TKP4140 Process Control
Department of Chemical Engineering NTNU
Autumn 2022 - Exercise 13

Due date: Friday 25 November at 23:59

Problem 1: Decoupling

Consider a 2×2 process

$$y_1 = g_{11}u_1 + g_{12}u_2 \tag{1}$$
$$y_2 = g_{21}u_1 + g_{22}u_2 \tag{2}$$

controlled with two single loop controllers

$$u_1' = c_1(y_{1s} - y_1) \tag{3}$$
$$u_2' = c_2(y_{2s} - y_2) \tag{4}$$

Consider the three cases:

1. Decentralized control

$$u_1 = u_1' \tag{5}$$
$$u_2 = u_2' \tag{6}$$

2. Standard decoupling

$$u_1 = u_1' + D_{12}u_2' \tag{7}$$
$$u_2 = u_2' + D_{21}u_1' \tag{8}$$

3. Inverse decoupling

$$u_1 = u_1' + D_{12}u_2 \tag{9}$$
$$u_2 = u_2' + D_{21}u_1 \tag{10}$$

For each case, derive the transfer function from the controller outputs (u_1', u_2') to the process outputs (y_1, y_2).

Assume that ideal decoupling is possible, that is:

$D_{12} = -\frac{g_{12}}{g_{11}}$ and $D_{21} = -\frac{g_{21}}{g_{22}}$ are realizable.

1. $\underline{\underline{y_1 = g_{11} u_1' + g_{12} u_2', \quad y_2 = g_{21} u_1' + g_{22} u_2'}}$

2. $y_1 = g_{11}\left(u_1' - \frac{g_{12}}{g_{11}} u_2'\right) + g_{12}\left(u_2' - \frac{g_{21}}{g_{22}} u_1'\right)$

$y_1 = \left(g_{11} - \frac{g_{12} g_{21}}{g_{22}}\right) u_1' + 0 \cdot u_2'$

$y_2 = g_{21}\left(u_1' - \frac{g_{12}}{g_{11}} u_2'\right) + g_{22}\left(u_2' - \frac{g_{21}}{g_{22}} u_1'\right)$

$y_2 = \left(g_{22} - \frac{g_{21} g_{12}}{g_{11}}\right) u_2' + 0 \cdot u_1'$

$\Rightarrow \underline{\underline{y_1 = \left(g_{11} - \frac{g_{12} g_{21}}{g_{22}}\right) u_1', \quad y_2 = \left(g_{22} - \frac{g_{12} g_{21}}{g_{11}}\right) u_2'}}$

3. $y_1 = g_{11}\left(u_1' - \frac{g_{12}}{g_{11}} u_2\right) + g_{12} u_2$

$= g_{11} u_1'$

$y_2 = g_{21} u_1 + g_{22}\left(u_2' - \frac{g_{21}}{g_{22}} u_1\right)$

$= g_{22} u_2'$

$\Rightarrow \underline{\underline{y_1 = g_{11} u_1', \quad y_2 = g_{22} u_2'}}$

Problem 2: Water mixer - RGA

Consider the mixing process from Exercise 12 with

$$G = \begin{bmatrix} k_1 & k_2 \\ 1 & 1 \end{bmatrix}$$

Compute the RGA at steady state. What pairing does this suggest? Does it agree with your suggestion from Exercise 12?

$$\lambda_{11}^* = \frac{k_1^*}{k_1^* - \frac{k_2^* \cdot 1}{1}} = \frac{1}{1 - \frac{k_2^*}{k_1^*}} = \frac{2}{3}$$

$$k_1^* = \frac{60-40}{1} = 20 \qquad k_2^* = \frac{30-40}{1} = -10$$

Using that the sum of each column and row $= 1$ for RGA:

$$\text{RGA} = \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix},$$ selecting the pairings closest to 1. $\Rightarrow$ $q_h \leftrightarrow T$, $q_c \leftrightarrow h$