Problem 1: Feedforward control

Consider the following process:

$$y = gu + g_d d \tag{1}$$

with a measured disturbance $d_m = g_{dm} d$. Propose a realizable feedforward controller when:

$$g(s) = 5\frac{e^{-2s}}{5s+1} \tag{2}$$

$$g_d(s) = \frac{3}{5s+1} \tag{3}$$

$$g_{dm}(s) = \frac{1}{0.5s+1} \tag{4}$$

Inserting $d_m = g_{dm} d$

$$\Rightarrow d = \frac{d_m}{g_{dm}}$$

$$y = gu + g_d \cdot \frac{d_m}{g_{dm}}$$

An ideal feedforward controller can be found from $y=0$

$$\Rightarrow u = -\frac{g_d}{g \cdot g_{dm}} \cdot d_m$$

$$\Rightarrow C_{FF,ideal} = -\frac{g_d}{g \cdot g_{dm}} = -\frac{\frac{3}{\cancel{5s+1}}}{5 \cdot \frac{e^{-2s}}{\cancel{5s+1}} \cdot \frac{1}{0{,}5s+1}} = -\frac{3}{5}(0{,}5s+1)e^{2s}$$

$$c_{FF}(s) = k\frac{(T_1 s+1)\cdots}{(\tau_1 s+1)(\tau_2 s+1)\cdots}e^{-\theta s}$$

where must have at least as many τ's as T's

For the controller to be realizable, the pole-polynomial must have higher or equal order than the zero polynomial, also θ must be ≥ 0

selecting $(0{,}5s+1) \Rightarrow$ $\underline{\underline{C_{FF} = \frac{C_{FF,ideal}(\theta=0)}{(0{,}5s+1)} = -\frac{3}{5} = -0{,}6}}$

Problem 2: Cascade control

The process below describes cascade control with controllers c_1 and c_2 based on measurements y_1 and y_2

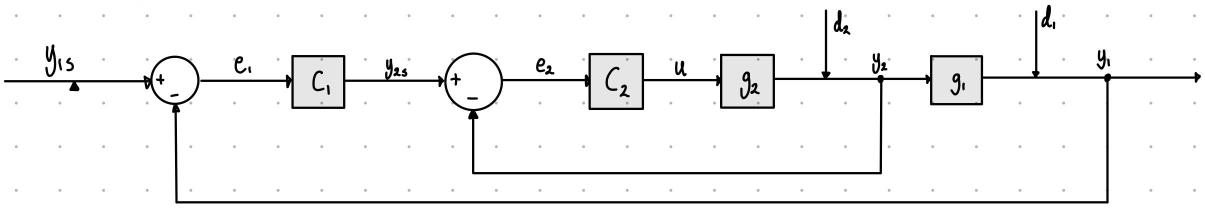
$$y_2 = g_2 u + d_2 \tag{5}$$

$$y_1 = g_1 y_2 + d_1 \tag{6}$$

$$u = c_2(y_{2s} - y_2) \tag{7}$$

$$y_{2s} = c_1(y_{1s} - y_1) \tag{8}$$

1. Draw a block diagram of cascade control.



2. Answer if the following statements are true or false. Justify your answers.

   Cascade control is recommended when ...

   (a) d_1 is the main disturbance
   (b) d_2 is the main disturbance
   (c) g_1 has a large effective delay
   (d) g_2 has a large effective delay
   (e) g_1 is nonlinear
   (f) g_2 is nonlinear

a) False, then we don't have a benefit of controlling y_2.

b) True, then the fast control will ensure that d_2 is neglected.

c) True, then we will have a fast inner loop, and a slower outer loop.

d) False, then the inner loop will have a lot of instability.

e) False
f) True
When g_2 is non-linear, then the outer loop will "see" g_2 as linear, as the controller is handeling the non-linearity. This is not the case if g_1 is nonlinear.

3. Now, you are going to tune the controllers for the cascade control configuration.

   In all cases, select $\tau_c =$ effective delay.

