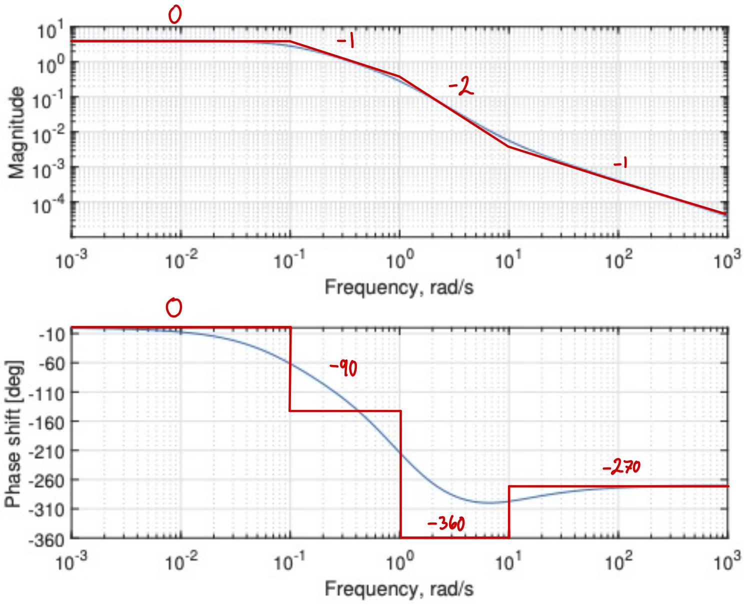
Problem 1: Bode diagram 1

Given the transfer function

$$g(s) = 4\frac{(1-s)(0.1s+1)}{(10s+1)(s+1)^2}. \tag{1}$$

Find the poles and zeros and draw the asymptotes into the Bode diagram in Figure 1.



Looking at g(s), we have:

Zeros: 1, -10

Poles: -1, -0,1

Using

Rule for asymptotic Bode-plot, L = k(Ts+1)/(τs+1)….. :

1. Start with low-frequency asymptote (s→0)
   (a) If constant (L(0)=k):
   Gain=k (slope=0)
   Phase=0°
   (b) If integrator (L=k'/s):
   Gain slope= -1 (on log-log plot). Need one fixed point, for example, gain=1 at ω=k'
   Phase: -90°.

2. Break frequencies (order from large T to small T):

	Change in gain slope	Change in phase
ω=1/T (zero)	+1	+90° (-90° if T negative)
ω=1/τ (pole)	-1	-90° (+90° if τ negative)

3. Time delay, $e^{-\theta s}$. Gain: no effect, Phase contribution: -ωθ [rad] (-1 rad = -57° at ω=1/θ)

As L(0) = 4, the gain is 4, and slope = 0

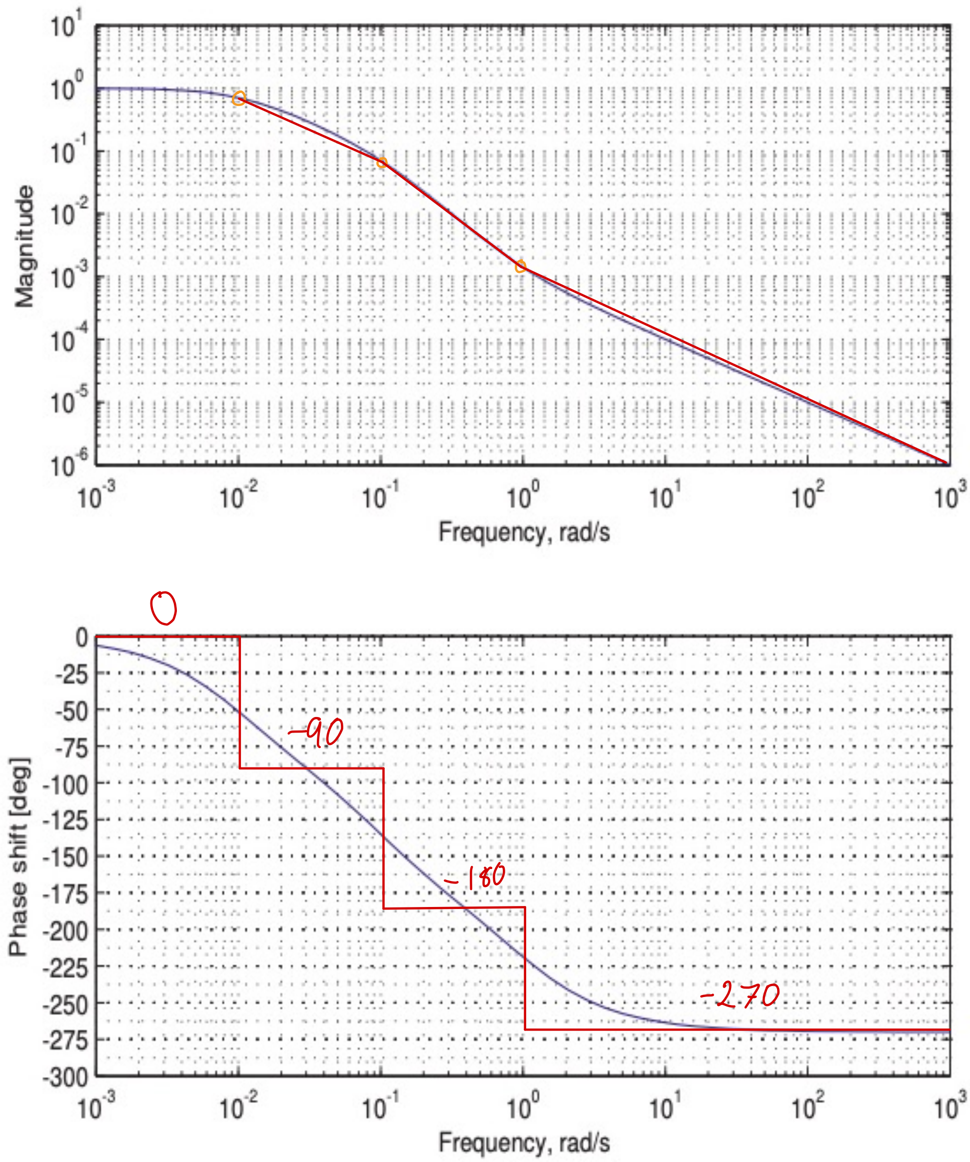
Phase = 0

Then, from the table above:

		Slope	Phase
	$W = 0$	0	0
(Pole)	$W = \frac{1}{10} = 0{,}1$	-1	-90
(Zero + Pole²)	$W = \frac{1}{1} = 1$	= -1 + 1 - 2 = -2	= -90 - 90 - 2·90 = -360
(Zero)	$W = \frac{1}{0{,}1} = 10$	= -2 + 1 = -1	= -360 + 90 = -270

Problem 2: Bode diagram 2

Identify the break frequencies in Figure 2 and whether they are related to a pole or to a zero. What transfer function is shown in Figure 2?



Marking "breaks" on the plot in orange

$W=0$ gives $10^0 \Rightarrow$ Gain $=1=k$

There is a break at $w=0{,}01$

The gain of the curve is ≈ -1

There is a break ab $w=0{,}1$

The gain is ≈ -2

The is a break at $w=1$

gain ≈ -1

At $w=0{,}01 \Rightarrow \frac{1}{w}=100$, ΔSlope: -1, $\Delta\angle g: -90 \Rightarrow$ Pole, $\tau>0$

$w=0{,}1 \Rightarrow \frac{1}{w}=10$, ΔSlope: -1, $\Delta\angle g: -90 \Rightarrow$ Pole, $\tau>0$

$w=1 \Rightarrow \frac{1}{w}=1$, ΔSlope: $+1$, $\Delta\angle g: -90 \Rightarrow$ Zero, $T<0$

$$\Rightarrow g(s) = 1 \cdot \frac{(-s+1)}{(10s+1)(100s+1)}$$

Problem 3

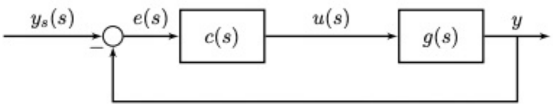


Figure 3: Closed loop control system

Given the system in from Exercise 9, Problem 1.2 c.), see Figure 3. The plant transfer function is:

$$g(s) = \frac{1}{(s+1)^3}, \tag{2}$$

and the PI controller with SIMC rules is

$$c(s) = 0.5\frac{(1+1.5s)}{1.5s}. \tag{3}$$

1. Draw the bode plot of the open loop ($L = gc$) for $\omega = 10^{-2}$ to $\omega = 10^{2}$ rad/s. Use Figure 4.

2. Fill in the following table.

	$\omega = 0.1$ rad/s	$\omega_c =$	$\omega_{180} =$	$\omega = 10$ rad/s
$\lvert L\rvert$		1		
$\angle L$			$-180°$	

3. How much dead time must we add to L to have $\angle L = -180°$ at the frequency ω_c where $|L| = 1$?

Comment: In Exercise 11, we will use the results from 2 and 3 to compute gain margin, phase margin and delay margin.

1.

$$L = g \cdot C = \frac{1}{(S+1)^3} \cdot 0{,}5 \cdot \frac{(1+1{,}5s)}{1{,}5s} = \frac{1}{3} \cdot \frac{(1+1{,}5s)}{s(s+1)^3}$$

Zeroes: $-\frac{2}{3}$ poles: $0, -1$

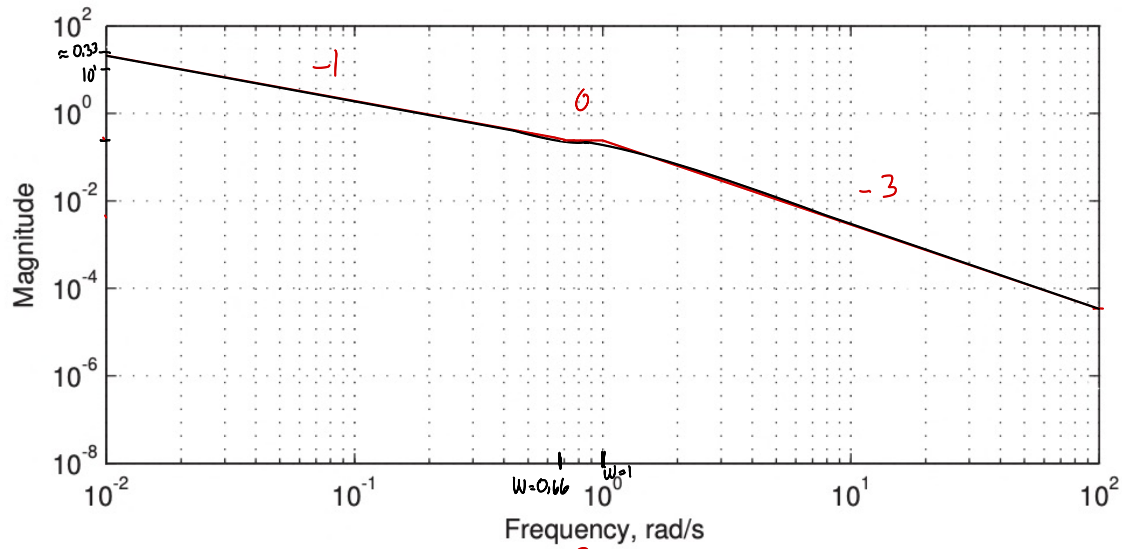
Due to there being an integrator $\left(\frac{1}{3}\right)$, we will start at slope $= -1$, $\angle g = -90$

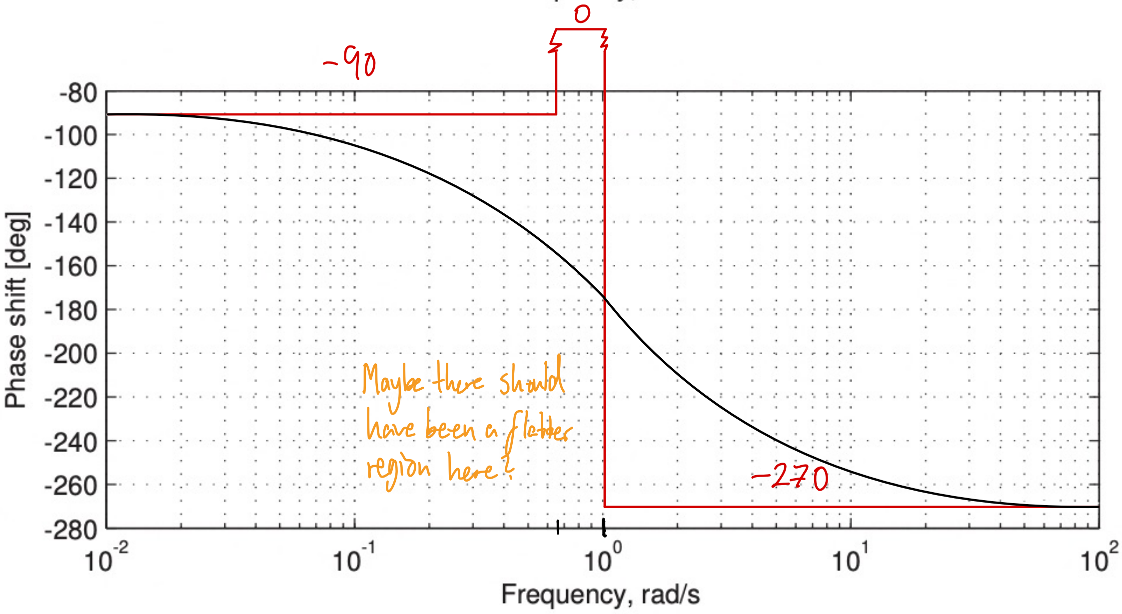
"Fixed point" at the start: As $w \to 0$, $L(wj) \to \frac{1}{3jw}$

$$|L(wj)| = \frac{1}{3w} = \frac{1}{3 \cdot 10^{-2}} = \underline{33{,}33}$$

There will be "breaks" at:

	Slope	$\angle L$
(at the start) $W < 0{,}66$ (Integrator)	-1	-90
(zero) $W = \frac{1}{1{,}5} = \frac{2}{3} = 0{,}66$	$= -1+1 = 0$	$= -90+90 = 0$
(3x pole) $W = \frac{1}{1} = 1$	$= 0 - 3 \cdot 1 = -3$	$= 0 - 3 \cdot 90 = -270$





2. $L = \frac{1}{3}\frac{(1+1{,}5s)}{s(s+1)^3}$,

Using that:

$$g(s) = k\,\frac{g_1\;g_2}{g_3\;g_4}\,e^{-\theta s}$$

$$|g| = k\,\frac{|g_1||g_2|}{|g_3|\;|g_4|}$$

$$\angle g| = \angle g_1 + \angle g_2 - \angle g_3 - \angle g_4 - \omega\theta$$

Let $g_1 = 1+1{,}5s \Rightarrow g_1(jw) = 1 + 1{,}5wj \Rightarrow |g_1| = \sqrt{1{,}5^2 w^2 + 1}$, $\arctan(1{,}5w)$

$g_2 = s \Rightarrow g_2(jw) = jw \Rightarrow |g_2| = w$, $\angle g_2 = 90^\circ = \frac{\pi}{2}$ (always complex)

$g_3 = (s+1) \Rightarrow g_3(jw) = jw+1 \Rightarrow |g_3| = \sqrt{w^2+1}$, $\angle g_3 = \arctan(w)$

Then $|L| = \frac{1}{3} \cdot \frac{|g_1|}{|g_2| \cdot |g_3|^3} = \frac{1}{3} \cdot \frac{\sqrt{1{,}5^2 w^2 + 1}}{w \cdot (w^2+1)^{3/2}}$

And: $\angle L = \arctan(1{,}5w) - 90 - 3 \cdot \arctan(w)$

For the given w, inserting 0,1 and 10 gives:

$w = 0{,}1 \Rightarrow |L| = 3{,}32$

$\angle L = -98{,}6^\circ$

$w = 10 \Rightarrow |L| = 4{,}94 \cdot 10^{-4}$

$\angle L = -256{,}7^\circ$

For w_c, $|L| = 1$

$$\Rightarrow \frac{1}{3} \cdot \frac{\sqrt{1{,}5^2 w^2 + 1}}{w \cdot (w^2+1)^{3/2}} = 1$$

$$\frac{\sqrt{1{,}5^2 w^2 + 1}}{w \cdot (w^2+1)^{3/2}} = 3 \qquad /n^2$$

$$\frac{1{,}5^2 w^2 + 1}{w^2 \cdot (w^2+1)^3} = 9$$

$$1{,}5^2 w^2 + 1 = 9w^8 + 27w^6 + 27w^4 + 9w^2$$

$$9w^8 + 27w^6 + 27w^4 + (9 - 1{,}5^2)\, w^2 - 1 = 0$$

Solving numerically gives $w_c = 0{,}32$

Inserting into $\angle L(w) \Rightarrow \angle L = -117{,}6^\circ$

For W_{180} : $\angle L = -180$

$$\arctan(1{,}5w) - 90 - 3\cdot\arctan(w) = -180$$

$$\arctan(1{,}5w) - 3\cdot\arctan(w) = -90 = -\frac{\pi}{2}$$

Again, solving numerically gives: $W_{180} = 1{,}21$

Then, taking $|L(1{,}21)| = 0{,}15$

Filling in the table, finally, we get:

	$\omega = 0.1$ rad/s	$\omega_c = 0{,}32$	$\omega_{180} = 1{,}21$	$\omega = 10\ rad/s$
$\lvert L\rvert$	3,32	1		
$\angle L$	-98,6	$-117{,}6^\circ$	-180°	-256,68

3. From:

$$\angle g| = \underbrace{\angle g_1 + \angle g_2 - \angle g_3 - \angle g_4}_{\angle L \text{ (without delay)}} - \omega\theta$$

And the table, the delay can simply be calculated from the values for W_c:

$$-180^\circ = -117{,}6^\circ - w_c\theta = -117{,}6^\circ - 0{,}32\,\theta$$

$$\Rightarrow \theta = \frac{-62{,}4^\circ}{-0{,}32} \cdot \frac{\pi}{180^\circ} = 3{,}4\ s$$