Optimum control for maximum power delivery

In the system considered above a simple power take-off device was assumed, where the load force is proportional to the oscillating velocity (with $-R_u$ as the coefficient of proportionality).

Let us now assume a power take-off device which is technologically more advanced, and for which a load force [cf. eq. (6.21)]

$$F_u(\omega) = -Z_u(\omega)u(\omega) \quad \text{or} \quad F_{u,t}(t) = -z_u(t)*u_t(t) \qquad [T113]$$

may be achieved. Then eq. (6.26) ~~[T110]~~ is to be replaced by

$$(Z_i(\omega) + Z_u(\omega))\, u(\omega) = F_e(\omega) = f(\omega)A(\omega) \qquad [T114]$$

We now wish to investigate how the load impedance $Z_u(\omega)$ or its inverse Fourier transform $z_u(t)$ should be chosen in order to maximise the converted useful power. The converted useful energy is

$$E_u = -\int_{-\infty}^{\infty} F_{u,t}(t)u_t(t)dt = -\frac{1}{2\pi}\int_{-\infty}^{\infty} F_u(\omega)u(-\omega)d\omega \qquad [T115]$$

The last expression here is obtained by using the frequency convolution theorem (2.150) ~~[T26]~~ and setting $\omega = 0$, or by using the so-called Parseval's formula. Now, since $F_{u,t}(t)$ and $u_t(t)$ are real functions, eq. (2.109) ~~[T16]~~ is applicable. Further, using also eq. [T113], we may rewrite eq. [115] as,

$$E_u = -\frac{1}{2\pi}\int_0^{\infty} (F_u(\omega)u^*(\omega) + F_u^*(\omega)u(\omega))d\omega = \qquad (6.29)\ [T116]$$

$$= -\frac{1}{\pi}\int_0^{\infty} \mathrm{Re}\{F_u(\omega)u^*(\omega)\}d\omega = \frac{1}{\pi}\int_0^{\infty} \mathrm{Re}\{Z_u(\omega)u(\omega)u^*(\omega)\}d\omega \qquad [T117,118]$$

Using the dynamic equation [T114] this becomes

$$E_u = \frac{1}{\pi}\int_0^{\infty} \frac{|F_e(\omega)|^2\ \mathrm{Re}\{Z_u(\omega)\}}{|Z_i(\omega) + Z_u(\omega)|^2}\, d\omega \qquad [T119]$$

With a given incident wave and a given linear wave-absorbing system $F_e(\omega)$ and $Z_i(\omega)$ are given functions. Now in order to maximise the useful energy E_u we have to choose the load impedance

$$Z_u(\omega) = -Z_i^*(\omega) \qquad [T120]$$

in order to maximise the (non-negative) integrand in eq. [T119] for every ω. Cf. chapter 3 ~~A~~, section 3.5 ~~A.9~~. Thus, it is required that

$$\mathrm{Re}\{Z_u(\omega)\} = \mathrm{Re}\{Z_i(\omega)\} = R_i(\omega) \qquad [T121]$$

and that

$$\mathrm{Im}\{Z_u(\omega)\} = -\mathrm{Im}\{Z_i(\omega)\} = -X_i(\omega) \qquad [T122]$$

This latter condition corresponds to resonance. Cf. eq. ~~[A76]~~ (3.42). If condition [T120] is satisfied, eq. [T114] gives

$$u(\omega) = \frac{F_e(\omega)}{2\mathrm{Re}\{Z_i(\omega)\}} = \frac{F_e(\omega)}{2R_i(\omega)} \qquad [T123]$$

corresponding to eq. ~~[D199b]~~ (6.18) in the case of a regular incident wave. Eq. [T123] implies for each frequency that u is in phase with F_e.

The impulse response functions corresponding to the intrinsic mechanical impedance $Z_i(\omega)$ and to the load impedance $Z_u(\omega)$ are

$$z_i(t) = \mathcal{F}^{-1}\{Z_i(\omega)\} \quad \text{and} \quad z_u(t) = \mathcal{F}^{-1}\{Z_u(\omega)\} \qquad [T124]$$

respectively. More explicitly we have, by using also eqs. [T121] and [T122],

$$z_i(t) = \mathcal{F}^{-1}\{R_i(\omega) + i\,X_i(\omega)\} = r_i(t) + x_i(t) \qquad [T125]$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\{R_i(\omega) + i\,X_i(\omega)\}e^{i\omega t}d\omega \qquad [T126]$$

