~~near-field region. It will be shown later, in chapter P, that this energy difference is related to the radiation reactance and hence to the added mass through the relation~~

$$\frac{1}{2} m_{jj} |\hat{u}_j|^2/2 = \frac{1}{4} (X_{jj}/\omega) |\hat{u}_j|^2 = \mathcal{E}_k - \mathcal{E}_p \qquad \text{(Cf. eq. 5.188)}$$

~~This relation, valid for the case of oscillation in one mode, only, can be generalised to the case of oscillation in several modes, simultaneously. The relation is given here without proof. However, refering to the discussion in the conclusion of section 2.3 A.4. the given relation seems very reasonable.~~

~~The radiation reactance is related to reactive power from the wave-generating oscillating body when the stored potential energy does not balance the stored kinetic energy in the near field region of the body.~~

Subsection 5.2.3 (p. 128) is an extension of the following simpler example.

D.4 A WAVE GENERATOR WITHOUT ADDED MASS

We shall consider generation of a harmonical wave

$$\hat{\eta} = A\, e^{-ikx} \qquad (4.89) \quad [D44]$$

in a wave channel with vertical walls parallel to the x axis. See fig. D.4. An absorbing beach at the other end of the wave channel ensures that there is no reflected wave. Alternatively we could assume that the wave channel is infinitely long. We assume that the wave is plane, that is, there is no cross wave in the channel.

The corresponding velocity potential is

$$\hat{\phi} = \frac{-g}{i\omega} A\, e(kz) e^{-ikx} \qquad (4.92) \quad [D45]$$

The particle velocity has an x component

$$\hat{v} = \frac{\partial\hat{\phi}}{\partial x} = \frac{gk}{\omega} A\ e(kz)e^{-ikx} \quad [D46]$$

(cf. eq. 4.44)

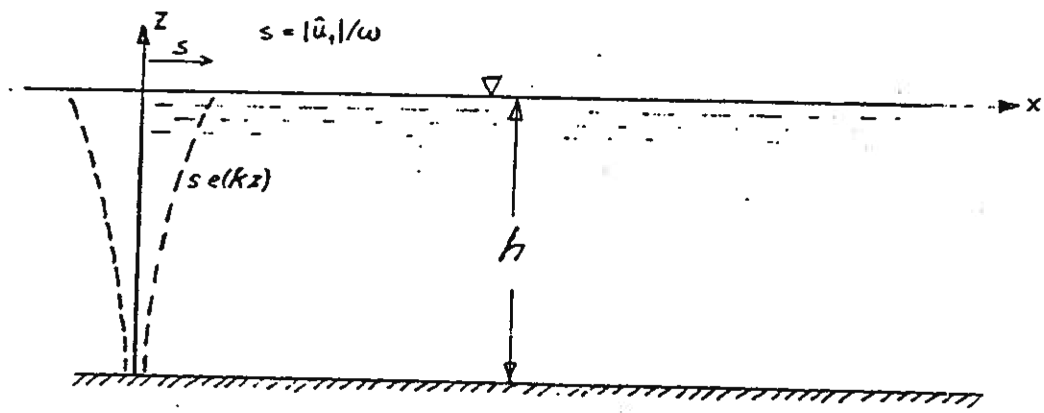

Fig. D.4. A vertical flexible flap as a generator of a two-dimensional wave.

Surging wavemaker

We consider a wave generator in the form of a vertical plate at $x = 0$. The plate is flexible and performs small horizontal oscillations where the oscillating speed is the following function of the depth

$$\hat{u} = \hat{u}(z) = \hat{u}_1 e(kz) \quad [D47]$$

where $\hat{u}_1$ is a constant. The maximum excursion of the plate is indicated by dashed curves in fig. D.4. Note that for $z = 0$ we have $\hat{u} = \hat{u}_1$. The boundary condition $\hat{v}_x = \hat{u}_x$ at $x = 0$ is satisfied by the plane wave [D46] and this special $\hat{u}(z)$ - given by eq. [D47] - because u_x and v_x vary with z in the same manner. The boundary condition gives

$$\hat{u}_1 e(kz) = \hat{u}_x = \hat{v}_x\Big|_{x=0} = \frac{gk}{\omega} A\ e(kz)e^{-i0} \quad [D48]$$

Accordingly we have

$$A = \frac{\omega}{gk}\hat{u}_1 \quad [D49]$$

$$\hat{\eta} = \frac{\omega}{gk}\hat{u}_1 e^{-ikx} \quad [D50]$$

(32)

(32)

$$\hat{\phi} = \frac{-1}{ik}\hat{u}_1 e(kz)e^{-ikx} = \varphi_1 \hat{u}_1 \qquad [D51]$$

$$\varphi_1 = \frac{-1}{ik} e(kz)e^{-ikx} \qquad [D52]$$

With this special wave generator we have obtained a far-field which is valid even close to the wave generator. Accordingly there is no near-field in this special case.

