Multi-room occupancy estimation through adaptive gray-box models

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***Abstract*— We consider the problem of estimating the occupancy level in buildings using indirect information such as CO_2 concentrations and ventilation levels. We assume that one of the rooms is temporarily equipped with a device measuring the occupancy. Using the collected data, we identify a gray-box model whose parameters carry information about the structural characteristics of the room. Exploiting the knowledge of the same type of structural characteristics of the other rooms in the building, we adjust the gray-box model to capture the CO_2 dynamics of the other rooms. The occupancy estimators are then designed using a regularized deconvolution approach which aims at estimating the occupancy pattern that best explains the observed CO_2 dynamics. We evaluate the proposed scheme through extensive simulation using a commercial software tool, IDA-ICE, for dynamic building simulation.**

***Index Terms*— Occupancy estimation, Maximum Likelihood, CO_2 dynamics, inference, building automation**

I. Introduction

The estimation of occupancy levels in buildings has important implications in efficient control of Heating, Venting and Air Conditioning (HVAC) systems and diagnostics [1], [2], [3], [4], [5], [6], [7]. Instrumenting buildings with dedicated hardware such as camera systems may raise privacy concerns and be economically disadvantageous, in particular when this requires retrofitting old structures. On the other hand, there is an increasing interest in understanding the effectiveness of estimating occupancy using non-dedicated information sources in buildings, such as CO_2 concentration and air inlet actuation levels.

There are two main strategies to estimate occupancy in buildings. The first utilizes direct occupancy measurements collected by people-counting devices (see [8], [9] for a survey). The second strategy exploits non-dedicated sensor and devices. The typical approach is to design occupancy estimators by inverting the CO_2 dynamics. The model relating the CO_2 concentration with occupancy can be derived either using physics-based concepts (e.g., mass-balance equations) or by employing data-based modeling techniques. As for the physics-based CO_2 models, assuming well-mixed air in the room, authors in [10] derived a bilinear model which has similarities with the model presented in this paper. Still assuming well-mixed air, [11], [12] and [5] make use of mass-balance equations and linear models for the CO_2 dynamics. More detailed models are considered in [13]. Regarding data-based modeling techniques, [14] uses methods of moments, while [15] proposes both linear parametric and nonparametric identified models, from which estimators based on deconvolution are designed. A novel approach is proposed in [16], where, using blind system identification techniques as in [17], no training sets including occupancy measurements are required. Other types of estimators use black box models identified using, e.g., neural networks or hidden Markov models, and potentially including several sources of information (e.g., temperature, humidity, concentrations, door statuses and light status, sound and motion, electricity consumption patterns) [18], [19], [20], [21], [22], [23], [24], [25]. This literature focuses on occupancy estimation in single rooms. Besides a few studies dealing with modeling and estimation of occupancy movements across buildings (see e.g., [26], [19]), the multi-room case has not received as much attention as the single-room case.

In this paper, we take an important step towards the extension of single-room occupancy estimators to the multi-room case. Our fundamental question is whether the information on the CO_2 dynamics gathered in one room can be exploited to design occupancy estimators for other rooms of the same building. To answer such a question, we assume that one room of the building is temporarily equipped with an occupancy measurement device. We use the data collected by this device, together with CO_2 concentration and ventilation data, to identify a nonlinear gray-box model via Maximum Likelihood (ML) [27]. The structure of the gray-box model is derived from first principles [28] and it permits defining a one-to-one correspondence between the model parameter vector and the physical parameters characterizing the room (i.e., room volume and size of the ventilation system). Exploiting this correspondence, we adapt the gray-box model to the characteristics of the other rooms, and we design an occupancy estimation based on regularized environmental signal deconvolution, similarly to the strategy proposed in [15]. The role of regularization here is to promote piecewise constant occupancy patterns.

We evaluate the proposed estimation scheme on a simu-

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lated environment generated using the commercial software IDA-ICE [29]. The simulated environment models a building on the KTH campus. The generated data are validated by comparison with a dataset available on the [30].

The paper is structured as follows. In Section II, we derive and identify the physics-based gray-box model which models the CO_2 dynamics. In Section III, we introduce the occupancy estimator based on the identified model. In Section IV we present the problem of extending the occupancy estimator to the multi-room case. Experiments are reported in Section V. Some conclusions end the paper.

