Problem 1

Consider the following plant transfer function

$$g(s) = \frac{1}{(s+1)^3}, \tag{1}$$

and the PI controller with SIMC rules

$$c(s) = 0.5\frac{(1+1.5s)}{1.5s}. \tag{2}$$

Using the results of Problem 3 in Exercise 10, compute:

a) Gain margin

b) Phase margin

c) Delay margin

Indicate these margins in the Bode plot in Figure 1. Note: if you have not solved problem 3 in exercise 10, you may use

$$L(s) = g(s)c(s) = 0.5\frac{(1+1.5s)}{1.5s(s+1)^3}$$

From last exercise:

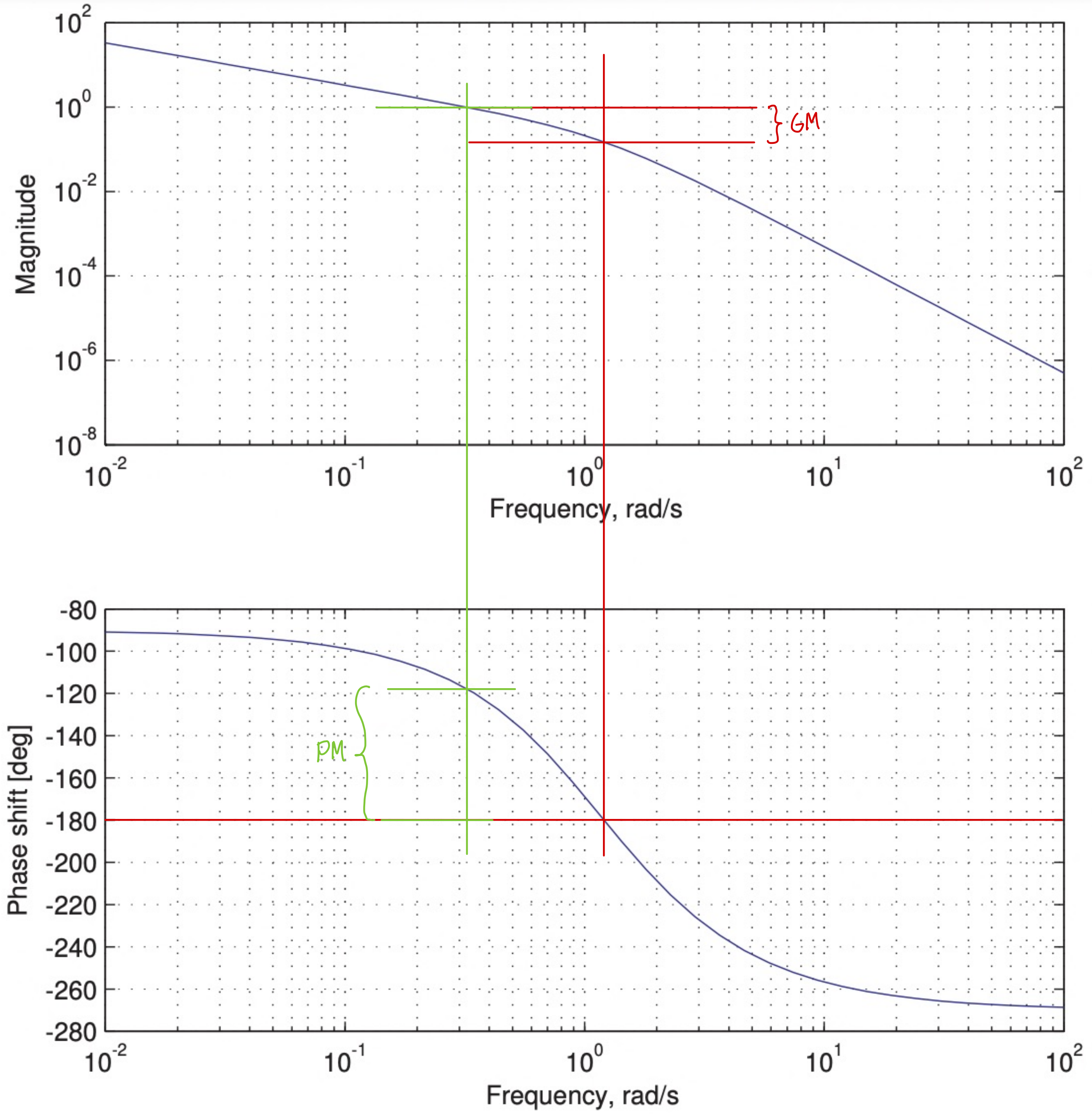
	$\omega = 0.1$ rad/s	$\omega_c = 0{,}32$	$\omega_{180} = 1{,}21$	$\omega = 10\ rad/s$
$\lvert L\rvert$	3,32	1	0,15	$4{,}94 \cdot 10^{-4}$
$\angle L$	-98,6	$-117{,}6^\circ$	-180°	-256,68

