

Qualitative modeling of pressure vs. pain relations in women suffering from dyspareunia

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Abstract: Genital pain / penetration disorders affect a significant portion of the female population and diminish significantly the quality of life of the subjects. Treatments, that often consist in stretching opportunely the vaginal duct by means of opportune vaginal dilators, are known to be invasive, lengthy and uncomfortable. Designing better treatments (e.g., more efficient locations and levels of pressures) nonetheless requires understanding better how the pressure developed in the vaginal channel affects the patient and leads to subjective pain. Here we take a control-oriented approach to the problem, and aim at describing the dynamics of the pressure vs. pain mechanisms by means of opportune state space representations. In particular, we first collect and discuss the medical literature, that describes how the variables that are involved in the treatment of genital pain / penetration disorders with vaginal dilators, are logically related. After this we translate (and complete) this set of logical relations into a qualitative model that allows control oriented analyses of the dynamics. The obtained state space model is then proved to both mimic correctly what is expected from logical perspectives and reproduce behaviors measured in clinical settings.

Keywords: genital pain / penetration disorders, state space models

1. INTRODUCTION

Female genital pain / penetration disorders affect a great number of persons (being estimated that 30-40% of women suffer from painful experiences during sexual intercourses (Goldstein et al., 2009, Chap. 2)), and arise from a great variety of causes (biological, psychological, social reasons, and the union of these (Goldstein et al., 2009, Chap. 3), with interpersonal and relational factors such as hostile partner responses and other psychosocial factors contributing to maintenance, exacerbation and chronicization of genital pain). Affected persons are known to be likely to develop comorbid sexual difficulties, negative affect, and relationship concerns (Goldstein et al., 2009, Chap. 3), all factors that diminish significantly the quality of life.

Treatments of genital pain / penetration disorders follow combinations of psychological perspectives (e.g., Cognitive Behavioral Therapies (CBTs) such as *exposure, modifying, hypervigilance* and *catastrophizing* strategies) and physiological perspectives (such as stretching the vaginal duct, desensitizing the vestibulum, and relaxing the pelvic

floor) (Binik et al., 2006; Bergeron et al., 2008; Goldstein et al., 2011). These therapies are nonetheless known to be invasive, lengthy, intimate and scarce at the point that several patients rather prefer to go untreated and live with these debilitating disorders. The societal need is thus to adapt the treatments in order to increase their appealingness.

Towards this goal, here we focus on the specific and common physiological treatment of stretching the vaginal duct by applying a certain pressure by means of opportune vaginal dilators. This strategy is indeed commonly used for treating disorders with components that are both psychosomatic (e.g., vaginismus) and physiological (e.g., complications after cervix cancer surgeries, vaginal radiotherapies, Mayer-Rokitansky-Küster-Hauser syndromes, or male-to-female gender reassignment surgeries).

We thus develop qualitative and quantitative models of the dynamics of the quantities involved in the treatment, i.e., how the pressure developed in the vaginal channel (plus some ancillary variables) transform into subjective pain. Our aim is thus to provide a mathematical tool that can be used to design novel vaginal pressure patterns (location, level of pressure, way of building up the pressure) with higher effectiveness than the current practice.

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Delaying the discussion on the relevant literature to Section 3, our contributions consist in collecting the existing medical literature on the subject, translating and completing it into a state-space description of the dynamics of the relevant variables, and eventually analyzing the dynamics from mathematical perspectives (e.g., the positivity of the system, its equilibria and their stability properties).

The paper is structured as follows: Section 2 lists and discusses the variables involved in the treatment of genital pain / penetration disorders. Section 3 reviews the medical literature that describes qualitative relations among the variables listed in Section 2. Section 4 defines which variables above should be included and which ones should be excluded in a state-space representation of the system under investigation. Section 5 drafts the co-implications among the variables in our state-space representations, and Section 6 completes our translation efforts by presenting our final state-space model plus a mathematical analysis of it in Section 7. Section 8 concludes the paper with some remarks and indications for future research directions.

2. INVOLVED QUANTITIES

The following list contains the variables that are usually involved in the treatment of genital pain / penetration disorders by means of vaginal dilators. Note that this paper focusses on a qualitative model without considering exact mathematical values. Hence, the typical order of magnitude or dimension of the variables are not of interest.

$\mathbf{u}_{\text{pressure}}$:= *intravaginal pressure*, physically corresponding to an actuation pressure signal. It can be measured numerically by means of opportune pressure sensors, but it is still unknown how a measured vaginal pressure relates to the muscular tension in the pelvic floor;

$\mathbf{u}_{\text{vibration}}$:= *vibration levels* of the vaginal dilator, physically corresponding to an actuation signal. As for $\mathbf{u}_{\text{pressure}}$, it is numerically known but it is still unknown how it relates to the muscular tension in the pelvic floor;

$\mathbf{u}_{\text{stimulus}}$:= *visual sexual stimuli*, physically corresponding to erotic movies with different arousal-level contents, and corresponding to an actuation signal measured subjectively through questionnaires;

\mathbf{x}_{pain} := *affective-motivational dimension of the perceived level of pain*, a subjective state that is measured subjectively¹;

¹ Importantly, *pain* is not a mono-dimensional feeling; one may indeed distinguish the following two dimensions:

sensory-discriminative dimension of pain: dimension of pain relative to the sense of the intensity, location, quality and duration of the pain;

affective-motivational dimension of pain: dimension of pain relative to the unpleasantness and urge to escape the unpleasantness.