II. Modeling and identification of the CO_2 dynamics

A. A physics-based model

The CO_2 concentration of room j, denoted by $\overline{c}_j(t)$, can be modeled from mass-conservation considerations, assuming well-mixed air (see [28]):

$$v_j \frac{d\overline{c}_j(t)}{dt} = \left(\dot{Q}_j^{\text{vent,sup}} + \dot{Q}_j^{\text{leak,in}}\right)c - \left(\dot{Q}_j^{\text{vent,exh}} + \dot{Q}_j^{\text{leak,out}}\right)\overline{c}_j(t) + g\ o_j(t). \quad (1)$$

Here, v_j is the volume of the room, and c is the outdoor air CO_2 concentration, which we assume constant and equal to 420 ppm; $\dot{Q}_j^{\text{vent,sup}}$ and $\dot{Q}_j^{\text{vent,exh}}$ represent the supply and exhaust mechanical ventilation rates and $\dot{Q}_j^{\text{leak,in}}$ and $\dot{Q}_j^{\text{leak,in}}$ the inflow and outflow air leakages through doors and windows. The term $go_j(t)$ models the occupants CO_2 generation in the room, where g is the CO_2 generation rate per person and $o_j(t)$ is the number of occupants at time t. In the case of balanced ventilation it is reasonable to assume that $\dot{Q}_j^{\text{vent,sup}} \approx \dot{Q}_j^{\text{vent}} \approx \dot{Q}_j^{\text{vent,exh}}$ and that $\dot{Q}_j^{\text{leak,in}} \approx \dot{Q}^{\text{leak}} \approx \dot{Q}_j^{\text{leak,out}}$. Equation (1) can then be simplified to

$$\frac{d\overline{c}_j(t)}{dt} = \frac{\dot{Q}_j^{\text{vent}}}{v_j}\left(c - \overline{c}_j(t)\right) + \frac{\dot{Q}^{\text{leak}}}{v_j}\left(c - \overline{c}_j(t)\right) + \frac{g}{v_j}o_j(t). \quad (2)$$

In the ventilation system considered in this work, there is a constant ventilation flow in the zones, which can be increased if the indoor CO_2 concentration is above a certain threshold. Under these assumptions, (2) can be rewritten as

$$\frac{d\overline{c}_j(t)}{dt} = \frac{\dot{Q}_j^{\text{vent,min}} + (\dot{Q}_j^{\text{vent,max}} - \dot{Q}_j^{\text{vent,min}})u_j(t)}{v_j}\left(c - \overline{c}_j(t)\right) + \frac{\dot{Q}^{\text{leak}}}{v_j}\left(c - \overline{c}_j(t)\right) + \frac{g}{v_j}o_j(t). \quad (3)$$

Since $\dot{Q}_j^{\text{vent,min}}$ does not depend on the ventilation control signal $u_j(t)$, we rewrite (3) as

$$\frac{d\overline{c}_j(t)}{dt} = \frac{\dot{Q}_j^{\text{u}}u_j(t)}{v_j}\left(c - \overline{c}_j(t)\right) + \frac{\dot{Q}_j^{\text{c}}}{v_j}\left(c - \overline{c}_j(t)\right) + \frac{g}{v_j}o_j(t), \quad (4)$$

with $\dot{Q}_j^{\text{u}} = \dot{Q}_j^{\text{vent,max}} - \dot{Q}_j^{\text{vent,min}}$ and $\dot{Q}_j^{\text{c}} = \dot{Q}_j^{\text{vent,min}} + \dot{Q}^{\text{leak}}$.

We discretize the continuous-time model (4) using the backward Euler discretization[1], so we obtain

$$\frac{\overline{c}_j(k) - \overline{c}_j(k-1)}{T} = \frac{\left(\dot{Q}_j^{\text{u}}u_j(k) + \dot{Q}_j^{\text{c}}\right)}{v_j}\left(c - \overline{c}_j(k)\right) + \frac{g}{v_j}o_j(k), \quad (5)$$

where T is the sampling time. We define $c_j(k) := \overline{c}_j(k) - c$ and the parameter vector $\boldsymbol{\theta}_j^T := \left[\theta_j'\ \theta_j''\ \theta_j'''\right]$, where

$$\begin{cases} \theta_j' := \dfrac{v_j}{v_j + T\dot{Q}_j^{\text{u}}} \\ \theta_j'' := \dfrac{Tg}{v_j + T\dot{Q}_j^{\text{u}}} \\ \theta_j''' := \dfrac{T\dot{Q}_j^{\text{c}}}{v_j + T\dot{Q}_j^{\text{u}}} \end{cases}. \quad (6)$$

We assume that the measurements of $c_j(k)$ are corrupted by additive noise. Then, the measured CO_2 concentration, denoted by $y_j(k)$, can be expressed through the measurement model $y_j(k) = c_j(k) + e_j(k)$, where $e_j(k)$ is the noise, assumed white and Gaussian. The overall model for the CO_2 dynamics can be rewritten as the nonlinear Output Error (OE) system

$$\begin{cases} c_j(k) = \dfrac{\theta_j'}{1 + \theta_j'''u_j(k)}c_j(k-1) + \dfrac{\theta_j''}{1 + \theta_j'''u_j(k)}o_j(k) \\ y_j(k) = c_j(k) + e_j(k). \end{cases} \quad (7)$$

B. Identification of the gray-box model

In this section we describe a procedure for identifying the parameter vector $\boldsymbol{\theta}_j$ characterizing the model (7). Here, we assume that we have collected the dataset of information from room j

$$\mathcal{D}_j := \left\{y_j(k), u_j(k), o_j(k)\right\}_{k \in \mathcal{K}_j}, \quad (8)$$

containing recorded occupancy levels plus environmental information from the building supervisory control and data acquisition (SCADA) system for a set of time indexes $\mathcal{K}_j$,

Let us introduce the auxiliary notation

$$a_j(k) := \frac{\theta_j'}{1 + \theta_j'''u_j(k)}, \qquad b_j(k) := \frac{\theta_j''}{1 + \theta_j'''u_j(k)}, \quad (9)$$

so that (7) becomes

$$c_j(k) = a_j(k)c_j(k-1) + b_j(k)o_j(k). \quad (10)$$

Expanding recursively (10) back in time, and defining the quantities

$$\widetilde{c}_j(k) := c_j(k) - c_j(0)\prod_{\tau=0}^{k-1} a_j(k-\tau), \quad (11)$$

$$B_j(k, k-h) := b_j(k-h)\prod_{\tau=0}^{h-1} a_j(k-\tau) \quad (12)$$

[1] This choice is motivated by the fact that backward Euler discretization led to better identification and estimation performance than the forward Euler discretization.

(with the convention that $\prod_{\tau=0}^{-1} \star = 1$ for every possible $\star$), it follows that

$$\begin{bmatrix} \widetilde{c}_j(1) \\ \vdots \\ \widetilde{c}_j(k) \end{bmatrix} = \begin{bmatrix} B_j(1,1) & & 0 \\ \vdots & \ddots & \\ B_j(k,1) & \cdots & B_j(k,k) \end{bmatrix} \begin{bmatrix} o_j(1) \\ \vdots \\ o_j(k) \end{bmatrix}. \quad (13)$$

Given $u_j(1), \ldots, u_j(k)$, $o_j(1), \ldots, o_j(k)$ and our Gaussian assumptions on the noise $e_j(k)$ in (7), we have that

$$\begin{aligned} \widehat{c}_j\,(k;\boldsymbol{\theta}_j) := {} & y_j(0) \prod_{\tau=0}^{k-1} a_j(k-\tau) \\ & + \begin{bmatrix} B_j(k,1) & \cdots & B_j(k,k) \end{bmatrix} \begin{bmatrix} o_j(1) \\ \vdots \\ o_j(k) \end{bmatrix}, \end{aligned} \quad (14)$$

is the best estimator of $c_j(k)$ for the parameter guess $\boldsymbol{\theta}_j$. This estimator can then be used for defining the ML estimator for the parameters $\boldsymbol{\theta}_j$ given the dataset $\mathcal{D}_j$. The estimate $\widehat{\boldsymbol{\theta}}_j$ is obtained solving

$$\widehat{\boldsymbol{\theta}}_j := \arg \min_{\widetilde{\boldsymbol{\theta}}_j \in \mathbb{R}^3} \sum_{k \in \mathcal{K}_j} \left(y_j(k) - \widehat{c}_j\left(k; \widetilde{\boldsymbol{\theta}}_j\right) \right)^2. \quad (15)$$

Even if problem (15) is nonlinear, it involves only three decision variables and can be effciently solved using standard interior point methods [31].

III. Estimating occupancy levels by regularized deconvolution

In this section we revise the occupancy estimation approach proposed in [15], with some modifications to adjust it to the nonlinear gray-box model (7). We assume that we have the estimate $\widehat{\boldsymbol{\theta}}_j$ of the parameters of room j, and that, for each time instant k, we have access to $y_j(k)$, $y_j(k-1)$ and $u_j(k)$, but not to $o_j(k)$.

From the assumption of Gaussianity of the measurement noise $e_j(k)$ in (7), the best unbiased estimator of $o_j(k)$ corresponds to a Least Squares (LS) estimator. However, since we know that candidate occupancy patterns are piecewise constant, more effective estimators can be obtained by applying regularized estimators. Let us introduce the following matrix and vector notation

$$\widetilde{y}_j(k) := y_j(k) - y_j(0) \prod_{\tau=0}^{k-1} a_j(k-\tau), \quad (16)$$

$$\widetilde{\boldsymbol{y}}_j := \begin{bmatrix} \widetilde{y}_j(1) \\ \vdots \\ \widetilde{y}_j(k) \end{bmatrix}, \quad \boldsymbol{o}_j := \begin{bmatrix} o_j(1) \\ \vdots \\ o_j(k) \end{bmatrix}, \quad B_j := \begin{bmatrix} B_j(h,\tau) \end{bmatrix}.$$

Furthermore, we introduce the discrete derivative of $o_j(k)$ as

$$\Delta o_j(\tau) := o_j(\tau) - o_j(\tau-1), \quad \tau = 1, \ldots, k-1,$$

$$\Delta \boldsymbol{o}_j := \left[\Delta o_j(1), \ldots, \Delta o_j(k-1) \right]$$

In [15], the following estimator for occupancy was introduced:

$$\widehat{\boldsymbol{o}}_j = \left\lfloor \arg \min_{\widetilde{\boldsymbol{o}}_j \in \mathbb{R}_+^k} \ \|\widetilde{\boldsymbol{y}}_j - B_j \widetilde{\boldsymbol{o}}_j\|_2^2 + \lambda_j \left\|\Delta \widetilde{\boldsymbol{o}}_j\right\|_1 \right\rceil, \quad (17)$$

where $\lfloor \cdot \rceil$ denotes the vector-wise rounding operator, which is used to obtain integer solutions. This estimator is composed of a LS-type part, which favors adherence to data, and a ℓ_1 (component) depending on the derivative of the unknown occupancy. This latter component promotes piecewise constant solution patterns. The parameter λ_j allows a trade-off between the two components. We refer to [15] for further details.

Problem (17) is usually called fused-lasso. More elaborated theoretical analysis on the performance of these estimators can be found in [32] and [33].

It is straightforward to modify (17) to obtain an online estimator which considers only a fixed-length (say, N) data window of the past. At each time instant, the estimator is run by constructing the vectors $\widetilde{\boldsymbol{y}}_j$ and $\boldsymbol{o}_j$ using the latest N data of the past. The length N is chosen so that the computational complexity is low enough to allow a real-time solution of (17) and so that the discarded information does not influence significantly the outcomes of the estimator. A reasonable choice for tuning λ_j in (17) is to use the value $\widehat{\lambda}_j$ that leads to the best estimation performance on the dataset used for training the parameters $\widehat{\boldsymbol{\theta}}_j$ in Section II-B. The performance index can be chosen as

$$\|\boldsymbol{o}_j - \hat{\boldsymbol{o}}_j(\lambda_j)\|_2\,,$$

where $\boldsymbol{o}_j$ is constructed from the dataset (8) and $\hat{\boldsymbol{o}}_j(\lambda_j)$ is the occupancy pattern obtained using λ_j in the estimator (17).

IV. From single-room to multi-rooms estimators

This section is dedicated to the extension of the estimator (17) to a generic room in a building that is not instrumented with occupancy sensors, by exploiting the information on the CO_2 dynamics obtained in one room of the same building which is instrumented with occupancy sensors. We assume that the sampling time T and the volume v_j are known for all the rooms of interest.

Assume that every single room in a generic building is instrumented with sensors measuring CO_2 and HVAC actuation levels (which are generally available in standard HVAC systems). We also assume that only one of the rooms, denoted by $j = 0$, is instrumented with occupancy sensors for a short period. For this room the dataset $\mathcal{D}_0$ defined in (8) is available and thus it is possible to identify the CO_2 dynamic of the room by estimating the unknown parameters $\boldsymbol{\theta}_0$ through (15) and estimate the occupancy levels invoking estimator (17).

However, for the rooms without occupancy measurements, the estimator (15) cannot be used to find the CO_2 dynamic due to the lack of training set. We call these rooms untrained rooms. Define $\mathcal{I}$ as the set of rooms without occupancy measurements or untrained rooms. The question is now how to extend the estimator (17) to these rooms.

To implement the estimator (17) one needs to know the CO_2 dynamics of the room, i.e., (7), or alternatively, $\boldsymbol{\theta}_j$ and the regularization parameter λ_j. Finding the variables $\boldsymbol{\theta}_j$ for a room, in turn, requires either a training set $\mathcal{D}_j$ or the knowledge of $\dot{Q}_j^{\text{c}}, \dot{Q}_j^{\text{u}}, g$ since there is a one-to-one correspondence between $\boldsymbol{\theta}_j$ and $\dot{Q}_j^{\text{c}}, \dot{Q}_j^{\text{u}}, g$. Since for the rooms $j \in \mathcal{I}$ the set $\mathcal{D}_j$ is not available, we need to infer the triplet $\left(\dot{Q}_j^{\text{c}}, \dot{Q}_j^{\text{u}}, g\right)$ and the regularization parameter λ_j from the training room and other available information.

A. Estimating $\left(\dot{Q}_j^{u}, \dot{Q}_j^{c}, g\right)$

We pose the following assumptions

$$\frac{\dot{Q}_j^{\text{vent,max}}}{\dot{Q}_0^{\text{vent,max}}} = \frac{M_j}{M_0}, \quad \forall j, \tag{18a}$$

$$\frac{\dot{Q}_j^{\text{vent,min}}}{\dot{Q}_0^{\text{vent,min}}} = \frac{M_j}{M_0}, \quad \forall j, \tag{18b}$$

where M_j and M_0 are parameters proportional to the ventilation inlet area serving the rooms j and the training room, respectively. According to the assumptions the maximum and minimum ventilation air flows are proportional to the total inlet area in a room and the parameters M_j can be obtained easily by physical inspection of the rooms. However, the assumptions are made for the purpose of this paper and might not be always applicable. The main reason is that the ventilation system design also depends on the room usage. We will study the implications of these assumptions through a simulated example.

Due to the assumptions, we can write

$$\dot{Q}_j^{\text{u}} = M_j \dot{Q}^{\text{u}}, \quad \forall j, \tag{19a}$$

$$\dot{Q}_j^{\text{c}} = M_j \dot{Q}^{\text{c}}, \quad \forall j, \tag{19b}$$

where $\dot{Q}^{\text{u}}$ and $\dot{Q}^{\text{c}}$ are constant values. For (19b) we used the fact that the value of $\dot{Q}^{\text{leak}}$ is negligible compared to $\dot{Q}_j^{\text{vent,min}}$ and thus $\dot{Q}_j^{\text{c}} \approx \dot{Q}_j^{\text{vent,min}}$. Based on (19), we can readapt the gray-box model of the training room to the characteristics of the other rooms. It is sufficient to estimate the triplet $\left(\dot{Q}_j^{\text{c}}, \dot{Q}_j^{\text{u}}, g\right)$ for one room to be able to find it for all rooms. In order to estimate the triplet $\left(\dot{Q}_j^{\text{c}}, \dot{Q}_j^{\text{u}}, g\right)$ for any room $j \in \mathcal{I}$, we can thus start from the training room and estimate the unknown parameters $\widehat{\theta}_0$ through the single-room estimator (17). The parameters $\dot{Q}_0^{\text{c}}, \dot{Q}_0^{\text{u}}$ and g can be obtained invoking (6).

Once the triplet $\left(\dot{Q}_0^{\text{c}}, \dot{Q}_0^{\text{u}}, g\right)$ is obtained, one can use the information on M_j and v_j together with (6) to find $\widehat{\theta}_j$ for all $j \in \mathcal{I}$ and therefore the CO_2 dynamic of the untrained rooms. It is then straightforward to estimate the occupancy levels using the estimator (17).

B. Estimating λ_j

The regularization parameter λ_j is connected to the usage of the room as well as its structural characteristics. For instance, if a crowd size in a room is changing frequently, then λ_j should be small (and vice versa). The problem of generalizing λ_j to untrained rooms is an open problem and cannot be answered without additional assumptions on the usage of the room. In the following we analyze two different cases, corresponding to two specific hypotheses on the usage patterns in buildings. Although the suggested strategies are not immediately applicable in practical situations, they can produce some basic ideas on the choice of λ_j.

1) Assuming the same usage pattern for the rooms: Assume $\lambda_j = \lambda$ for all j; in this case λ should be estimated by coupling the tuning procedures described in Section III by finding the best $\widehat{\lambda}$ in the occupancy estimator for the training room.

2) Assuming the usage patterns to depend on the size of the room: One may assume

$$\lambda_j = \lambda v_j, \tag{20}$$

i.e., the usage pattern depends linearly on the size (for simplicity, the volume, assuming that the ceilings heights are equal among different rooms). This simple assumption leads to the strategy

$$\frac{\lambda_j}{\lambda_0} = \frac{v_j}{v_0}, \tag{21}$$

where λ_0 is obtained by solving the tuning problem in Section III for the training room. Once λ_0 in (20) has been found, then generalizing to other untrained rooms is immediate, as soon as one knows the relative room volumes v_j.

V. Assessing the extended occupancy estimators

We evaluate the effectiveness of our derivations, through a building simulations tool. The dedicated experiments are described below.

A. Simulation software environment

Simulations have been performed using IDA-ICE 4.6, a commercial program for dynamic simulations of energy and comfort in buildings [29]. The program features equation-based modeling (NMF-language [34], [35] or Modelica language [36]) and is equipped with a variable time step differential-algebraic solver [37].

B. Geometry of the simulated building

The simulated indoor environment in Figure 1 represents the ground floor of a seven-storey university building in the KTH main campus in Stockholm. The rooms considered for our simulations are the labeled ones, and have different dimensions and use. The rooms have a Variable Air Volume (VAV) ventilation strategy where the mechanical ventilation airflow $u_j(k)$ varies depending on the current CO_2 concentration in the room. In all rooms the ventilation is provided by a central fan active between 8:00 and 18:00. Room dimensions range from the 40 m^2 of a small workshop (A:231) to the 130 m^2 of a lecturing room (A:213), see

Table I. The rooms have different occupancy and uses, which is reflected in the specific ventilation flows in the rooms; for instance, the project room (A:235) has more regular occupancy patterns than the conference hall (B:213), where periods of zero and high occupancies are alternated.

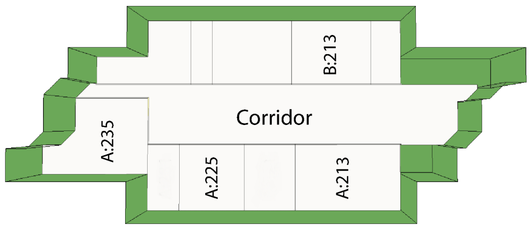

Fig. 1. Floor plan modeled in IDA-ICE.

Room name	Room size [m^2]	Min air flow [l/(s m^2)]	Max air flow [l/(s m^2)]
A:235	125	0.6	2.16
A:225	81	0.93	3.34
A:213	130	0.58	2.1
B:213	96	1.05	13.13

TABLE I

ROOM FOOTPRINT, MINIMUM AND MAXIMUM ROOM MECHANICAL VENTILATION AIR FLOWS PER UNIT AREA. AIR FLOWS PER UNIT AREA ARE INDICATIVE OF THE NEED FOR VENTILATION AND THE LEVEL OF ACTIVITY IN THE ROOM.

C. Simulation setup

Room environment simulations were carried out for a period of two weeks between July 13 and July 26 in 2014. The climate file was a weather file for Bromma airport in Stockholm.

Air infiltrates through the windows and doors depending on the external wind speed and the air leakage area; Table II gathers the main infiltration parameters used in the simulations. The air tightness of the building is assumed to be 0.5 Air Changes per Hour (ACH) @ 50 Pa.

Room name	Windows surface [m^2]	Air leakage area [m^2]
A:235	10.6	0.015
A:225	2.3	0.008
A:213	3.4	0.014
B:213	0	0.009

TABLE II

TOTAL EXTERNAL WINDOWS AREA AND AIR LEAKAGE AREA IN THE ROOMS.

Each room has a different profile for the occupants; the level of activity of the occupants was set to 1.8 Metabolic Equivalent of Task (MET), corresponding to a light physical activity, such as typical office working conditions; CO_2 emissions per person (parameter g), which is proportional to the activity, resulted in 15.4 mg_{CO_2}/s, corresponding to $8 \cdot 10^{-6}$ $m^3_{CO_2}/s$.

D. Validation of the data generation mechanism

To assess the accuracy of the IDA-ICE physical model with respect to the real room dynamics, we compare measured and simulated CO_2 data in Figure 2, under the same conditions of occupancy and ventilation levels [38]. The real data are collected from the laboratory room A:225 [30]. The two sets of measured and simulated data show that the physical model is capable to capture the main CO_2 dynamics. The mismatch between the two curves is attributed to events whose effect, though minor, is not simple to account for; examples of such events are doors kept open and non-logged window openings.

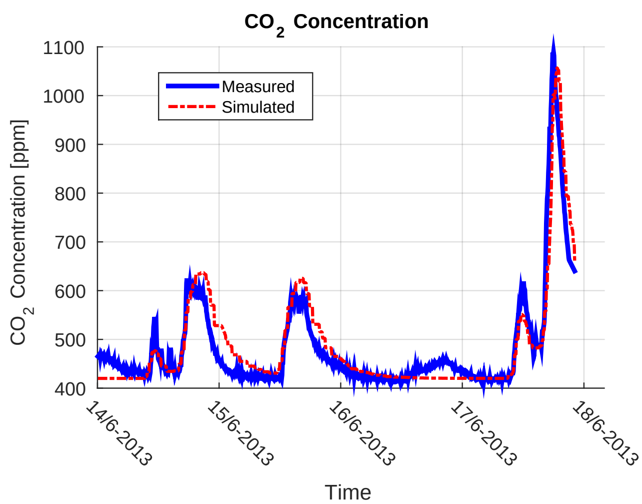

Fig. 2. Validation of the IDA-ICE model. CO_2 levels from room measurements and from simulation are compared, from [38].

E. Assessing the single-room occupancy estimation algorithm

Here we compare the predictive capabilities of the single-room model (7) against numerical representations of the rooms. This assessment is performed to check out whether the proposed model reproduces the internal and not accessible CO_2 model of IDA-ICE. To this aim we:

1) collect the dataset $\mathcal{D}_j = \{c_j(k)u_j(k), o_j(k)\}_{k\in\mathcal{K}_j}$ for each room from the virtual room built in IDA-ICE;
2) add to $c_j(k)$ some artificial white Gaussian noise (whose variance is estimated from the real data used in SectionV-D and is equal to 35) and build the dataset
$$\mathcal{D}_j := \{y_j(k), u_j(k), o_j(k)\}_{k\in\mathcal{K}_j}; \qquad (22)$$
3) identify the model, i.e., estimate the unknown part of $\boldsymbol{\theta}$ through the ML strategy discussed in Section II-B. This step corresponds to estimate the parameters $\widehat{\boldsymbol{\theta}}$ solving (15) and thus to obtain both the CO_2 estimator $\widehat{y}_j\left(k;\widehat{\boldsymbol{\theta}}\right)$ through (14) and the occupancy estimator $\widehat{o}_j(k)$ through (17);

The results for the estimated and measured CO_2 for one day for the room A:225 are plotted in Figure 3, where it is possible to see that the proposed model is able to reproduce the CO_2 generated by IDA-ICE. