kW to 5.4 kW. We observe that there is good agreement between the results.

Figure 11 shows duration curves for the mean power production for both buoys, based on one set of simulations. These curves show the percentage of the year for which the mean power production is above a certain value. The time resolution is three hours, the time period between the measurements of the sea state. The year-average power production is in this case 14.5 kW for the buoy with control and 5.4 kW for the buoy without control. For the buoy with control the power production is close to the year-average for a large portion of the year. The mean power production is between 10 kW and 20 kW for more than 70% of the time. Further, the power is larger than twice the year-average power less

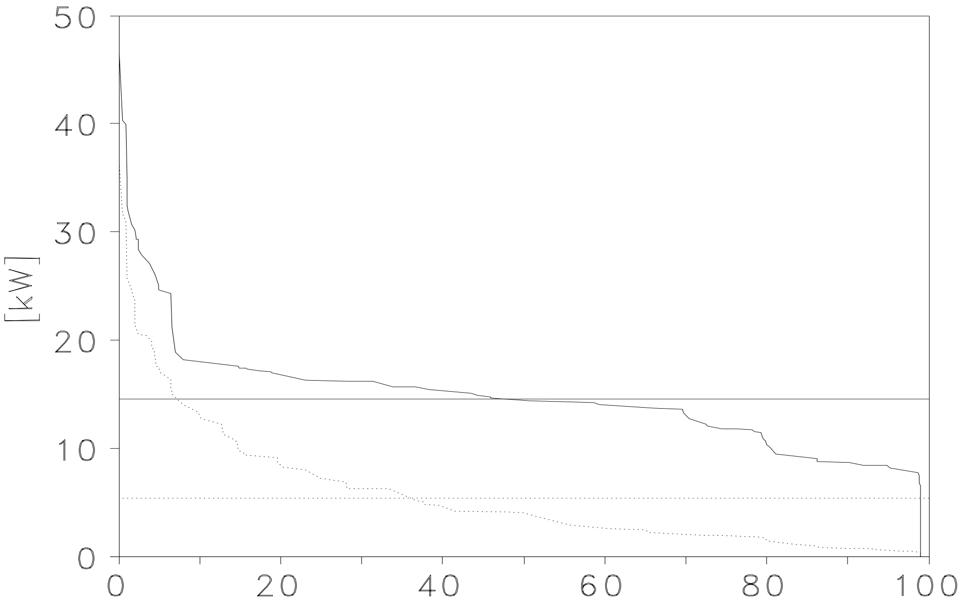

Figure 11. *Duration curve for the mean power production. The curve gives the percentage of the year the mean power production is above a certain level. The solid (dotted) curve is for the buoy with (without) control. The horizontal lines are the year-average power production for the two buoys.*

than 3% of the time, and the largest power production is approximately three times the year-average. This means that the sea states with the largest power production are not very important for the average power production.

For the buoy without control the power production has very large variation. Only 16% of the energy is produced in that half of the year when the power production is lowest, and 54% of the energy is produced in that 20% of the year when the power production is largest. This means that the sea states with largest production are very important for the average production.

4.4 Free oscillation

A calculation has been performed to determine the free oscillation of the buoy, with the hydraulic system with the control facility. The controllable valve is kept open, and the excursion of the device is so small that the check valves to the high pressure accumulator and the low pressure accumulator are never open. The friction resistance is set to $R_f = 200$ Ns/m. The initial excursion of the buoy is 0.75 m, it is released at $t = 0$, and there is no incident wave. The oscillation may be approximated by the usual formula

$$s(t) = A\, e^{-\delta t} \cos(\omega_d t + D) \qquad (8)$$

for free damped oscillations, where A, δ, ω_d and D are constants used to obtain the best possible fit between the formula and the simulation result. The constants A and D are given by the initial conditions. The angular frequency of the damped oscillation is given by ω_d, and

δ is the damping coefficient. With $\delta = 0.017\ s^{-1}$ and $\omega_d = 2.5$ rad/s good agreement is obtained between the simulation result and the formula, and since $\delta << \omega_d$ we have $D \approx 0$ rad and $A \approx s(0) = 0.75$ m. The damping coefficient δ seems to be most difficult to adjust. With the value given here, the oscillation described by equation (8) has an oscillation amplitude that is a somewhat too large in the beginning and then it gradually becomes too small as time progresses. This has probably to do with the parts of the mathematical model that are nonlinear.