a) $GM = \dfrac{1}{|L(\omega_{180})|} = \dfrac{1}{0{,}15} = \underline{\underline{6{,}67}}$

b) $PM = \angle L(\omega_c) + 180^\circ = -117{,}6^\circ + 180^\circ = \underline{\underline{62{,}4^\circ}}$

c) $DM = \dfrac{PM\,(rad)}{\omega_c} = \dfrac{62{,}4^\circ \cdot \pi}{180^\circ \cdot 0{,}32} = \underline{\underline{3{,}4s}}$ ← Corresponds to 3,3 from ex 10 (its the same calculation)

How do I draw DM?



Problem 2

1. A pure time delay process is:
$$g(s) = e^{-\theta s}$$
If a pure time delay process is controlled with a P-controller and the controller gain is increased, one eventually gets persistent oscillations (on the limit to instability). What is the period of these persistent oscillations?

2. A pure I-controller is:
$$c(s) = \frac{K_I}{s},$$
What is the period of oscillations if you use a pure I-controller and increase K_I to the limit of stability?

Hint: $P_u = \frac{2\pi}{\omega_{180}}$

1. For a P-controller, $c(s) = K_c$

$\Rightarrow L(s) = K_c e^{-\theta s}$, then by

$g = e^{-\theta s}$

Gain = $|g(j\omega)| = 1$

Phase shift = φ = angle$(g(j\omega)) = -\omega\theta$ [rad]

$|L| = K_c$

$\angle L = -w\theta$

From bode stability condition, oscillations occur if $|L(wj)| \geq 1$ at w_{180} as $|L(wj)| = K_c \;\forall w$, Then we get percistant oscillations for $\underline{\underline{K_c = 1}}$

To calculate the period: $P_u = \frac{2\pi}{w_{180}}$

$\angle L(jw_{180}) = 180 = -\pi$

$\Rightarrow -w_{180}\theta = -\pi$

$w_{180} = \frac{\pi}{\theta}$

Then $\underline{\underline{P_u = \frac{2\pi}{\pi/\theta} = 2\theta}}$

2. For pure I-control

$L(s) = \frac{K_I}{s} e^{-\theta s}$

$|L(jw)| = \frac{K_I}{w}$, $\angle L(jw) = -\arctan(\infty) - w\theta = -\frac{\pi}{2} - w\theta$

w_{180}: $-\frac{\pi}{2} - w_{180}\theta = -\pi \Rightarrow \underline{\underline{w_{180} = \frac{\pi}{2\theta}}}$

At instability limit, $w = w_{180}$, $\underline{\underline{P_u = \frac{2\pi}{\pi/2\theta} = 4\theta}}$

Problem 3

Consider a first-order process with delay,

$$g(s) = \frac{e^{-s}}{2s+1},$$

1. Derive the SIMC PI controller and find $L(s)$ for the two choices $\tau_c = \theta = 1$ and $\tau_c = 3$.
2. Plot the Bode-plot for the two choices.
   You can use Matlab; you would have to define the transfer function L(s) and then use `bode(L)`. Remember to change the magnitude units from dB to absolute.
3. Indicate the GM, PM and delay margin in the plot, for the two choices.
4. Using the values of $|L(j\omega_{180}|$, $\angle L(\omega_c)$, and ω_c from the plot, calculate analytically the GM, PM and delay margin, for both choices.
5. Simulate for the two cases: show y and u for setpoint change at $t = 0$ and input disturbance at $t = 20$.