The system's "intrinsic" impulse response $z_i(t)$ is real and causal, and it represents a force response due to a unit velocity impulse $u(t) = \delta(t)$. Moreover

$$r_i(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} R_i(\omega)e^{i\omega t}d\omega = r_i^*(t) = \frac{1}{\pi}\int_{0}^{\infty} R_i(\omega)\cos(\omega t)d\omega \qquad [T127]$$

is an even function of t and

$$x_i(t) = i\,\frac{1}{2\pi}\int_{-\infty}^{\infty} X_i(\omega)e^{i\omega t}d\omega = x_i^*(t) = -\frac{1}{\pi}\int_{0}^{\infty} X_i(\omega)\sin(\omega t)d\omega \qquad [T128]$$

is an odd function of t. It is implied that the functions $r_i(t)$ and $x_i(t)$ are not causal. Since $z_i(t)$ is causal, $z_i(t) = 0$ for $t < 0$, that is

$$x_i(t) = -r_i(t) \quad \text{for } t < 0.$$

Thus for $t > 0$

$$x_i(t) = r_i(t) \qquad [T129]$$

and hence

$$z_i(t) = 2r_i(t) = 2x_i(t) \quad \text{for} \quad t > 0 \qquad [T131]$$

The optimum condition for maximum useful power is, according to eqs. [T120], [T121], [T122], [T124], [T127] and [T128],

$$z_u(t) = \mathcal{F}^{-1}\{Z_i^*(\omega)\} = r_i(t) - x_i(t) \qquad [T132]$$

Consequently

$$z_u(t) = 0 \quad \text{for} \quad t > 0\,, \qquad [T133]$$

and

$$z_u(t) = 2r_i(t) = -2x_i(t) \quad \text{for} \quad t < 0. \qquad [T134]$$

In general

$$z_u(t) = r_i(t) - x_i(t) = r_i(-t) + x_i(-t) = z_i(-t) \qquad [T135]$$

Hence the optimum condition $Z_u(\omega) = Z_i^*(\omega)$ in the frequency domain corresponds to $z_u(t) = z_i(-t)$ in the time domain. Thus the optimum load impedance $Z_u(\omega)$ is not a causal transfer function. Rather, it is an anti-causal function, that is $z_u(t) = 0$ for $t > 0$.

In the time domain

$$(z_i(t) + z_u(t)) * u_t(t) = F_{e,t}(t) \qquad [T136]$$

is the dynamic equation corresponding to the frequency-domain equation [T114]. Hence the load force (cf. eq. [T113]) is

$$F_{u,t}(t) = -z_u(t) * u_t(t) = -F_{e,t}(t) + z_i(t) * u_t(t). \qquad [T137]$$

At optimum we have from eqs. [T120], [T121], [T125] and [T135]

$$Z_u(\omega) + Z_i(\omega) = 2R_i(\omega) \qquad [T138]$$

and

$$z_u(t) + z_i(t) = 2r_i(t) \qquad [T139]$$

Using also eqs. [T114] and [T136] gives

$$2R_i(\omega)\, u(\omega) = F_e(\omega)$$

(in agreement with the optimum condition [T123]) and

$$2r_i(t) * u(t) = F_{e,t}(t) \qquad [T141]$$

An alternative expression for the optimum load force is

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$$F_{u,t}(t) \equiv -z_u(t) * u_t(t) = -z_i(-t) * u_t(t) \qquad \text{[T142]}$$

Assume that $F_{e,t}(t)$ and $u_t(t)$ are known at time t together with their history for earlier times. (Remember that $z_i(t)$ is causal.) The optimum load force (control force) is

$$F_{u,t}(t) = -z_i(-t)*u_t(t) = -\int_{-\infty}^{\infty} z_i(-\tau)u_t(t-\tau)d\tau = -\int_{-\infty}^{0} z_i(-\tau)u_t(t-\tau)d\tau$$

$$= \int_{\infty}^{0} z_i(\sigma)\, u_t(t+\sigma)d\sigma = -\int_{0}^{\infty} z_i(\tau)\, u_t(t+\tau)d\tau \qquad \text{[T145,146]}$$

The optimum load force $F_{u,t}(t)$ is given by this integral if the oscillating velocity $u_t(t)$ is known not only at time t, but also at all future times $t + \tau$ $(0 < \tau < \infty)$. For an exact optimum control, it is necessary to predict the excitation force correctly into all future times! (Naito, 1985). For an approximate optimum control, however, prediction is needed only for a rather short time into the future, as the following discussion will indicate.