The following relation can be used to determine the natural angular frequency without damping

$$\omega_0^2 = \omega_d^2 + \delta^2 \tag{9}$$

which means that we obtain $\omega_0 = 2.5$ rad/s. The natural period of the buoy can then be determined to be approximately 2.5 s. The natural angular frequency without damping is also given as the square root of the stiffness of the system divided by the total mass of the system. The stiffness of the gas accumulator is found to be approximately 26.1 kN/m, when the adiabatic equation is linearised, which together with the hydrostatic stiffness gives the system a total stiffness of approximately 113 kN/m. The total mass of the system is the sum of the mass of the buoy and the added mass at the oscillation frequency. The added mass is estimated to be 7800 kg at this frequency, giving the system a total mass of 17500 kg. This gives $\omega_0 = 2.54$ rad/s, which is in good agreement with the result previously obtained.

An estimate for δ can be obtained as the sum of the radiation resistance and the friction resistance divided by two times the total mass of the system. The radiation resistance at the oscillation frequency is approximately 240 Ns/m, which gives us $\delta = 0.013\ s^{-1}$. This result is somewhat below the value previously obtained. However, in the simulation program the damping of the oscillation due to the energy loss in the valve was taken into consideration, and these losses have not been included here.

5 CONCLUSION

A mathematical model has been presented for a tight-moored heaving-buoy WEC, with a high-pressure hydraulic system for energy production and motion control. The model is based on linear hydrodynamic theory, but the forces from the end-stop device and the hydraulic system are non-linear. The buoy is, in general, exposed to an irregular incident wave, and oscillations in heave only are considered. When the buoy becomes fully submerged the model is not expected to give correct results. For comparison a quite similar hydraulic system, without the control facility, has also been investigated.

Calculations have been carried out for sinusoidal waves to determine how the buoy should be controlled to obtain maximum power production, when the excursion is limited to ±2.0 m. That is, the end-stop device should preferably never come into operation. It is determined that when the oscillation amplitude is unconstrained the best control is to have coinciding extrema of buoy velocity and excitation force. This is obtained by opening the controllable valve approximately a quarter of the natural period of the buoy before the extremum of the excitation force. When the oscillation amplitude is constrained, a phase shift should be introduced so that the extremum of the buoy velocity is after the extremum of the excitation force. In this way the excursion of the buoy does not exceed the design limit. The excursion could also, alternatively, have been constrained by having the extremum of the velocity before the extremum of the excitation force, but this results in a significantly lower power production.

Results are presented for a number of combinations of wave period and wave height, with sinusoidal waves. For the buoy without control the power production is approximately proportional to the square of the wave height. For short wave periods the increase is somewhat slower, and for long wave periods the increase is more rapid. For the buoy with control the power production increases rapidly with the wave height for small waves, for all wave periods. The excursion approaches the design limit already for relatively moderate wave height. For the largest wave heights the phase control was used in a way which kept the buoy from becoming fully submerged, which meant constraining the excursion more than what would have been necessary if only the stroke of the piston had been considered. This resulted in a small power production, especially for short wave periods, and an excursion considerably smaller than the design limit. This control was used only to obtain correct simulation results, and would not have been used for a real WEC. For the longest wave periods the power production is almost constant for wave heights from 1.5 m to 3.0 m. The power production in small waves could probably have been increased if the power take-off device could work with a diminished pressure difference across the hydraulic motor, that is a diminished pressure difference between the high-pressure accumulator and the low-pressure accumulator. The check valves would then open more easily, and power production could have been obtained with a reduced oscillation amplitude, and thereby less reradiated power due to the heave motion of the buoy.

For operation in irregular waves a control strategy is presented, which has the excitation force as input. It is assumed that the excitation force can be predicted a certain time interval into the future, and in this case an interval of approximately 4 s was used. The procedure searches for the extrema of the excitation force, and uses them to determine the opening instants of the controllable valve. If an extreme value is within certain limits, the valve is opened a quarter of the natural period of the buoy before the extremum. This will give approximately coinciding extrema of buoy velocity and excitation force. Otherwise, it is searched for an interval after the extremum where the excitation force is such that the excursion of the buoy is likely to be within the design limits. Using this strategy, the mean power production increases rapidly with the significant wave height for small wave heights, and for significant wave height larger than 1.5 m it continues to increase more slowly. This is when it is necessary to start constraining the excursion. For the buoy without control the power production is approximately proportional to the square of the significant wave height. The power production is considerably larger for the buoy with control than for the buoy without control, especially for sea states with significant wave height below 3 m. The control procedure is also reasonably successful in constraining the excursion, so that the end-stop device is not in regular operation. The control strategy presented here can probably be improved by determining a better opening instant for the controllable valve. This could increase the power production, and at the same time reduce the use of the end-stop device. The procedure could also check if the power absorbed during the next oscillation cycle is likely to be positive, before the oscillation cycle is started.