Let us now make a direct derivation of the radiation impedance. The hydrodynamical pressure is according to eq. (4.39) [B33]

$$\hat{p} = -i\omega\rho\hat{\phi} = \frac{\omega\rho}{k}\hat{u}_1 e(kz)e^{-ikx} \qquad [D53]$$

and the fluid particle velocity is

$$\hat{v}_x = \frac{\partial\hat{\phi}}{\partial x} = \hat{u}_1 e(kz)e^{-ikx} \qquad [D54]$$

On the wave-generating plate at $x = 0$ we have

$$\frac{1}{2}\hat{p}\hat{v}^* = \frac{1}{2}\frac{\omega\rho}{k} e^2(kz)|\hat{u}_1|^2 \qquad [D56]$$

Note that this is real. That means we have no reactive power. The power delivered by the wave generator is found by integration over the oscillating plate

$$\iint_S \frac{1}{2}\hat{p}\hat{v}^* dS = \frac{1}{2}|\hat{u}_1|^2 \frac{\omega\rho d}{k}\int_{-h}^{0} e^2(kz)\,dz$$

where d is the width of the wave-channel and the wave generator. The radiation impedance is according to eq. (5.46) [D42]

$$Z_{11} = \frac{\omega\rho d}{k}\int_{-h}^{0} e^2(kz)\,dz \qquad [D58]$$

Using Eq. (4.112) (B73) we find

$$Z_{11} = \frac{\omega\rho d}{2k^2} D(kh) \qquad [D59]$$

Note that

$$Z_{11} = R_{11} + i\omega m_{11} \qquad [D60]$$

is purely real. Hence, the added mass is zero, $m_{11} = 0$ for this particular wave generator.

$$Z_{11} = R_{11} = \frac{\omega\rho D(kh)d}{2k^2} \qquad [D61]$$

The radiated power is

$$P_r = \frac{1}{2} R_{11} |\hat{u}_1|^2 = \frac{\omega\rho D(kh)d}{2\cdot 2\cdot k^2} |\hat{u}_1|^2 =$$

$$= \left|\frac{\omega\hat{u}_1}{gk}\right|^2 \frac{\rho g^2 D(kh)d}{4\omega} = \frac{\rho g^2 D(kh)d}{4\omega} |\eta|^2 = Jd \qquad [D62]$$

in accordance with eq. ~~[B75]~~. (4.136)

Note that the added mass vanishes only at one particular frequency, namely that frequency for which k and hence, $e(kz)$ matches the designed flexible-plate wave generator.

Heaving wavemaker (extension to subsection 5.2.3)

We can use the obtained results to find the radiation impedance for a wave generator operating in the heave mode. Consider the heaving wedge-shaped body indicated in fig. D.5. The body is plane and vertical on the left hand side. On the right-hand side it is bounded by the wave-generating surface $x = be_1(kz)$. As argued below, we shall choose the function $e_1(kz)$ as

$$e_1(kz) = \frac{\sinh(kz+kh)}{\cosh(kh)} \qquad [D63]$$

in order to obtain vanishing added mass. The constant b is assumed to be small of the same order as the wave amplitude $|\hat{\eta}|$ is small. Then it is consistent, in linear theory, to make the approximation of applying the boundary condition (5.11) [D18] at $x = 0$ instead of at the wave-generating surface $x = be_1(kz)$.

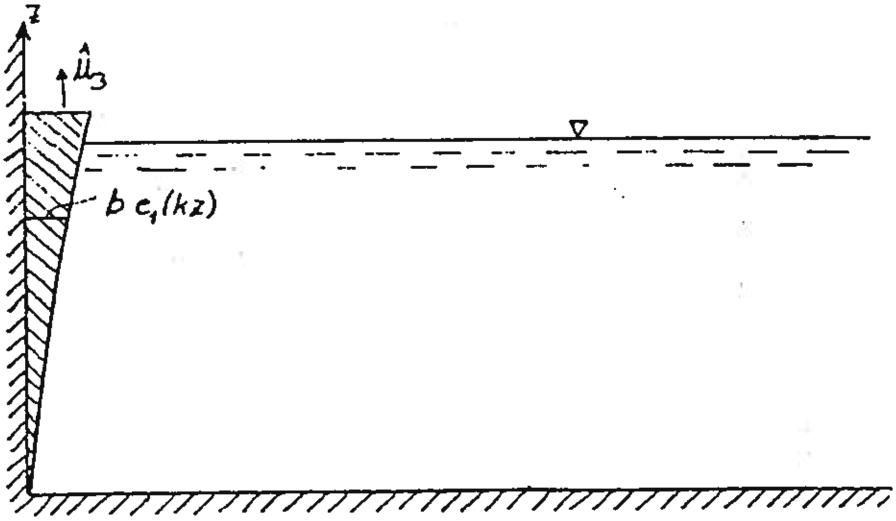

Fig. D.5. A heaving wedge as wave generator.

We have (cf. fig. D.6)

$$\tan\alpha = b\,\frac{de_1(kz)}{dz} = bk\,e_1'(kz) \qquad [D64]$$

Since b and, hence, α are small, we have $\sin\alpha \approx \alpha \approx \tan\alpha$.