Using transformation pairs from table ~~T1~~ 2.2 the inverse transform of the intrinsic mechanical impedance $Z_i(\omega)$, eq. [T112], is

$$z_i(t) = k(t) + R_f\delta(t) + (m(\infty) + m)\,\dot{\delta}(t) + SU(t) \qquad (5.335)\ \text{[T147]}$$

Before applying this to eq. [T146] we make the following remarks. Firstly, we note that

$$\int_{-\infty}^{\infty} \delta(t)\varphi(t)dt = \varphi(0) \quad \text{and} \quad \int_{-\infty}^{\infty} \dot{\delta}(t)\varphi(t)dt = -\dot{\varphi}(0) \qquad \text{[T148]}$$

where $\varphi(t)$ is an arbitrary function which is differentiable at $t = 0$. Secondly, we shall assume that

$$\Delta s \equiv s_t(\infty) - s_t(-\infty) = \int_{-\infty}^{\infty} u_t(t)dt = 0 \qquad \text{[T149]}$$

Practically, this means that the heaving body is assumed to have the same calm-sea equilibrium position e.g. before and after a stormy time interval. This means that

$$-\int_{t}^{\infty} u_t(\tau)d\tau = \int_{-\infty}^{t} u_t(\tau)d\tau = s_t(t) \qquad \text{[T150]}$$

Using this, the optimum load force is, according to eqs. [T146] and [T147],

$$F_{u,t}(t) = -\int_{0}^{\infty} k(\tau)u_t(t+\tau)d\tau - R_f u_t(t) + (m(\infty) + m)\,\dot{u}(t) + Ss_t(t) \qquad \text{[T151]}$$

If the body's heave position $s_t(t)$, velocity $u_t(t)$ and acceleration $\dot{u}_t(t)$ are measured at all times, instantaneous knowledge is available on the contributions to the optimum load force represented by the last three terms on the right-hand side of eq. [T151]. In contrast, the value of the first term (the integral) at instant t requires knowledge of the heave velocity $u_t(t+\tau)$ at future instants $t + \tau$. The contribution to the integral is, however, negligible for times τ exceeding the characteristic time for the impulse response function $k(t)$. Since this impulse response is a decaying function, an acceptable, however still approximate, control is obtained provided a reasonably good prediction into a certain future time interval is available. We have to predict as long into the future as long as the system's impulse response $k(t)$ "remembers" into the past.

Note that eq. [T151] gives the optimum load force in terms of the optimum body motion. The relation between the optimum motion and the given excitation force is determined by

$$F_{e,t}(t)/2 = k_e(t) * u_t(t) + R_f u_t(t) + S\Delta s/2 \qquad [T152]$$

which is obtained by using eq. ~~[T112]~~ (5.109, 5.320) and by applying the inverse Fourier transformation to eq. [T123]. Here

$$k_e(t) = \mathcal{F}^{-1}\{R(\omega)\} \qquad [T153]$$

is the even part of the causal impulse response $k(t)$. Note that the last term of eq. [T152] vanishes in view of the assumption [T149]. By using eq. [T147] the general (i.e. not optimum) load force given by eq. [T137] may be written as

$$F_{u,t}(t) = -F_{e,t}(t) + k(t) * u_t(t) + R_f u_t(t) + (m(\infty) + m)\,\dot{u}_t(t) + Ss_t(t) \qquad [T154]$$

which is simply the dynamic equation of motion. By subtraction between eqs. [T154] and [T151], the optimum condition [T152] is obtained, when we observe that the first right-hand term in eq. [T151] may be written as $-k(-t)*u_t(t)$. By addition of the two equations [T154] and [T151] the optimum load force may be written as

$$F_{u,t}(t) = -F_{e,t}(t)/2 + k_o(t) * u_t(t) + (m(\infty) + m)\,\dot{u}_t(t) + Ss_t(t) - \frac{S}{2}\Delta s \qquad [T155]$$

where

$$k_o(t) = k(t) - k_e(t) = \mathcal{F}^{-1}\{K(\omega) - R(\omega)\} \qquad [T156]$$

is the odd part of the causal impulse response $k(t)$. Eq. [T155] indicates how the optimum load force together with half of the excitation force contributes to the reactive power associated with the optimum heave oscillation.