An estimate for the year-average power production is computed on the basis of a scatter table (significant wave height versus zero-upcross period). From four different simulations, the year-average power production is estimated to be between 14.5 kW and 14.9 kW for the buoy with control. Further, a duration curve is presented, based on the mean power production for each sea state, which shows that the mean power production is between 10 kW and 20 kW for 70% of the year. This means that the sea states with largest power production are not essential for the year-average output. From five different simulations the year-average power production for the buoy without control is estimated to be between 5.2 kW and 5.4 kW. From a duration curve it is found that more than half of the annual energy production is obtained in less than 20% of the year. This shows that control will both increase the mean power production, and give a much more smooth output.

The basic idea of operation of the WEC with control is that it is important to have optimum phase in small waves, in order to produce as much energy as possible. In large waves the excursion should be constrained, in order to protect the machinery. This is why a strict constraint has been imposed on the piston-stroke. However, the design of the WEC should be optimised with respect to the wave climate at the selected site, taking both energy production and survivability into consideration. The results also suggest that it is desirable to have a power take-off system designed to work with a larger range of pressure differences across the hydraulic motor. The pressure across the hydraulic motor should be determined by the sea state, and it should be possible to adjust it on a relatively short time scale. This could increase the power production in relatively small waves.

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Appendix A. The hydraulic systems

The paper discusses the use of two different hydraulic systems, one which can be used to control the motion of the WEC, and another system without the control facility. The piston-and-cylinder is the same for both systems. The cylinder is envisaged to be 5 m long, of which 0.5 m in each end is used for an end-stop device.

The hydraulic system for the buoy with control, as proposed by Budal,[25] is shown in figure 4. The hydraulic system can be used both for a single oscillating body, moving relative to a fixed reference, and for two oscillating bodies, where the relative motion between the bodies is used to absorb energy. This system is based on discrete control of the motion (latching), and not continuous control. How the system is supposed to control the motion of the WEC, and produce useful energy, has been described earlier.[25] Phase control is obtained by means of an operable valve which closes or opens the connection from the cylinder to a gas accumulator (*A*) placed inside the hull of the buoy. Amplitude control is achieved through two check valves (or operable valves) between the cylinder and one high pressure gas accumulator (*B*) and one low pressure gas accumulator (*C*). We note that these valves should be open only when the controllable valve is closed, that is, when the connection from the cylinder to accumulator *A* is closed. The pressure difference between these accumulators is used to run a hydraulic motor, and produce useful energy. How the buoy will typically move is shown in figure 6.

There is a fourth gas accumulator (*D*), which was not included in the original proposal, connected directly to the cylinder. This accumulator is small, and is used to smoothen the pressure in the system, and to avoid pressure peaks. This is desirable, so that the components of the hydraulic machinery are not subject to very rapid changes in pressure. In the mathematical model this accumulator can also be used to simulate the compressibility of the oil in the hydraulic cylinder. The pressure in this accumulator is assumed to be equal to the pressure in the cylinder.

It has been suggested that it is desirable to store energy corresponding to the energy production during one hundred seconds,[26] in order to give an electric output that is not varying too rapidly. This means that the gas accumulators will have to be large, and it will not be desirable to place them inside the hull of the buoy. Alternatively, accumulators *B* and *C* could be placed outside the buoy and they could be common for a group of buoys.[27] These accumulators could then be placed on shore, on the sea floor or in a floating structure. In that case the connection between the two gas accumulators and the two check valves could be accomplished by means of a pair of hoses. However, the volume flow from the cylinder to these gas accumulators will be large and occur only during parts of the wave cycle. This means the hoses will have to have large cross-section, if losses are to be kept at a minimum. This problem can be solved by having smaller gas accumulators placed inside the buoy, connected to larger external accumulators, which are common for a group of buoys. The accumulators inside the buoy smoothen the volume flow over one wave period, and reduces the maximum instantaneous fluid flow through the hoses. The larger accumulators outside the buoy store energy over several wave periods.

The pressure and volume of the gas accumulators are assumed to be related by the following formula

$$p\,V^{\gamma} = \text{constant} \tag{10}$$

When the process is adiabatic, we have $\gamma = 1.4$, and when the process is isothermal, $\gamma = 1.0$.

Heat transfer has not been included in the model.

The dimensions of the hydraulic system are determined as follows. The force from the hydraulic piston-and-cylinder shall keep the floating body in the desired equilibrium position. The piston shall, in this case, at equilibrium, act on the body with a force of 173 kN downwards, which is determined by the mass of the body and the submerged volume at equilibrium. Assuming an equilibrium pressure of 10 MPa, it is necessary to have a net piston area $A_p = 0.0173\ m^2$. The check valves to the low-pressure and high-pressure accumulators should open when the excursion of the buoy is approximately 1 m from the equilibrium position and the controllable valve is closed. This suggests an initial pressure of 5 MPa for the low-pressure accumulator and 15 MPa for the high-pressure accumulator. During normal operation the pressure in the high-pressure accumulator should not be allowed to drop below 15 MPa, and the pressure in the low pressure accumulator should not rise above 5 MPa.

The maximum displacement volume of the hydraulic motor is determined by the maximum mean flow delivered by the cylinder to the high pressure accumulator. If we assume that the piston has a maximum stroke of 4.0 m, but only delivers oil to accumulator *B* from a 1 m long part of the stroke, and assuming a wave period of 5 s, the maximum mean flow is $Q = 0.0035\ m^3/s$. If the pressure difference across the motor is 10 MPa the maximum power is approximately 35 kW. This is a reasonable number for a WEC of this size. Further, when the piston moves 2.0 m out from its equilibrium position in either direction, with the controllable valve open, the check valves to accumulator *B* and *C* should not open. This means that the check valves will open only when the controllable valve is closed. Assuming that the gas in accumulator *A* can be described by the adiabatic equation (10), and that the equilibrium pressure in the accumulator is 10 MPa, the gas volume in the accumulator in the equilibrium position can be chosen to be $0.16\ m^3$. The total volume of the accumulator can then for instance be chosen to be $0.25\ m^3$.

The stiffness of the system should be approximately the same in both directions, when the check valves to gas accumulator *B* and *C*, respectively, are open. This means that equal changes in the excursion of the buoy should produce equal changes in the pressure in the cylinder. By linearisation of the adiabatic equation the following relation is obtained between the pressure change and the volume change in an accumulator

$$\Delta p = -\gamma \frac{p}{V} \Delta V \tag{11}$$

This means that $\gamma\, p/V$ should be approximately equal for the two accumulators under typical working conditions. These accumulators should also be able to store energy equal to maximum production of approximately 30 s, which is roughly $0.1\ m^3$. This is a compromise between the desire to be able to smoothen the output power by being able to store energy, and the desire to keep the accumulators as small as possible, in order to keep the weight as low as possible. It is important to note that the combined oil volume of accumulator *B* and *C* should be almost constant during operation, otherwise the mean position of the buoy will move away from the equilibrium position. If the pressure in accumulator *B* is allowed to rise to 20 MPa when $0.1\ m^3$ of oil is stored in it, the gas volume at equilibrium should be $0.52\ m^3$, and the total volume of the accumulator could be $0.6\ m^3$. If the gas volume of accumulator *C* at equilibrium is chosen to be $0.092\ m^3$, it has the desirable stiffness, when typical working pressure has been established, and the pressure drops to approximately 1.8 MPa when $0.1\ m^3$ of oil is removed from it. The total volume of the accumulator could then be $0.2\ m^3$.

Regarding the choice of hydraulic motor, it has to have variable displacement volume, high efficiency and be able to work at the desired pressure and volume flow. For this purpose axial piston motors and wing motors seem most suitable. The motor recently proposed by Salter might also be suitable.[28] When a particular motor has been chosen, it might be necessary to change the specifications of the hydraulic system, so that the pressure over the motor and the liquid flow gives the best possible efficiency. In the present work, the displacement volume has been controlled so that at any instant the high pressure accumulator would be back to the initial state in 30 s if the displacement volume was kept constant, and no more oil was allowed to get into the accumulator.

When the capacity of the hydraulic motor and the electric generator is to be decided, economic considerations have to be taken. The motor should run at full capacity for as much of the time as possible. Calculations will have to be made which give the percentage of the year in which the absorbed power exceeds indicated values, and on this basis it will be possible to determine the most cost efficient size of motor and generator.

In these calculations it is assumed that the flow between the gas accumulators is determined by the valves, and that the system of pipes is of minor importance. The valves are modelled as orifices, and the pressure drop (Δp) (difference between static upstream and downstream pressure) and flow (Q) are related as follows

$$Q = \mu A_o \sqrt{2\Delta p/\rho_o} \tag{12}$$

where μ is the orifice (discharge) coefficient, A_o is the orifice area and ρ_o the density of the hydraulic oil. This equation describes turbulent flow through the orifice. We consider a circular orifice with sharp edges. If the orifice area is much smaller than the area of the pipe, the discharge coefficient is $\mu = 0.611$. For low temperatures and small pressure differences, the flow through an orifice can also be laminar. However, this is not considered here. With this model of the hydraulic system, all the losses are associated with the flow through the valves and the linear friction force.

The hydraulic system for the buoy without the control facility is shown in figure 5. The hydraulic piston-and-cylinder for this system is the same as for the system with control. Gas accumulators *B* and *C* are connected to the cylinder, underneath the piston, by check valves. When the piston moves down, oil will flow from the cylinder into accumulator *B*, and when the piston moves up, oil will flow out of accumulator *C* into the cylinder. The gas volumes of the accumulators are chosen so that the excursion of the WEC is approximately symmetric around the equilibrium position. As for the other system, a small gas accumulator (*D*) is connected directly to the cylinder. Oil flows from accumulator *B* to *C* through a hydraulic motor. For the results presented here the hydraulic motor has been simulated by an orifice placed between the high pressure accumulator and the low pressure accumulator, with orifice area 0.00005 m^2. The power production is represented by the power dissipated by the orifice. The diameter of the orifice is the same for all the simulations, and was chosen to maximise the mean power production for a wave with height 1.5 m and period 6.0 s. For other wave heights and periods the power production could probably have been larger, if a different orifice diameter had been chosen.

With this hydraulic system the force from the piston-and-cylinder on the buoy will have an almost constant value when the buoy moves upwards, and another almost constant value when the buoy moves downwards. This is so because this force is determined only by the pressures in the accumulators, which will be almost constant over one wave period. However, the pressures will change on a longer time scale, as the sea state changes.

A summary of values of constants for both systems, used for these calculations, are given in table 1, and initial values of variables are given in table 2. Note that the check valves and the operable valve are all assumed to have an area of $0.0079\,m^2$.

Appendix B. The end-stop device

It is sometimes necessary to include a deceleration cushion at the end of the stroke, and this function is carried out by the end-stop device. This device dissipates kinetic energy of the load gently, and reduces the possibility of mechanical damage to the cylinder. The force from the end-stop device can be composed of a spring term and a damping term, where the spring term represents storage of energy and the damping term represents dissipation of energy. The spring force term can be described by the following formula, when we envisage one spring placed in each end of the cylinder,

$$F_{spring}(t) = \begin{cases} -k_{-}(s(t)-s_{-}) & when\ s(t) < s_{-} \\ 0 & when\ s_{-} < s(t) < s_{+} \\ -k_{+}(s(t)-s_{+}) & when\ s(t) > s_{+} \end{cases} \tag{13}$$

where k_- and k_+ give the stiffness of the springs, and s_- is a negative constant and s_+ is a positive constant. This allows different spring stiffnesses in the two directions, and the end-stop device can start working at different excursions in positive and negative direction. This expression uses a linear spring (working only in one direction from its equilibrium position), but it should also be possible to use a nonlinear spring if that is desirable.

The damping force term can be described by the following formula

$$F_{fric}(t) = \begin{cases} B_{-}u(t)u(t) & when\ s(t) < s_{-}\ and\ u(t) < 0 \\ 0 & when\ s_{-} < s(t) < s_{+} \\ -B_{+}u(t)u(t) & when\ s(t) > s_{+}\ and\ u(t) > 0 \end{cases} \tag{14}$$

This models a piston-and-cylinder part of the end-stop device, with one cylinder in each end of the main cylinder of the WEC. When oil is forced out of these cylinders it flows through orifices. This creates a pressure in the cylinder, and thereby a force which tends to stop the motion of the WEC. The flow through the orifices has here been modelled to be turbulent. A linear term could also have been included to take into account laminar flow. However, this has not been done in the present model. It is assumed that the spring force is sufficient to reset the damping mechanism, or otherwise that the oil can flow freely back into the cylinder when the WEC starts moving in the opposite direction. The constants B_- and B_+ are determined by the design of the end-stop piston-and-cylinder.

Appendix C. Constrained oscillation in sinusoidal waves

For a body described by the mathematical model given in chapter 2, but with a control force that can be given an arbitrary variation with time, it can be shown that the mean power input to the conversion machinery is given by[1]

$$P = \frac{1}{2}|\hat{F}_e||\hat{u}|\cos(\beta) - \frac{1}{2}(R_r + R_f)|\hat{u}|^2 \qquad (15)$$

when both incident wave and buoy motion are sinusoidal. Here $\hat{F}_e$ is the complex amplitude of the excitation force, $\hat{u}$ is the complex velocity amplitude and β is the phase angle between the excitation force and the velocity. Further, R_r is the radiation resistance and R_f is the friction resistance. When the oscillation is unconstrained, the optimum velocity, which maximise the power, is given by

$$\hat{u} = \frac{\hat{F}_e}{2(R_r + R_f)} \qquad (16)$$

for which the power is

$$P_{\max} = \frac{|\hat{F}_e|^2}{8(R_r + R_f)} \qquad (17)$$

If the oscillation is constrained, and the buoy is oscillating sinusoidally with excursion amplitude equal to the maximum value, l_{max}, and velocity amplitude $|\hat{u}| = \omega l_{\max}$, the power is given by

$$P = \frac{\omega l_{\max}}{2}|\hat{F}_e|\cos(\beta) - \frac{1}{2}(R_f + R_r)(\omega l_{\max})^2 \qquad (18)$$

which has its maximum value for $\beta = 0$. This means that the excitation force and buoy velocity should be in phase. It is then assumed that the control force necessary to establish this oscillation is available. Table 7 gives the maximum power input to the conversion machinery as function of wave height and wave period, obtained from equation (17) and (18), for the same problem as investigated in the main text.

Appendix D. Procedure for control in irregular waves

For operation of a wave-energy converter with a control facility, in irregular waves, it is necessary to develop some sort of strategy on how to absorb as much energy as possible, and at the same time protect the machinery in large waves. In this case, this means that we need a strategy on how to operate the controllable valve. The procedure given here has the excitation force as input, but it could also have been based on the incident wave. A schematic description of the procedure is given in figure 12, and the numbers beside the boxes correspond to the numbers in parentheses in the following text. The intention of this procedure is to have coinciding extrema of excitation force and buoy velocity when the waves are small. For larger waves the excursion is constrained by opening the controllable valve later, which means that the velocity phase is delayed.

T [s]	H [m]					
	0.5	1	1.5	2	2.5	3
4	3.5*	13.4	24.6	35.8	47	58.2
5	6.9	20.2	33.5	46.8	60.1	73.5
6	9.9	23.7	37.5	51.4	65.2	79
7	11.2	24.7	38.3	51.8	65.4	78.9
8	11.4	24.4	37.4	50.4	63.3	76.3
9	11.2	23.4	35.7	47.9	60.1	72.3

Table 7. *Maximum power input to conversion machinery for constrained oscillation,in kW, as given by equation (18). For the result marked with * the oscillation is unconstrained, and the result is given by equation (17).*

For each time step in the main program, a decision is made on whether the controllable valve should be open or closed during the next time step, and the procedure making this decision can be described as follows. When entering the procedure, the present time of the simulation is given by t. If the valve is closed, when the procedure is entered, it should be determined if it should remain closed or if it should be opened (2). It is assumed that the excitation force has been predicted a certain interval into the future, T_{pred} , and this can be considered as the input to the procedure. First, it is decided whether the previous extremum should be used to determine the opening instant of the controllable valve, as is the case when the valve is opened after the extremum (3). The previous extremum is chosen if it was a maximum and $F_e(t) > F_{max,1}$, or if it was a minimum and $F_e(t) < F_{min,1}$. The constants $F_{max,1}$ and $F_{min,1}$ are design specific. If one of these conditions is fulfilled, the opening of the valve relative to the previous extremum has been delayed, to constrain the excursion. Otherwise, the first three future extrema of the excitation force are located (3). This is done by examining the time derivative of the excitation force. If there are less than three extrema in the interval where the excitation force is known, the remaining extrema are assumed to be after the end of the interval. If the first and third extremum, which are in the same direction, are separated by more than a certain predetermined time interval (in this case half the natural period of the buoy (T_0 /2) has been used) the first extremum is chosen as input to the control. If the extrema are close, it will not be possible for the buoy to move significantly in this ("interextreme") time interval, and either the first or third extremum is chosen as input to the control. If the first extremum is a maximum, the extremum with highest maximum value (usually positive) is chosen, or if the first extremum is a minimum the extremum with lowest minimum value (usually negative) is chosen. The time of the extremum used for the control, is denoted t_{extr} (3).

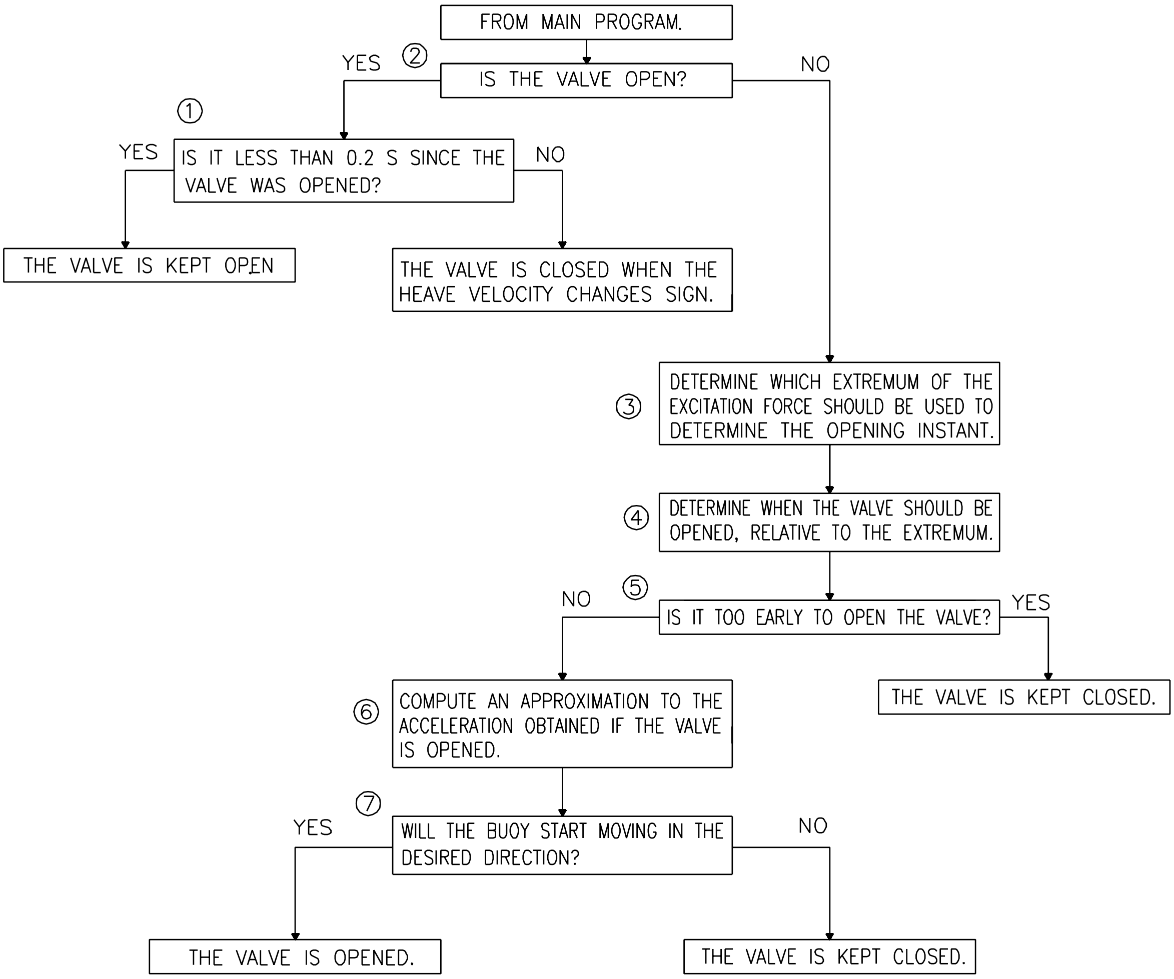

Figure 12. *Schematic description of the procedure that controls the operable valve.*