For example, placebos may modulate the affective-motivational dimension while leaving the sensory-discriminative one unchanged (Melzack and Casey, 1968). Intensity and unpleasantness of a painful stimulus should thus be measured and treated independently; in this paper nonetheless we consider only the affective-motivational dimension since it is the unique one dimension usually measured in tests through dedicated devices called *algometers*.

$\mathbf{x}_{\text{pleasure}}$:= *perceived and subjective level of pleasure*, that for convenience is measured with the same measurement units with which \mathbf{x}_{pain} is measured;

$\mathbf{x}_{\text{subj.arousal}}$:= *subjectively perceived sexual arousal levels*, corresponding to the *perceived* current interest in sexual activity, and measured through opportune questionnaires (notice that in our settings we ignore the type of sexual activity the subject is interested into) or by means of opportune Visual Analogue Scale (VAS). Notice that $\mathbf{x}_{\text{subj.arousal}}$ is different from the physiological arousal level $\mathbf{x}_{\text{phys.arousal}}$ defined below, that instead corresponds to the *physiological* genital arousal (that translates into lubrication levels, blood congestion levels, etc.). Importantly, women may be physiologically aroused but not subjectively aroused; this implies that there may be low concordance between the two signals $\mathbf{x}_{\text{subj.arousal}}$ and $\mathbf{x}_{\text{phys.arousal}}$;

$\mathbf{x}_{\text{phys.arousal}}$:= *physiological genital arousal level*, a quantity that merges different and distinct physiological signals such as $\mathbf{x}_{\text{lubrication}}$ (the vaginal lubrication levels) and $\mathbf{x}_{\text{vasocongestion}}$ (the vaginal vasocongestion levels, that can be measured through thermography, by Doppler considerations on ultrasounds, or using Magnetic Resonance Imagings (MRIs) techniques). In this manuscript we lump the various signals together for simplicity, assuming implicitly that they have large concordance;

$\mathbf{x}_{\text{muscles}}$:= *pelvic muscles tension / activity*, known to be influenced by the intravaginal pressure $\mathbf{u}_{\text{pressure}}$ but currently still not understood how;

\mathbf{x}_{fear} := *subjective fear level status*, a situation-related quantity that is in principle measurable through opportune questionnaires or a VAS;

$\mathbf{x}_{\text{touch}}$:= *perceived and subjective touch intensity*, assuming that the intensity of the stimulus is beyond the threshold for which the stimulus is perceived, and also that the stimulus is neither associated to pain nor pleasure.

The following list instead contains some other ancillary mathematical quantities that are useful to define the logical relations among the variables listed above.

$\mathcal{T}_{\text{touch}}$:= *touch perception threshold*, which measurement unit depends on the type of stimulus applied to the patient (e.g., Pascal or mmHg if it is a pressure);

$\mathcal{T}_{\text{pain}}$:= *pain perception threshold*, which measurement unit depends on the type of stimulus applied to the patient (e.g., Pascal or mmHg if it is a pressure);

$\mathcal{S}_{\text{pain}}$:= *pain sensitivity*, i.e., the function that maps a certain stimulus (considered as a time-dependent signal) into a subjective pain level (considered again as a time-dependent signal);

$\mathcal{S}_{\text{touch}}$:= *touch sensitivity*, for which the same considerations for the pain sensitivity $\mathcal{S}_{\text{pain}}$ also apply.

3. LITERATURE REVIEW

We now describe the known qualitative relationships among the variables introduced in Section 2.

3.1 Mechanisms leading to painful penetrative intercourses

$x_{\text{phys. arousal}} > 0 \implies (x_{\text{lubrication}} > 0, x_{\text{vasocongestion}} > 0)$, which reads $x_{\text{phys. arousal}}$ greater than 0 implies that $x_{\text{lubrication}}$ greater than 0 and $x_{\text{vasocongestion}}$ greater than 0; more precisely, when $x_{\text{phys. arousal}}$ starts increasing then this initiates genital blood flow leading to vasocongestion of the vestibular bulbs (Puppo, 2013) and vaginal lubrication (Levin, 2002). Indeed non-null $x_{\text{lubrication}}$ and $x_{\text{vasocongestion}}$ are indirect measures of a non null $x_{\text{phys. arousal}}$;

$u_{\text{vibration}} \in \text{"suitable range"}$ may $\implies x_{\text{pleasure}} > 0$; more precisely, stimulation of the inner labia together with the vestibular bulbs (summarizable in $u_{\text{vibration}}$) can facilitate and intensify orgasms (Puppo, 2011);