It may be of interest to find an explicit expression for the optimum heave velocity. By using eq. ~~[T111]~~ (5.320) in the optimum condition [T123] we obtain

$$u(\omega) = \frac{F_e(\omega)\,/2}{R(\omega)+R_f} \quad \text{for } \omega \neq 0 \qquad [T157]$$

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Moreover, we require $u(\omega) = 0$ for $\omega = 0$. This is consistent with our assumption [T149], that $\Delta s = 0$. Also we expect the excitation force $F_e(\omega)$ to have a vanishing frequency component for $\omega = 0$. Observing that the loss resistance R_f is assumed to be finite and independent of frequency, while the radiation resistance $R(\omega)$ tends to zero as $\omega \to \infty$ (and as $\omega \to 0$), we rewrite eq. [T157] as:

$$u(\omega) = \left(\frac{1}{2R_f} - \frac{R(\omega)}{2R_f(R(\omega)+R_f)}\right) F_e(\omega) \quad \text{for} \quad \omega \neq 0$$

Defining a function

$$V(\omega) = \begin{cases} \frac{R(\omega)}{R_f(R(\omega)+R_f)} & \text{for } \omega \neq 0 \\ \frac{1}{R_f} & \text{for } \omega = 0 \end{cases} \qquad \text{[T159]}$$

we then write for all ω,

$$u(\omega) = \left(\frac{1}{2R_f} - V(\omega)/2\right) F_e(\omega) \qquad \text{[T160]}$$

Then the optimum heave velocity is

$$u_t(t) = \frac{F_{e,t}(t)}{2R_f} - v(t)*F_{e,t}(t)/2 \qquad \text{[T161]}$$

where

$$v(t) = \mathcal{F}^{-1}\{V(\omega)\} \qquad \text{[T162]}$$

Note that $V(\omega)$ and $v(t)$ are even functions of ω and t, respectively. Hence, $v(t)$ is not a causal impulse response. We expect, however, that the decay time of this impulse response, is of the same order of magnitude as the decay time of $k(t)$. Because of the non-causality of the function $v(t)$, a prediction of the excitation force $F_e(t)$ a certain time into the future is required to find the optimum heave velocity $u_t(t)$.

For a reasonably successful optimum operation, prediction (of the incident wave and/or the wave-energy converter's oscillation) is required some seconds into the future. The coherence time of real ocean waves is probably sufficiently long to make such a prediction feasible in practice.

REFERENCES (TO CHAPTER T) 13, 14, 21, 34, 49, 50, 51 +

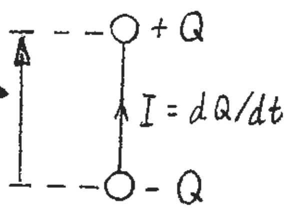
Oltedal, G.: Pneumatic wave-power conversion by a phase-controlled buoy. Dr.ing. thesis, Institutt for eksperimentalfysikk, NTH, Trondheim 1985.

Oltedal, G.: Simulation of a pneumatic wave-power buoy with phase control, D.V. Evans and A. Falcão (eds.): Hydrodynamics of Ocean Wave-Energy Utilization, IUTAM Symposium, Lisbon, Portugal 1985, pp.303-313, (Springer Verlag Berlin 1986).

Sarmento, A.J.N.A. and Perdigão, J.N.B.A.: On the time-domain analysis of OWC devices for wave-energy absorption. Offshore Mechanics and Arctic Engineering, OMAE88, Houston, USA (1988).

Stråling frå oscillerande elektrisk dipol.

Oscillerande dipol

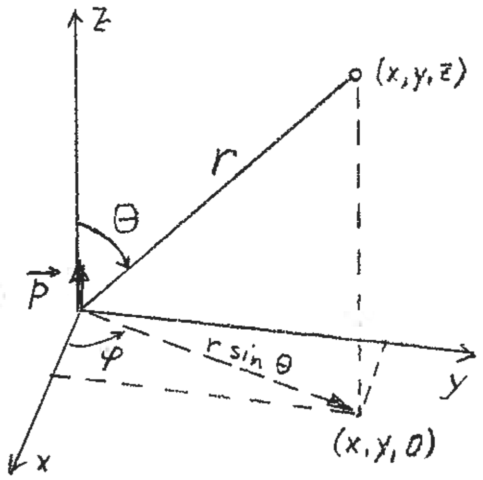


$$p = Ql = \hat{p}e^{i\omega t} \qquad \text{(A84)}$$

$$\hat{p} = \hat{Q}l$$

$$\hat{I}l = i\omega\hat{Q}l = i\omega\hat{p} = ikc\hat{p} \qquad \text{(A86)}$$

Frå elektromagnetisk feltteori kan utleiast (t.d. ved bruk av retarderte potensial) at feltet frå ein oscillerande dipol har følgjande komponentar