   Consider the following transfer functions:

$$g_1(s) = \frac{2e^{-s}}{10s+1} \tag{9}$$

$$g_2(s) = \frac{3e^{-0.2s}}{15s+1} \tag{10}$$

- First-order + delay model for PI-control

$$G(s) = \frac{k}{\tau_1 s+1}\, e^{-\theta s}$$

- PI-controller (based on first-order model)

$$c(s) = K_c\left(1 + \frac{1}{\tau_I s}\right) = K_c \frac{\tau_I s+1}{\tau_I s}$$

For cascade form PID controller:

$$K_c = \frac{1}{k}\frac{\tau_1}{\tau_c+\theta} = \frac{1}{k'}\cdot\frac{1}{\tau_c+\theta}$$

$$\tau_I = \min\{\tau_1, \frac{4}{k'K_c}\} = \min\{\tau_1, 4(\tau_c+\theta)\}$$

(a) Tune a SIMC PI controller $c_2(s)$ for $g_2(s)$

$$K_c = \frac{1}{3}\cdot\frac{15}{2\cdot 0{,}2} = \frac{5}{0{,}4} = 12{,}5$$

$$\tau_I = \min(15, 4\cdot(2\cdot 0{,}2)) = 1{,}6$$

$$\Rightarrow \underline{\underline{C_2(s) = 12{,}5\cdot\frac{1{,}6s+1}{1{,}6s}}}$$

(b) Obtain the closed loop transfer function $T_2(s)$ for the inner loop.
Hint: the closed loop response of the inner loop $T_2(s)$ can be approximated to a first order with time delay process. This can be used to tune controller $c_1(s)$ in part *(c)* and *(d)*,

$$T_2(s) = \frac{c_2(s)g_2(s)}{1+c_2(s)g_2(s)} \approx \frac{e^{-\theta s}}{\tau_c s+1} \tag{11}$$

Using eq. (11): $T_2(s) \approx \dfrac{e^{-0{,}2s}}{0{,}2s+1}$

(c) Obtain the transfer function $g_1'(s)$ for the "new" process from y_{2s} to y_1.

$$g_1'(s) = T_2\cdot g_1$$

$$= \frac{e^{-0{,}2s}}{0{,}2s+1}\cdot\frac{2e^{-s}}{10s+1}$$

$$\underline{\underline{g_1'(s) = 2\cdot\frac{e^{-1{,}2s}}{(10s+1)(0{,}2s+1)}}}$$

(d) Tune a SIMC PI controller $c_1(s)$ for $g_1'(s)$.

Using the half rule approximation: $\theta = \theta_0 + \frac{\tau_2}{2} = 1{,}2 + 0{,}1 = 1{,}3$

$\tau_1 = 10 + \frac{0{,}2}{2} = 10{,}1$

$$\Rightarrow g_1'(s) \approx 2\cdot\frac{e^{-1{,}3s}}{10{,}1s+1}$$

Then, the SIMC-rules gives ($\tau_c = \theta$)

$$K_c = \frac{1}{2} \cdot \frac{10{,}1}{2 \cdot 1{,}3} = 1{,}94$$

$$\tau_I = \min(10{,}1 ,\ 8 \cdot 1{,}3) = 10{,}1$$

Then, $\underline{\underline{C_1 = 1{,}94 \cdot \frac{10{,}1\,s + 1}{10{,}1\,s}}}$

(e) What would the SIMC PI controller be **without** the cascade? In other words, tune a controller $c_3(s)$ for $g_3(s) = g_1(s)g_2(s)$.

$$g_3 = \frac{2e^{-s}}{10s+1} \cdot \frac{3e^{-0{,}2s}}{15s+1} = \frac{6e^{-1{,}2s}}{(15s+1)(10s+1)}$$

Using the half rule:

$$\tau_1 = 15 + \frac{10}{2} = 20 ,\ \theta = 1{,}2 + \frac{10}{2} = 6{,}2$$

$$g_3 \approx 6 \cdot \frac{e^{-6{,}2s}}{20s+1}$$

Then, using SIMC-rules:

$$\underline{\underline{K_c = \frac{1}{6} \cdot \frac{20}{2 \cdot 6{,}2} = 0{,}269}}$$

$$\underline{\underline{\tau_I = \min(20 ,\ 4 \cdot (2 \cdot 6{,}2)) = 20}}$$

Problem 3: Water mixer

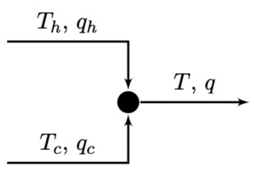


Figure 1: Mixer system

Consider the process of mixing hot and cold water, as shown in Figure 1. The process has inputs $u_1 = \Delta q_h$ [ℓ/s], $u_2 = \Delta q_c$ [ℓ/s], and outputs $y_1 = \Delta T$ [°C], $y_2 = \Delta q$ [ℓ/s].

The control objective is to have a mixing temperature $T = 40$°C and a total flow leaving the mixer of $q = 1$ ℓ/s. At the nominal operating point we have $T_c = 30$°C and $T_h = 60$°C.

1. Formulate the energy and mass balances. The dynamics of this process are very fast; so, a steady-state model is sufficient to get T and q.

Mass balance: $\boxed{q = q_h + q_c}$

Energy balance: $q \cancel{C_{p,w}} T = q_c \cancel{C_{p,w}} \cdot T_c + q_h \cancel{C_{p,w}} \cdot T_h$ Assume $C_{p,w} = \text{const}$

$$\boxed{T = \frac{q_c \cdot T_c + q_h \cdot T_h}{q} = \frac{q_c \cdot T_c + q_h \cdot T_h}{q_c + q_h}}$$

2. Linearize the model and show that the linear model can be written $y = Gu$, where:

$$G = \begin{bmatrix} k_1 & k_2 \\ 1 & 1 \end{bmatrix}$$

with: $k_1 = (T_h^* - T^*)/q^*$ $k_2 = (T_c^* - T^*)/q^*$ $u = [u_1 \quad u_2]^T$

The symbol * denotes the steady state value.

$\Rightarrow$ Want $G = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}$

Introducing deviation variables: $\Delta\psi = \psi - \psi^*$

$\Delta q = \Delta q_c + \Delta q_h \Rightarrow g_{21} = g_{22} = 1$

$\Delta T = g_{22} \Delta q_c + g_{21} \Delta q_h$

Using the 1st order taylor expansion:

$$g_{12} = \frac{\partial T}{\partial q_c} = \frac{T_c \cdot (q_c + q_h) - (q_c T_c + q_h T_h) \cdot 1}{(q_c + q_h)^2} = \frac{T_c}{q_c + q_h} - \underbrace{\frac{q_c T_c + q_h T_h}{(q_c + q_h)^2}}_{= \frac{T}{q_c + q_h}} = \boxed{\frac{T_c - T}{q_c + q_h} = k_2}$$

I expect a similar result for g_{21} due to how the expression looks:

$$\boxed{g_{11} = \frac{T_h - T}{q_c + q_h} = k_1}$$

In conclusion: $\Delta q = 1 \cdot \Delta q_c + 1 \cdot \Delta q_h$

$$\Delta T = \underbrace{\frac{T_c - T}{q_c + q_h}}_{k_2} \cdot \Delta q_c + \underbrace{\frac{T_h - T}{q_c + q_h}}_{k_1} \Delta q_h$$

Can be written as $y = Gu$, where $y = \begin{bmatrix} \Delta T \\ \Delta q \end{bmatrix}$, $u = \begin{bmatrix} \Delta q_h \\ \Delta q_c \end{bmatrix}$

$$\boxed{G = \begin{bmatrix} k_1 & k_2 \\ 1 & 1 \end{bmatrix} \quad \text{QED}}$$

3. What are the steady state values for q_c and q_h?

(1) $q^* = 1 = q_c^* + q_h^* \Rightarrow q_c^* = 1 - q_h^*$

(2) $T^* = 40 = \dfrac{q_c^* \cdot T_c^* + q_h^* \cdot T_h^*}{q^*} = \dfrac{(1-q_h^*)30 + q_h^* 60}{1}$

$40 = 30 + 30 q_h^* \Rightarrow \boxed{q_h^* = \frac{1}{3} \overset{(1)}{\Rightarrow} q_c^* = \frac{2}{3}}$

4. Find the gain matrix G at the nominal operating point.

$k_1^* = \dfrac{60-40}{1} = 20 \qquad k_2^* = \dfrac{30-40}{1} = -10$

$$\boxed{G^* = \begin{bmatrix} 20 & -10 \\ 1 & 1 \end{bmatrix}}$$

5. Based on G, which stream (q_h or q_c) would you use to control the temperature (T)? Explain briefly.

I would use q_h, as it has the highest gain due to the larger temperature difference, which gives a better/larger/faster response than for q_c.