The boundary condition ~~[D18]~~ (5.11) gives

$$\frac{\partial\hat{\phi}}{\partial n} = \hat{v}_n = \hat{u}_3 n_3 = \hat{u}_3\cos(n,z) = -\hat{u}_3\sin\alpha = -\hat{u}_3\alpha \qquad [D65]$$

At the wave-generating surface S we have

$$\hat{v}_x\big|_S = \hat{v}_n\cos\alpha \approx \hat{v}_n = \hat{u}_3 n_3 \approx -\hat{u}_3\alpha$$

[D66]

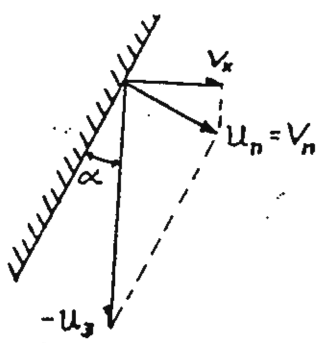

Fig. D.6. Enlarged portion of the heaving wedge. (Sketch to derive the wedge-shape function $e_1(kz)$)

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We now require the following condition to hold at x = 0 , D

$$\hat{v}_x\big|_{x=0} \approx \hat{v}_x\big|_S \approx -\hat{u}_3\alpha \approx -\hat{u}_3 bk\, e_1'(kz) \qquad [67]$$

We wish to match this condition to a solution of the form where $\hat{v}_x$ is proportional to $e(kz)e^{-ikx}$. In order to obtain this we may choose, for instance,

$$e_1(kz) = \frac{\sinh(kz+kh)}{\cosh(kh)}$$

such that

$$e_1'(kz) = e(kz) = \frac{\cosh(kz+kh)}{\cosh(kh)}$$

$$v_x\big|_{x=0} \approx -\hat{u}_3 bke(kz)$$

Hence,

$$v_x = -\hat{u}_3 bke(kz)e^{-ikx}$$

In the previously derived formulas we have to replace $\hat{u}_1$ by $-\hat{u}_3 bk$. The radiated power is now

$$\iint \frac{1}{2}\,\hat{p}\hat{v}^* dS = \frac{1}{2}\,|bk\hat{u}_3|^2\,\frac{\omega\rho d}{k}\int_{-h}^{0} e^2(kz)dz \qquad [D69]$$

and the radiation impedance is

$$Z_{33} = \frac{\omega\rho b^2 d}{2}\,D(kh) \qquad [D70]$$

Note that this is real. The added mass m_{33} vanishes. However, this is true only at one particular frequency, as mentioned above.

~~D.5.~~ ~~WAVE INTERACTION BY AN OSCILLATING VERTICAL PLATE~~

~~Wave radiation~~

~~A vertical plate as indicated in Fig.~~ 5.4 ~~D.7~~ ~~is forced to perform horizontal oscillating motion. Thereby it generates a wave which propagates from the plate at x = 0 . As a generalisation of~~

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Illustration to eq. (5.79)

See textbook page 131

On the contrary, if we, as indicated in fig. D.8, choose $c(z) = \cos(m_1 z + m_1 h)/\cos(m_1 h)$ we generate only one evanescent mode and no progressive wave. This shows us that it is, in fact, possible to make oscillations in the water without producing a propagating wave. This may be of use if it is desired that an oscillating body in the sea shall not generate undesirable waves. Cf. R. Meir, The development of the oscillating water column, pp. 35-42, Proc. Wave Energy Conference, Heathrow Hotel, London, 22-23 Nov. 1978.

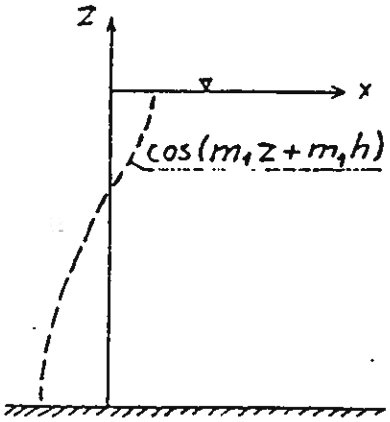

Fig. D.8. A wave generator with velocity distribution function $c(z) = \cos(m_1(z+h))/\cos(m_1 h)$ produces only an evanescent wave and no propagating wave.

~~Radiation resistance and excitation force of vertical stiff plate~~

~~Finally, let us consider the case of a stiff surging vertical plate. That is, we choose $c(z) \equiv 1$. Then, in general, $c_n \neq 0$ for all n. Cf. problem 5.3 ~~D1~~. In particular, we have~~

$$c_o = (i\omega^2/gk^3)\sqrt{2k/D(kh)}$$

~~The far-field coefficient is~~

$$a_1^+ = 2\omega^3/(g^2k^2D) \qquad (5.83)$$

~~and the radiation resistance is~~

$$R_{11} = \frac{2\omega^5\rho d}{g^2k^4D(kh)} \qquad (5.82)$$

~~If a wave~~

$$\hat{\eta}_o = A\, e^{ikx} \qquad (5.83)$$

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(37)