$$\hat{E}_\varphi = 0$$

$$\hat{H}_r = 0 \qquad \text{(A87)}$$

$$\hat{H}_\theta = 0$$

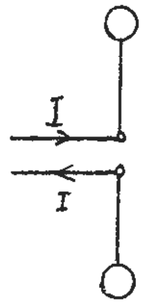
$$\hat{E}_r = \frac{(ik)^3\hat{p}}{4\pi\varepsilon_o}\left(\frac{2}{(ikr)^3} + \frac{2}{(ikr)^2} + 0\right)\cos\theta\, e^{-ikr}$$

$$\hat{E}_\theta = \frac{(ik)^3\hat{p}}{4\pi\varepsilon_o}\left(\frac{1}{(ikr)^3} + \frac{1}{(ikr)^2} + \frac{1}{ikr}\right)\sin\theta\, e^{-ikr} \qquad \text{(A88)}$$

$$\hat{H}_\varphi = \frac{(ik)^3c\hat{p}}{4\pi}\left(\underbrace{0 + \frac{1}{(ikr)^2}}_{\text{"nærfelt"}} + \underbrace{\frac{1}{ikr}}_{\text{fjernfelt}}\right)\sin\theta\, e^{-ikr}$$

Ein slik dipol $\hat{p}$ kan vera modell for eit atom der (middelpunktet av) den negative elektronladningen svingar att og fram i forhold til den positive kjerneladningen, som nærmast er i ro.

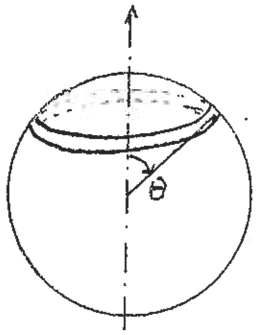
Dipolen $\hat{p}$ kan og vera modell for ein kort radioantenne med lengd $l \ll \lambda$ og matestraum



$$\hat{I} = i\omega\hat{p}/l. \qquad \text{(A89)}$$

Dette blir i radioteknikken kalla ein Hertz-dipol.

Effekten P_r som strålar ut frå dipolen, finn me ved å integrera Poyntingvektoren $\vec{I} = \bar{\bar{S}} = \frac{1}{2}\,\mathrm{Re}(\hat{\vec{E}} \times \hat{\vec{H}}^*)$ over ei kule omkring dipolen. Jfr. (A4) og (A51). Merk at felta er uavhengige av φ



$$P_r = \int_0^{\pi} I_r 2\pi r \sin\theta\, r d\theta =$$

$$= 2\pi \int_0^{\pi} I_r r^2 \sin\theta d\theta \qquad \text{(A90)}$$

Då mediet (vakuum eller luft) kan reknast for tapsfritt, må integralet vera uavhengig av radien. Det er difor berre fjernfeltet som gjev tilskot til P_r.

Me har altså bruk for intensiteten berre i fjernfeltet

$$I_r = \frac{1}{2}\,\mathrm{Re}(\hat{E}_\theta \times \hat{H}^*_\varphi) = \frac{1}{2}\,\mathrm{Re}\left(\frac{k^6 c \hat{p}\hat{p}^*}{16\pi^2\varepsilon_o}\,\frac{1}{k^2 r^2}\,\sin^2\theta\right) + O\{r^{-4}\}$$

NB: Leddet $O\{r^{-3}\}$ er reint imaginært.

$$I_r = \frac{ck^4\sin^2\theta}{32\pi^2\varepsilon_o r^2}\,|\hat{p}|^2 \qquad \text{(A91)}$$

$$P_r = 2\pi\,\frac{k^4 c|\hat{p}|^2}{32\pi^2\varepsilon_o}\int_0^{\pi}\sin^2\theta\sin\theta d\theta$$

$$\int_0^{\pi}\sin^3\theta d\theta = \int_{\theta=\pi}^{\theta=0}(1-\cos^2\theta)d(\cos\theta) = \int_{-1}^{1}(1-x^2)dx = 2(1-\tfrac{1}{3}) = \tfrac{4}{3}$$

Utstrålt effekt frå dipolen er altså

$$\boxed{P_r = \frac{k^4 c|\hat{p}|^2}{12\pi\varepsilon_o}} = \frac{\mu_o \omega^4 |\hat{p}|^2}{12\pi c} \qquad \text{(A92)}$$

Dipolantenne.

Effekten som strålar ut frå Hertz-dipolen finn me ved å bruka (A86)

$$P_r = \frac{k^4 c|\hat{I}|^2 (l/kc)^2}{12\pi\varepsilon_o} = \frac{1}{2}\,\frac{(kl)^2}{6\pi\varepsilon_o c}\,|\hat{I}|^2$$

Samanliknar me med (A60b) ser me at Hertz-dipolen har ein strålingsresistans

$$\boxed{R_{r,el} = \frac{(kl)^2}{6\pi\varepsilon_o c} = \frac{\eta_o c(kl)^2}{6\pi} = \frac{Z_o}{6\pi}(kl)^2 = (kl)^2\,\frac{377\Omega}{6\pi}} \qquad \text{(A94)}$$

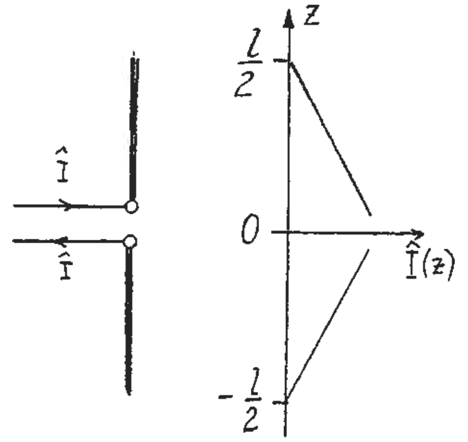
$$R_{r,el} = (kl)^2 20{,}0 \text{ ohm} \qquad \text{(A95)}$$

Når dipolføresetnaden $l \ll \lambda$ eller $kl \ll 1$ ikkje er oppfylt trengst ein modifisert analyse. Jfr. antenneteorien.

Formelen (A95) gjeld altså for så korte dipolantenner at $R_{r,el} < 20$ ohm.

Vi har utleidd formelen under føresetnad av at straumen $\hat{I}$ er konstant over heile lengda l. For ein tynn tråd er straumen null på enden.
Me kan definera ei effektiv lengd l_{eff} ved

$$\hat{I} l_{eff} = \int_{-l/2}^{l/2} \hat{I}(z)\,dz$$



der $\hat{I}$ er straumen ved matepunktet. Det resulterande dipolmomentet er

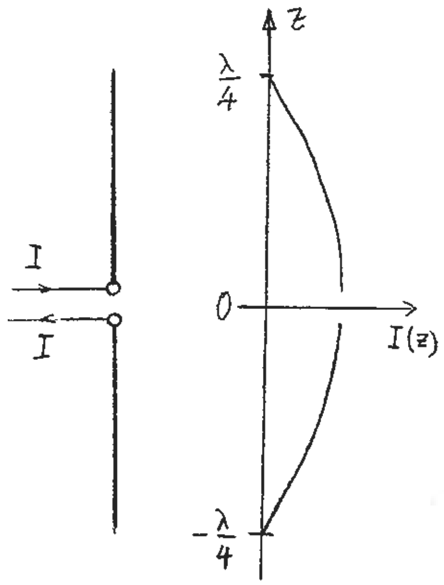
$$\hat{p} = \frac{\hat{I} l_{eff}}{i\omega}$$

slik at strålingsresistansen for dipolen er

$$R_{r,el} = (kl_{eff})^2\ 20{,}0\ \text{ohm} \qquad \text{(A95a)}$$

Merk at når $kl \ll 1$ vil fjernfelta frå alle antenneelementa dz interferera konstruktivt. For lengre antenner vil det bli destruktiv interferens og utrekninga av det resulterande fjernfeltet må ta omsyn til det. For ei tynn halvbølgje-antenne ($l = \lambda/2$) eller ein "halvbølgjedipol" gissar ein på ei straumfordeling

$$\hat{I}(z) = \hat{I} \cos kz$$



der $\hat{I}$ er straumen i matepunktet. Resultantfeltet blir som frå ein superposisjon av Hertz-dipolar med minkande dipolmoment utover mot enden av antenna. Ved integrasjon vil ein finna

$$R_{r,el} = 73{,}1\ \text{ohm}$$

Målte verdiar for praktiske halvbølgjedipolar ligg gjerne i området 65 til 72 ohm. Strålingsreaktansen som heng saman med nærfeltet (lagringsfeltet) er gjerne ca. 40 ohm.

Formelen (A95) brukt ukritisk gir for stor verdi når $kl = \pi$ ($\not\ll 1$), nemleg: $\pi^2 20$ ohm = 197 ohm.

Absorpsjonstverrsnitt for ein Hertz-dipol.

Me går ut frå at ei innfallande plan bølgje med vertikalpolarisert elektrisk felt $\hat{E}_i$ induserer ei eksitasjonsspenning

$$\hat{U}_e = \hat{E}_i l$$

i dipolen. Med bruk av (A94) og innsetjing i ~~(A83)~~ (3.45) får me

$$P_{max} = \frac{|\hat{E}_i l|^2}{8(kl)^2} \frac{6\pi\varepsilon_o c}{} = \frac{3\pi\varepsilon_o c}{4k^2} |\hat{E}_i|^2$$

Merk at l fell ut av samanhengen. Antennedimensjonen er ikkje avgjerande (anna enn at $l << \lambda$). ~~Etter (A50)~~ Som kjent. er den innfallande intensiteten

$$I_i = \frac{1}{2} \varepsilon_o c |\hat{E}_i|^2$$

Absorpsjonstverrsnittet blir

$$A_a = \frac{P_{max}}{I_i} = \frac{3}{2} \frac{\pi}{k^2} = \frac{3}{2} \frac{\lambda^2}{4\pi}$$

For ein rundstrålar er $A_a = \frac{\lambda^2}{4\pi}$. Jfr. ~~(A72) og oppg. A2.~~ problem 3.5
At me her får 50% større absorpsjonstverrsnitt heng saman med at Hertz-dipolen ikkje strålar isotropt, og vidare med at me her har gått ut frå den mest gunstige retningsorienteringa av mottakarantenna, nemleg med dipolen retta parallelt med polarisasjonsretninga for det elektriske feltet i den innfallande plane bølgja

Med $\nu = \omega/2\pi = 3 \cdot 10^6$ Hz er $\lambda = \frac{c}{\nu} = 100$ m og

$$A_a = \frac{3}{2} \frac{100^2}{4\pi} = 1194 \text{ m}^2 \qquad \text{(A96)}$$

trass i at antennelengda l er mykje mindre enn 100 m. Me ser at den absorberte effekten ved resonansavstemt og resistanstilpassa antennekrets ikkje er korrelert med dimensjonen på antenna.

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Vekselverknad mellom lys og eit atom (klassisk, og ikkje-kvantemekanisk teori her).

Ein molekyl eller eit atom blir elektrisk polarisert når der er eit elektrisk felt.

Det elektriske feltet i lyset induserer ein elektrisk dipol i partikkelen som oscillerer med lysfrekvensen. Den oscillerande dipolen sender ut lys i ymse retningar (mest i retninga $\theta = \frac{\pi}{2}$ - jfr. (A91) - der $\theta = 0$ svarar til dipol-retninga, d.v.s. retninga til feltet i det innfallande lyset som i atmosfæren er upolarisert lys).

Det spreidde lyset frå luftmolekylane er ansvarleg for at himmelen ser blå ut, trass i at sollyset har ein etter måten svak styrke i den blå enden av spektret. Det blå lyset, som har størst ω, dominerer likevel i det spreidde lyset p.g.a. faktoren ω^4 i (A92). Det raude lyset blir spreidd minst. Dette forklarer kvifor himmelen er raud-farga nær solrenning og solefall.

Desse effektane forsvinn ved overskya ver fordi partik-lane (vassdropane) i skyene er så store at føresetnaden for (A92), nemleg $l \ll \lambda$ ikkje gjeld. Skyene er kvite eller grå.

Sterkare vekselverknad mellom lys og atom, blir det når lyset er i resonans med atomet:

Me ser på ein klassisk harmonisk oscillator (oscillerande dipol) som ein modell for eit lys-absorberande og lys-emitterande atom. Atomet har altså ein eigenfrekvens ω

(svarande til spektrale absorpsjons- og emisjonslinjer).

I nullte orden lyder den oscillerande dipolen differen-siallikninga

$$m_e(\ddot{s} + \omega^2 s) = 0$$

Her er m_e - elektronmassen og s posisjonen til elektronet (eller middelposisjonen til "elektronskya"). Denne har ei løysing $s = s_o e^{i\omega t}$ svarande til udempa frie svingingar.

$$m_e(\ddot{s}\dot{s} + \omega^2 s\dot{s}) = 0$$

$$\frac{1}{2} m_e(\dot{s})^2 + \frac{1}{2}\omega^2 m_e s^2 = W = \text{konst}$$

$$W_k + W_p = W$$

): Totalenergien fordeler seg på kinetisk og potensiell energi. Utsvinget kan uttrykkjast ved dipolmomentet

$$p = es$$

$$W_k = \frac{1}{2} m_e(\dot{s})^2 = \frac{1}{2}\frac{m}{e^2}(\dot{p})^2$$

$$W_p = \frac{1}{2}\omega^2 m_e s^2 = \frac{1}{2} m \left(\frac{\omega}{e}\right)^2 p^2$$

$$W = W_k + W_p = \frac{1}{2}\frac{m}{e^2}(\dot{p}^2 + \omega^2 p^2) = \frac{1}{2}\frac{m}{e^2}\omega^2 p_o^2$$

Men dei frie svingingane er ikkje udempa. Energitapet pr. tidseining er etter (A92)

$$\dot{W} = -P_r = -\frac{\mu_o \omega^4 p_o^2}{12\pi c}$$

$$\frac{\dot{W}}{W} = -\frac{\mu_o \omega^4 \; 2e^2}{12\pi c \; m\omega^2} = -\frac{2\mu_o e^2 \omega^2}{12\pi c m}$$

Energien minkar altså eksponensielt med tida

$$W \propto e^{-\frac{2\mu_o e^2\omega^2}{12\pi c m}t}$$

$$s_o \propto p_o \propto W^{\frac{1}{2}} \propto e^{-\frac{\mu_o e^2 \omega^2}{12\pi c m}t} = e^{-\delta t}$$

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Dempingskoeffisienten

$$\delta = \frac{\mu_o e^2 \omega^2}{12\pi c m}$$

For atom er $\delta \ll \omega$.

Ser me på tvungne svingingar for oscillatoren, vil me ha ei skarp resonanskurve med halvverdibreidd

$$\Gamma = 2\delta = \frac{\mu_o e^2 \omega^2}{6\pi c m}$$

[Cf. (3.51), p. 53]

Jfr. t.d. Westin, Svingningslære, kap. 3., s. 12. Dette er den naturlege linjebreidda for spektrallinja. Den relative halvverdibreidda er

$$\frac{\Gamma}{\omega} = \frac{\mu_o e^2 \omega}{6\pi c m_e}$$

For synleg lys er $\frac{\Gamma}{\omega} \ll 1$.

$$\lambda = 2\pi/k = 2\pi c/\omega$$

$$\left|\frac{\Delta\lambda}{\lambda}\right| = \left|-\frac{\Delta\omega}{\omega}\right| \approx \left|-\frac{\Gamma}{\omega}\right|$$

Naturleg linjebreidd:

$$(\Delta\lambda)_{\text{naturleg}} = \frac{\Gamma}{\omega}\lambda = \frac{\mu_o e^2 \omega\lambda}{6\pi c m_e} = \frac{\mu_o e^2}{3m}$$

$$= \frac{4\pi \cdot 10^{-7} \cdot (1{,}6 \cdot 10^{-19})^2}{3 \cdot 9{,}11 \cdot 10^{-31}} = 1{,}18 \cdot 10^{-14}\,\text{m}$$

Synleg lys (gult) $\lambda = 0{,}56$ ηm

$$\frac{\Delta\lambda}{\lambda} = \frac{1{,}18 \cdot 10^{-14}}{0{,}56 \cdot 10^{-6}} = 2 \cdot 10^{-8} \qquad \text{(klassisk)}$$

I tillegg til den naturlege linjebreidda kan spektrallinja ha tilleggsbreidd (kollisjonsbreiing, dopplerbreiing).

Då $\delta = R/2m$ (jfr. Westin, Svingningslære) kan me gjerne definera ein <u>mekanisk strålingsresistans for atomet</u> [cf. (2.5) p. 5]

$$R_r = 2m_e\delta = 2m_e \frac{\mu_o e^2 \omega^2}{12\pi c m_e} = \frac{\mu_o e^2 \omega^2}{6\pi c}$$

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