It should then be determined when the valve should be opened relative to the extremum (4), and we will first consider a maximum. If the selected extremum is in the future, that is $t_{extr} > t$, and $F_e(t_{extr}) < F_{max,2}$, the opening instant for the valve, t_{open}, is set to be a quarter of a natural period before the extremum, that is t_{extr}-T_0 /4. The constant $F_{max,2}$ is design specific. The amplitude of the excitation force extremum is then so small that it is not necessary to constrain the excursion. Otherwise, it is searched for an opening instant, after t_{extr} , for which $F_e(t_{open}) < F_{max,2}$ and $F_e(t_{open}+T_0/2) < F_{max,1}$, or for which $F_e(t_{open}+T_0/4) < F_{max,1}/2$. The first condition will apply when the change in the excitation force is relatively slow, and when the excitation force fulfil this condition the excursion is likely to be somewhat below the design limit. However, if the excitation force changes more rapidly this condition will result in an excursion significantly below the design limit, because the opening instant is delayed to long.

This is why the second condition is included, to open the valve earlier if the change in the excitation force is rapid. Note that using the control strategy presented here, the time delay will always be $T_0/4$ or longer, relative to the unconstrained opening instant, when the oscillation is constrained. This is a result of the relative simplicity of the control strategy, and it will probably result in a slight reduction of the power production for certain wave conditions, compared to a control strategy where the delay can have any value.

Similar conditions apply if the selected extremum is a minimum. If the extremum is in the future, that is $t_{extr} > t$, and $F_e(t_{extr}) > F_{min,2}$, the opening instant for the valve is set to t_{extr}-$T_0/4$. The constant $F_{min,2}$ is design specific. Otherwise, it is searched for an opening instant, after t_{extr}, for which $F_e(t_{open}) > F_{min,2}$ and $F_e(t_{open} + T_0/2) > F_{min,1}$, or for which $F_e(t_{open} + T_0/4) > F_{min,1}/2$.

The parameters $F_{max,1}$ and $F_{max,2}$ are positive, and $F_{min,1}$ and $F_{min,2}$ are negative, and must be determined for each design. If the absolute values of these parameters are increased, the excursion of the buoy in large waves will increase, and the power production will increase. However, the end-stop device will also be in operation more frequently, which is not desirable. In this case the parameters have been determined by running the program several times, for different waves based on a one-parameter Pierson-Moskowitz spectrum. The parameters were changed to maximise the power production, and at the same time keep the excursion within the desired limits. The following parameters have been used in the present calculations $T_0 = 2.2$ s, $T_{pred} = 4.4$ s, $F_{max,1} = 40$ kN, $F_{max,2} = 50$ kN, $F_{min,1} = -35$ kN and $F_{min,2} = -45$ kN. Note that the value used for the natural period is slightly shorter than the value which was determined from the free oscillation. The value used here was found to give maximum power production for unconstrained oscillation in sinusoidal waves.

When the opening instant t_{open} has been determined, it should be determined if the valve should be open during the next time step. If $t < t_{open}$ the valve remains closed, because it is to early to open it (5). Otherwise, an approximation to what the acceleration of the buoy will be, if the valve were opened, is computed (6). This is done by assuming that the cylinder pressure will be equal to the pressure in accumulator *A* (see figure 4), when the valve is opened. If the buoy starts moving in the desired direction, that is, upwards if the extremum of the excitation force is a maximum and downwards if the extremum is a minimum, the valve is opened, otherwise it remains closed.

If the valve is open when entering the procedure, it should be determined whether the valve should remain open, or be closed. If it is less than a certain time interval since the valve was opened, in the present case 0.2 s is used, it remains open (1). This is done so that the buoy should have time to start moving in the desired direction, and since it is not desirable that the valve should open and close too often. Afterwards, the valve is closed when the (heave) velocity of the buoy changes sign, which is also when the excursion has its extreme value. In this way the flow through the valve is approximately zero at the instant of closing the valve.

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