1. Controller transfer function for PI: $C(s) = K_c \cdot \frac{\tau_I s + 1}{\tau_I s}$

SIMC:

$$K_c = \frac{1}{k} \cdot \frac{\tau}{\tau_c + \theta}$$

$$\theta = \min(\tau, 4(\theta + \tau_c))$$

We have $k = 1$, $\theta = 1$, $\tau = 2$

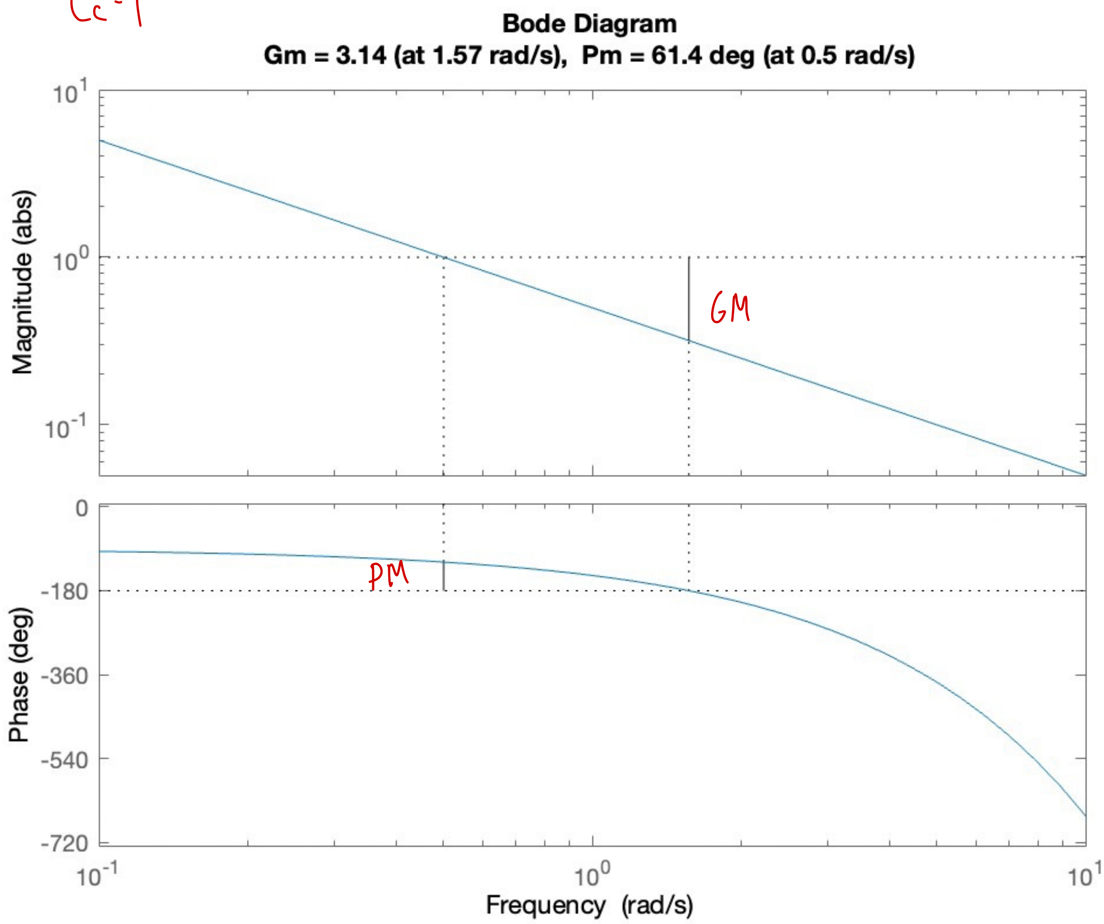
$$\Rightarrow \boxed{\begin{array}{l} \tau_c = 1,\ K_c = 1,\ \tau_I = 2 \\ \tau_c = 3,\ K_c = 0{,}5,\ \tau_I = 2 \end{array}}$$

Then, $\tau_c = 1$, $C_1(s) = \frac{2s+1}{2s} \Rightarrow L_1(s) = C_1 \cdot g = \frac{e^{-s}}{\cancel{2s+1}} \cdot \frac{\cancel{2s+1}}{2s} = \frac{e^{-s}}{2s} = \underline{\underline{0{,}5 \cdot \frac{e^{-s}}{s}}}$

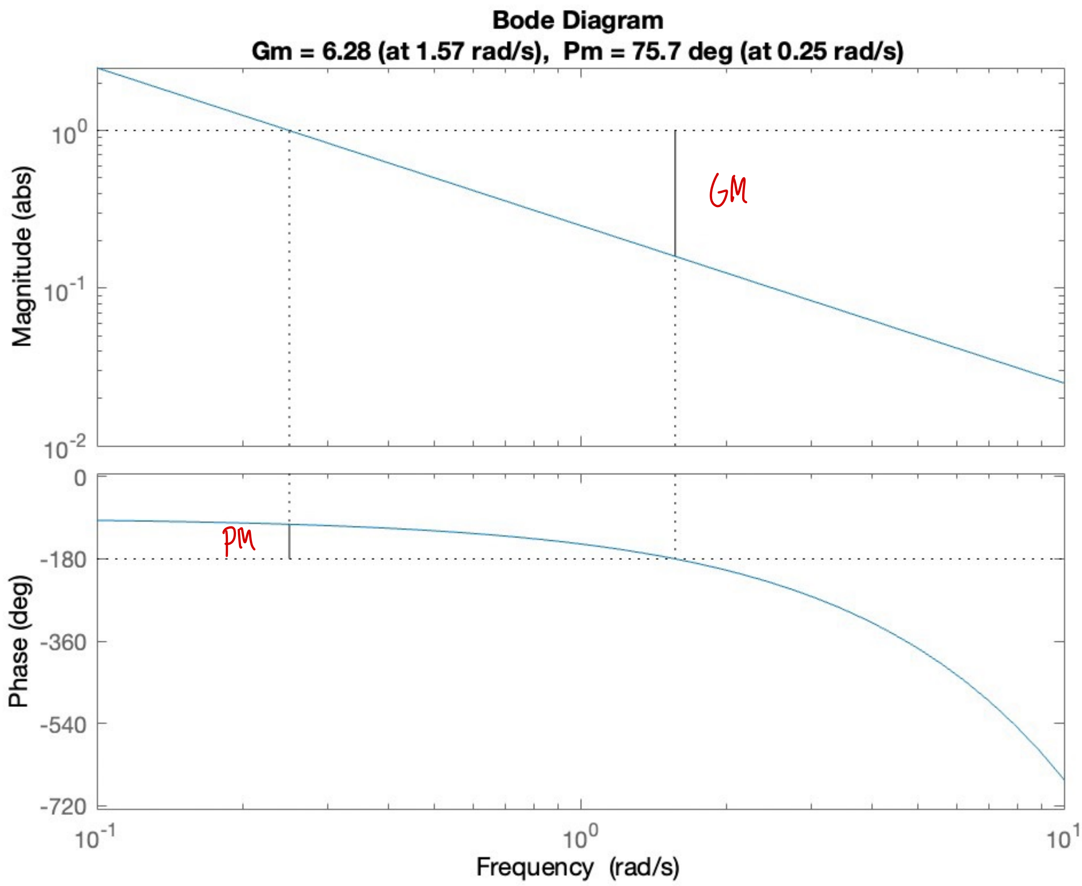
$\tau_c = 3$, $C_2(s) = 0{,}5 \cdot \frac{2s+1}{2s} \Rightarrow L_2(s) = \underline{\underline{0{,}25 \cdot \frac{e^{-s}}{s}}}$

2., 3. are on next page.

$\tau_c = 1$


Bode Diagram
Gm = 3.14 (at 1.57 rad/s), Pm = 61.4 deg (at 0.5 rad/s)
Magnitude (abs)
Phase (deg)
Frequency (rad/s)
GM
PM

$\tau_c = 3$


Bode Diagram
Gm = 6.28 (at 1.57 rad/s), Pm = 75.7 deg (at 0.25 rad/s)
Magnitude (abs)
Phase (deg)
Frequency (rad/s)
GM
PM

4. From the plots

	$\tau_c = 1$	$\tau_c = 3$
ω_c	0,5	0,25
ω_{180}	1,57	1,57
$\lvert L(j\omega_{180})\rvert$	0,318	0,159
$\lvert \angle L(j\omega_c)\rvert$	-119	-104

Then, using the expressions from problem 1

	$\tau_c = 1$	$\tau_c = 3$
GM	3,14	6,28
PM	61,4	75,7
DM	2,14	5,28

5) Simulating the responses