$x_{\text{fear}} > 0 \implies x_{\text{muscles}} > 0$, i.e., fear induces activity of the pelvic floor muscles (van der Velde et al., 2001; Both et al., 2012);

(if happening before or at the beginning of the penetrative act) $x_{\text{muscles}} > 0 \implies (x_{\text{lubrication}} \downarrow 0, x_{\text{vasocongestion}} \downarrow 0)$, where $x \downarrow 0$ denotes that x decreases till 0; more precisely, increasing the pelvic muscles activity before or at the beginning of the penetrative act may result in more pressure on the vulvar and vaginal skin, that in its turn allows less blood flow and lubrication (Lunsen, H.W. and Ramakers, 2002; Binik et al., 2006). Thus a non-negligible $x_{\text{muscles}}(0)$ (where we explicit the time index to remark it) may decrease $x_{\text{lubrication}}$ and $x_{\text{vasocongestion}}$. Unfortunately, though, this relationship depends on the timing: initial stages of arousal seems to lead indeed to an initial relaxation of the pelvic floor, i.e., $x_{\text{phys. arousal}}(0) > 0 \implies x_{\text{muscles}}(0) = 0$. Then, as the time passes and arousal increases, the deeper located pelvic floor muscles actually undergo contractions that are instrumental to achieve the orgasmic phase, Both et al. (2012), but these contractions seem to do not affect the lubrication and vasocongestion levels anymore, so that the dynamics seem in this case to decouple. Thus there is some dynamics on both how $x_{\text{phys. arousal}}$ influences x_{muscles} and x_{muscles} influences $x_{\text{lubrication}}$ and $x_{\text{vasocongestion}}$, but (to the best of our knowledge) no mathematical model seem to exist in the literature.

The previous implications thus suggest that a mathematical model of the quantities above should satisfy the logical relation ($x_{\text{phys. arousal}} \approx 0$ or $x_{\text{muscles}}(0) \gg 0$) $\implies x_{\text{pain}} > 0$ that says that penetrative activities with no or little arousal or initial activity of the pelvic floor muscles along with mechanical friction (due, e.g., to fear) may cause vulvar pain and dyspareunia. This logical chain in its turn agrees with other existing medical literature on the subject (Brauer et al., 2006; Farmer and Meston, 2007; ter Kuile et al., 2010).

3.2 Relations between $x_{\text{phys. arousal}}$ and $x_{\text{subj. arousal}}$

The existing literature orbits around the Basson's non-linear model of the female sexual response (Basson, 2000), stating that the subjective sexual arousal $x_{\text{subj. arousal}}$ is affected by several psychological inputs (e.g., satisfaction with the relationship, self-image, previous negative sexual experiences, negative cognitions, focus of attention), so

that $x_{\text{subj. arousal}}$ is highly influenced by the cognitive appraisal of the stimuli. Indeed the Basson's model assumes overlapping phases of physiological and subjective arousal in a variable and circular temporal sequence, plus considers relational factors and needs for emotional closeness as important motivators of sex, and focuses also on non-sexual outcomes instead of focusing on sexual urges and orgasmic release. In other words, in the Basson's model the goal of sexual activity for women is not necessarily orgasm, but rather personal and relational satisfaction/intimacy, which can manifest itself as physical satisfaction (orgasm and/or satisfactory sexual intercourse) and/or emotional satisfaction (e.g., a feeling of intimacy and connection with a partner) and self esteem.

Thus, concerning the relationship between $x_{\text{subj. arousal}}$ and $x_{\text{phys. arousal}}$, some psychological factors (e.g., memories, thoughts, desire for increased emotional closeness and intimacy, a particular overture from a partner in a long-term relationship, etc.) may trigger a predisposition to participate in sexual activity, i.e., leading to further increase of sexual arousal and desire conditions. At the same time, and concurrently, the physiological sexual arousal $x_{\text{phys. arousal}}$ may start via conversations, music, reading, fantasies, or viewing erotic materials, or physical stimulation. Once $x_{\text{phys. arousal}}$ is triggered, $x_{\text{subj. arousal}}$ may increase and motivate to continue the sexual activity (even if this is not always the case, since a subjective arousal may not necessarily be translated into sexual action, specially when the mere experience is already satisfying enough). Notice that while $x_{\text{phys. arousal}}$ and $x_{\text{subj. arousal}}$ are high, nonetheless, some other psychological factors (e.g., increased negative affect, disgust, fear, attentional bias) may work as turn-off and diminish up to vanishing $x_{\text{phys. arousal}}$ and/or $x_{\text{subj. arousal}}$, stopping thus the sexual fulfillment sensations. Notice that it is not stated elsewhere in the known literature that $x_{\text{subj. arousal}} \downarrow 0$ implied by turn-off factors will also imply physiological signals to decrease (i.e., ($x_{\text{lubrication}} \downarrow 0, x_{\text{vasocongestion}} \downarrow 0$)) up to a level that is sufficient to cause dyspareunia in case of penetrative intercourse.

Summarizing, there are currently no quantitative models relating direct interrelations between physiological and subjective arousal, and moreover the data available in the literature is insufficient to directly connect psychological factors with the mechanisms described above.

3.3 Relations among $x_{\text{phys. arousal}}$, u_{pressure} , x_{pain} , x_{pleasure} and x_{muscles}

To the best of our knowledge there are only two published papers describing these interrelationships. In particular,

(Gruenwald et al., 2007): under $u_{\text{vibration}} > 0$ assumptions, $x_{\text{phys. arousal}} \uparrow \implies (\mathcal{T}_{\text{pain}} \uparrow, \mathcal{T}_{\text{touch}} \uparrow)$, where $x \uparrow$ denotes that x increases; more precisely, in this experiments subjects were exposed to visual and auditory stimuli through erotic clips without manual or vibratory self-stimulation. After reaching a sufficiently high $x_{\text{phys. arousal}}$ state (considered here being equal to the lubrication level $x_{\text{lubrication}}$), subjects started experiencing vibrations $u_{\text{vibration}}$ (but also thermal differences) up to the moment that the patient perceived something. The conclusions drawn in that study was that touch sensitivity

of the vagina did not significantly change as a function of sexual arousal;

(Paterson et al., 2013): under $u_{\text{pressure}} > 0$ assumptions, $x_{\text{phys. arousal}} \uparrow \implies (\mathcal{T}_{\text{pain}} \uparrow, \mathcal{T}_{\text{touch}} \uparrow)$ and $x_{\text{phys. arousal}} \uparrow \not\Rightarrow x_{\text{pleasure}} \uparrow$; more precisely, the authors inserted a punctuated probe to exert a pressure u_{pressure} until the moment that the patients started feeling something or pain. They thus conclude that pain sensitivity of the vulvar vestibule increased during sexual arousal.

4. QUANTITIES INVOLVED IN OUR MODELS

Our aim is to have models that can be analyzed from quantitative perspectives; this means that the models that we are seeking are simple enough to admit analytical characterizations. Moreover we are also aiming at obtaining models with parameters that can be identified through data-driven approaches; since the available datasets usually comprise only a limited set of variables (i.e., measuring everything is usually infeasible). Thus, we must limit the complexity of the proposed models.

This implies that we can consider models that comprise only a subset of the quantities that are listed in Section 2. For this reason our subsequent models focus only on the following quantities, currently considered the most informative ones from therapeutic perspectives:

$$\begin{matrix} u_{\text{pressure}} & u_{\text{stimulus}} & x_{\text{pain}} & x_{\text{fear}} \\ x_{\text{muscles}} & x_{\text{pleasure}} & x_{\text{subj. arousal}} & x_{\text{phys. arousal}} \end{matrix} \quad (1)$$

5. A LOGICAL MODEL CONNECTING THE VARIABLES IN SECTION 4

To the best of our knowledge the existing literature does not offer models that describe the relations among all the quantities listed in (1), but only partial logical models suggesting co-implications among subsets of these variables. In this section we start filling this gap by connecting all the variables in (1) through the logical scheme of Figure 1. Parts of the scheme thus capture some implications that have been suggested in the existing literature; other parts of the scheme instead complete the picture through some intuition-driven guesses. These guesses are then made so that, overall, the complete scheme has a logically consistent qualitative behavior.

The right part of the scheme, called *Circle Of Fear (COF)*, models the feedback loop between fear, muscular tension and pain in a subject, and summarizes the relations listed in Sections 3.1 and 3.3. Summarizing, this part of the diagram indicates that when erotic stimuli are associated with fear (or also associated to some too weak pleasure, in a sense defined more precisely in Section 6) and the vaginal pressure is strong enough, then the patient may perceive pain. This in its turn causes fear, that eventually leads to an increased muscular tension (in its turn reinforcing the pain perception).

The left part of the diagram, called *Circle Of Pleasure (COP)*, models instead the feedback loop between subjective arousal, physiological arousal and pleasure in a subject, and summarizes plus completes the relations listed in Sections 3.2 and 3.3. More precisely we simplify Basson’s model to say that when erotic stimuli are strong enough,

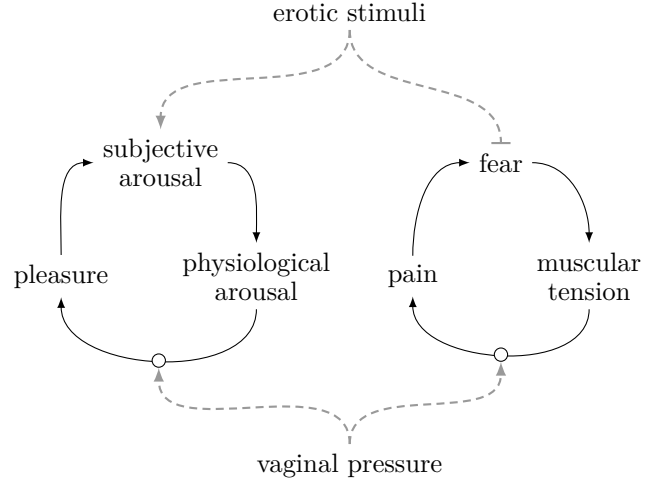


Fig. 1. Schematic representation of the logical dependencies among the various quantities defined in Section 2. Erotic stimuli work as enabling factors for the subjective arousal, while work as blocking factors for fear (i.e., tend to diminish the fear levels). Vaginal pressure stimuli, instead, work as enabling factors in both the implications between muscular tension and pain and between physiological arousal and pleasure. Vaginal stimuli may thus lead to either pain or pleasure, depending on the psychophysical status of the subject.

the patient may be brought in a state of subjective arousal, that then causes physiological arousal. We exploit instead Paterson et al. (2013) to say that if the vaginal pressure stimulus is strong enough, then, the patient may perceive pleasure (in its turn reinforcing the subjective arousal as in the Basson model). Note that it is reasonable to assume that erotic stimuli also directly influences the physiological arousal apart from the influence on the subjective arousal. However, in order to keep this model simple, we assume that erotic stimuli increases subjective arousal, which then leads to an increase in physiological arousal. Hence, the influence of erotic stimuli on physiological arousal is captured indirectly here. Importantly, and differently with the already existing literature, we ignore two important effects:

- (1) disturbing and turn-off factors: as for the validity of this simplification, we motivate it by recalling that the model is intended to be used for treatments that happen in clinical settings and when patients are in favorable psychological conditions (i.e., not during psychological breakdown periods), where disturbances should be minimized naturally by the environment;
- (2) effects of orgasmic phases: as stated in the literature review, pelvic floor muscle tension can co-affect pleasure and subjective arousal levels; nonetheless these co-dependencies are more remarked towards orgasmic phases, that we neglect due to the clinic nature of the model. This thus leads us to neglect muscular tension in the COP.

Summarizing, the proposed model does not want to capture daily-life conditions but rather what happens during specific treatments, and (importantly) being sufficiently simple to be prone to mathematical analysis.

6. A QUALITATIVE MODEL EXTENDING THE LOGICAL MODEL IN SECTION 5

In this section we transform the logical model in Figure 1 into a qualitative one capturing the dependencies between the quantities listed in (1). The proposed model does not capture actual time scales of the physiological and psychological human dynamics, but enables to find and analyze the dynamical behavior of the system.

In our model the external input signals u_{pressure} and u_{stimulus} are considered to be nonnegative (reflecting the fact that negative pressure or stimulus cannot be applied) and strictly less than 1 (interpretable as u_{pressure} and u_{stimulus} being normalised signals where $u_{\text{pressure}} = 1$ or $u_{\text{stimulus}} = 1$ corresponds to the maximal bearable pressure or stimulus, respectively).

6.1 Modeling the Circle Of Fear

We model the feedback loop between fear, muscular tension and pain through the dynamics

$$\begin{cases} \dot{x}_{\text{pain}} = -x_{\text{pain}} + \sqrt{x_{\text{muscles}}}u_{\text{pressure}}, & (2a) \\ \dot{x}_{\text{muscles}} = -x_{\text{muscles}} + x_{\text{fear}}, & (2b) \\ \dot{x}_{\text{fear}} = -x_{\text{fear}} - x_{\text{fear}}u_{\text{stimulus}} + x_{\text{pain}}. & (2c) \end{cases}$$

More precisely, equation (2a) describes what influences the level of pain over time. Our model captures the fact (analyzed in Section 3.1) that, at least qualitatively, pelvic muscles activity before or at the beginning of the penetrative act may lead to pain in the patient if there is some vaginal stimulus acting on the person. The proposed dynamics models the facts that: *i*) a constant level of vaginal pressure eventually leads to an equilibrium in the pain; *ii*); x_{muscles} logically acts as an “enabling” factor that grows sub-linearly since the influence of u_{pressure} on x_{pain} increases relatively more when x_{muscles} is small rather than high (Paterson et al., 2013); *iii*) when the pressure disappears then the pain exponentially decreases towards zero, since in this case the equation reduces to $\dot{x}_{\text{pain}} = -x_{\text{pain}}$.

Equation (2b) instead describes what influences the level of muscular tension over time. The dynamics of the muscle tension are effected by the feeling of fear, which leads to an increase of muscle tension. However, if the patient is not feeling fear (i.e., for $x_{\text{fear}} = 0$) then the muscle tension will exponentially decay towards zero.

Equation (2c) finally describes what influences the level of fear over time. Due to equation (2b), fear makes the muscle tension increase; the dynamics of fear instead are s.t. pain makes the fear increase. The proposed model moreover connects erotic stimuli with fear, and tries to mimic the implications listed in Section 3.2. More precisely we postulate that inducing an erotic stimulus can reduce fear, and capture this with the last term on (2c): if the patient is being stimulated (i.e., $u_{\text{stimulus}} > 0$) while feeling fear then $x_{\text{fear}}u_{\text{stimulus}}$ acts as a fear-reduction mechanism.

6.2 Modeling the Circle Of Pleasure

Our qualitative model of the feedback loop between physiological arousal, subjective arousal and pleasure is given by

$$\begin{cases} \dot{x}_{\text{phys.arousal}} = -x_{\text{phys.arousal}} + x_{\text{subj.arousal}}u_{\text{stimulus}}, & (3a) \\ \dot{x}_{\text{pleasure}} = -x_{\text{pleasure}} + x_{\text{phys.arousal}}u_{\text{pressure}}, & (3b) \\ \dot{x}_{\text{subj.arousal}} = -x_{\text{subj.arousal}} + x_{\text{pleasure}} + u_{\text{stimulus}}. & (3c) \end{cases}$$

More precisely, equation (3a) describes what influences the state of physiological arousal over time. The equation simplifies the Basson model by stating that the physiological arousal increases if the patient is being simultaneously sexually stimulated and subjectively aroused. In our approximated model, the physiological arousal changes more rapidly if the level of subjective arousal and/or the sexual stimulus are higher. Once again the model is designed so that if sexual stimulus or subjective arousal are missing (i.e., $x_{\text{subj.arousal}}u_{\text{stimulus}} = 0$) then the physiological arousal decreases naturally to zero. This mimics the fact that under no external stimulation the subject should tend to go towards a resting state.

Equation (3b) instead describes what influences the level of subjective pleasure over time. The intuition captured by the model is that if the patient is physiologically aroused then vaginal stimuli induce pleasurable sensations. Similarly to as before, our model states that if vaginal pressure or physiological arousal are missing (i.e., $x_{\text{phys.arousal}}u_{\text{pressure}} = 0$) then the pleasure decreases exponentially to zero.

Equation (3c) eventually describes what influences the level of subjective arousal over time. This equation models this level as increasing when the patient is sexually stimulated and when perceiving a pleasurable physical sensation. The two effects moreover are modeled as being independently adding to the subjective arousal levels. As in all the various equations above, also here if the various other variables are zero then the subjective arousal is modeled as exponentially vanishing in time.

Notice that equations (3b) and (3c) jointly capture the intuitions developed after Section 3.2. More precisely in our model we do not postulate a direct effect of the physiological arousal on the subjective one, but rather exploit the “intermediate” state variable x_{pleasure} . Notice also that the vice-versa is not true, i.e., in our model the subjective arousal affects directly the physiological one through equation (3a).

7. MATHEMATICAL ANALYSIS OF MODEL (2)-(3)

For notational simplicity we will exploit the notation

$$x_{\text{COF}} := \begin{bmatrix} x_{\text{pain}} \\ x_{\text{muscles}} \\ x_{\text{fear}} \end{bmatrix} \quad x_{\text{COP}} := \begin{bmatrix} x_{\text{phys.arousal}} \\ x_{\text{pleasure}} \\ x_{\text{subj.arousal}} \end{bmatrix}.$$

7.1 Positivity

For model (2)-(3) to be meaningful, all states of the system must be nonnegative if all the initial states and all the external inputs are nonnegative; i.e., the COF and COP should be positive systems.

Given that the maps (2) and (3) are continuously differentiable, to prove their positivity we can exploit the characterization in de Leenheer and Aeyels (2001) that a continuously differentiable dynamics $\dot{x} = f(x)$ with $x \in \mathbb{R}^n$ is positive if and only if for all x at the boundary

of \mathbb{R}_+^n it is true that $x_i = 0$ implies $f_i(x) \geq 0$. In our case this condition can be easily verified since, once assuming all the input signals to be nonnegative,

$$\begin{aligned}\dot{x}_{\text{pain}}|_{x_{\text{pain}}=0} &= \sqrt{x_{\text{muscles}}}u_{\text{pressure}} \geq 0, \\ \dot{x}_{\text{muscles}}|_{x_{\text{muscles}}=0} &= x_{\text{fear}} \geq 0, \\ \dot{x}_{\text{fear}}|_{x_{\text{fear}}=0} &= x_{\text{pain}} \geq 0, \\ \dot{x}_{\text{phys. arousal}}|_{x_{\text{phys. arousal}}=0} &= x_{\text{subj. arousal}}u_{\text{stimulus}} \geq 0, \\ \dot{x}_{\text{pleasure}}|_{x_{\text{pleasure}}=0} &= x_{\text{phys. arousal}}u_{\text{pressure}} \geq 0, \\ \dot{x}_{\text{subj. arousal}}|_{x_{\text{subj. arousal}}=0} &= x_{\text{pleasure}} + u_{\text{stimulus}} \geq 0\end{aligned}$$

7.2 Equilibria of the COF

The equilibria of (2) are

$$\begin{cases} x_{\text{fear}}^* = \frac{u_{\text{pressure}}^2}{(1 + u_{\text{stimulus}})^2} & (4a) \\ x_{\text{pain}}^* = \frac{u_{\text{pressure}}^2}{1 + u_{\text{stimulus}}} & (4b) \\ x_{\text{muscles}}^* = \frac{u_{\text{pressure}}^2}{(1 + u_{\text{stimulus}})^2} & (4c) \end{cases}$$

so that for a given u_{pressure} and u_{stimulus} the linearized system is

$$\Delta \dot{x}_{\text{COF}} = \begin{bmatrix} -1 & \frac{1}{2} \frac{u_{\text{pressure}}}{\sqrt{x_{\text{muscles}}^*}} & 0 \\ 0 & -1 & 1 \\ 1 & 0 & -(1 + u_{\text{stimulus}}) \end{bmatrix} \Delta x_{\text{COF}}.$$

Thus $u_{\text{pressure}} = 0$ or $u_{\text{pressure}} > 0$ discriminates between the trivial equilibrium $x_{\text{COF}}^* = 0$ and a non-trivial one $x_{\text{COF}}^* > 0$. As for the stability of these equilibria, for $x_{\text{muscles}}^* = 0$ the term $\frac{u_{\text{pressure}}}{\sqrt{x_{\text{muscles}}^*}}$ is unbounded, so that

choosing the working point $u_{\text{pressure}} > 0$ leads to an equilibrium $x_{\text{COF}}^* = 0$ that is unstable irrespectively of u_{stimulus} .

The equilibrium $x_{\text{COF}}^* > 0$ is instead stable as we only allow $u_{\text{stimulus}} \geq 0$ (indeed, since $\frac{1}{2} \frac{u_{\text{pressure}}}{\sqrt{x_{\text{muscles}}^*}} = \frac{1}{2}(1 + u_{\text{stimulus}}) < 1$, the eigenvalues of the linearized dynamics matrix have all strictly negative real parts).

7.3 Equilibria of the COP

The equilibria of (3) are

$$\begin{cases} x_{\text{pleasure}}^* = \frac{u_{\text{pressure}}u_{\text{stimulus}}^2}{1 - u_{\text{pressure}}u_{\text{stimulus}}} & (5a) \\ x_{\text{phys. arousal}}^* = \frac{u_{\text{stimulus}}^2}{1 - u_{\text{pressure}}u_{\text{stimulus}}} & (5b) \\ x_{\text{subj. arousal}}^* = \frac{u_{\text{stimulus}}}{1 - u_{\text{pressure}}u_{\text{stimulus}}} & (5c) \end{cases}$$

so that for a given u_{pressure} and u_{stimulus} the linearized system is

$$\Delta \dot{x}_{\text{COP}} = \begin{bmatrix} -1 & 0 & u_{\text{stimulus}} \\ u_{\text{pressure}} & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \Delta x_{\text{COP}},$$

that implies that the equilibrium is stable for $u_{\text{stimulus}} < 1$ and $u_{\text{pressure}} < 1$. The absence of erotic stimuli, i.e., $u_{\text{stimulus}} = 0$, moreover implies $x_{\text{COF}}^* = 0$.

7.4 Discussion of the equilibria

Summarizing, the equilibria of our model depend on u_{stimulus} and u_{pressure} as follows:

	$u_{\text{stimulus}} = 0$	$u_{\text{stimulus}} > 0$
$u_{\text{pressure}} = 0$	(resting state) $x_{\text{COF}}^* = 0$ $x_{\text{COP}}^* = 0$	(arousal state) $x_{\text{COF}}^* = 0$ $x_{\text{COP}}^* > 0$
$u_{\text{pressure}} > 0$	(fear state) $x_{\text{COF}}^* > 0$ $x_{\text{COP}}^* = 0$	(full state) $x_{\text{COF}}^* > 0$ $x_{\text{COP}}^* > 0$

As graphically indicated above, we can discriminate four different situations:

Resting state: if $u_{\text{pressure}} = u_{\text{stimulus}} = 0$ then the equilibrium in both circles is the origin, and it is in both cases stable: intuitively this means that without external stimuli a person is supposedly resting and supposedly remaining resting.

Arousal state: if $u_{\text{pressure}} = 0$ and $u_{\text{stimulus}} > 0$ then the origin is an *unstable* equilibrium for the COF, while the COP has a non-trivial stable equilibrium. The instability of the origin for x_{COF} in this specific case is actually a desired mathematical property, since this behavior captures the intuition that the resting state $x_{\text{COF}}^* = 0$ is tenuous when $u_{\text{pressure}} > 0$: as soon as some pressure is exerted a patient will eventually feel something (i.e., the COF will activate). The non-null equilibrium for the COP instead corresponds to the patient being sexually stimulated but not experiencing any vaginal pressure, which is modeled to lead to an absence of fear or pain. This state is called the ‘‘arousal state’’.

Fear state: if $u_{\text{pressure}} > 0$ and $u_{\text{stimulus}} = 0$ then the COP is modeled to remain in the stable trivial equilibrium $x_{\text{COP}}^* = 0$, while $x_{\text{COF}}^* > 0$. Hence, the patient is modeled to experience pain and fear without any sexual arousal or pleasure.

Full state: if $u_{\text{pressure}} > 0$ and $u_{\text{stimulus}} > 0$ then both circles have nontrivial and stable equilibria; our model thus admits a patient to experience simultaneously pleasure as well as pain, with amounts depending on the quantitative measures of applied pressure and stimulus. This is an important aspect as patients undergoing treatment must often learn to tolerate pressure as well as learn to associate pressure with pleasure (often referred to as ‘‘counter conditioning’’). Hence, being able to create a state where both circles are active is essential to understand the treatment of dyspareunia.

7.5 Examples

For completeness, we now plot and comment one example of system trajectories for each situation but the trivial resting state described in Section 7.4.

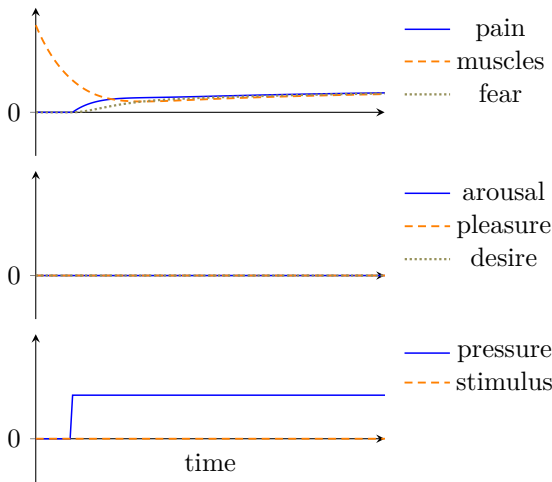


Fig. 2. (*fear state*) in this case the model/patient starts at time zero with a bit of muscular tension; after that it is subject to a pressure stimulus (but no erotic stimulus), so that the patient ends up with a pain that is proportional to the pressure stimulus. In this case the COP is never active, given that u_{stimulus} remains null for all times.

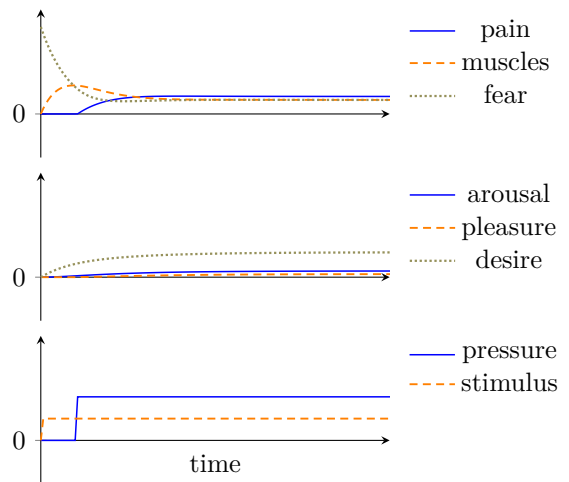


Fig. 4. (*full state*) in this situation the model/patient starts at time zero with a null COP but a non-null COF, in the sense that the model-patient starts at time zero with a bit of fear and with a “small” erotic stimulus, and after that it is subject to a pressure stimulus. Before the pressure starts the fear decays quickly thanks to the erotic stimulus. The non-null fear, nonetheless, activates the muscular tension, so that when the vaginal pressure starts, because of the non-null muscular tension, the patient ends up again in pain. In this case, nonetheless, the pain is not directly proportional to the pressure stimulus because it is also diminished by the presence of the erotic stimulus. Eventually, thus, in this case the circle of pleasure is active because the erotic stimulus activated it at the beginning.

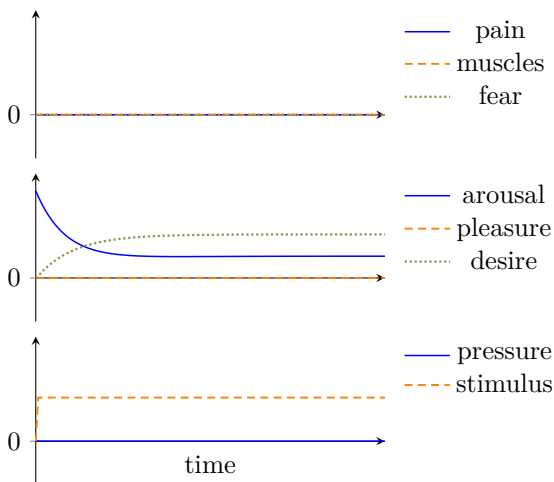


Fig. 3. (*arousal state*) the model/patient starts at time zero with a bit of physiological arousal and is subject to an erotic stimulus but no vaginal pressure. In this case, as expected, the COP is partially active and the patient ends up in an aroused state but without physical pleasure, while the COF stays null because of the opportune input signals.

8. CONCLUSIONS

We presented a simplified and deterministic control-oriented model of the dynamics of the main quantities involved in the treatment of genital pain / pelvic disorders by means of vaginal dilators. The model is meant to capture clinical setting situations, and for this reason neglects psychological factors and other disturbances; on the other hand it simultaneously captures the typical behaviors of the average patient and is prone to mathematical characterizations, offering thus insights on how to design better treatments.

The model moreover opens some possibilities of new research directions; among others, next research steps are to perform system identification and verify the predictive capability of the model when tuned based on real data. For this, as a next step, a quantitative model with adequate parameters has to be developed. Then, the parameters can be determined using system identification methods and measurement data.

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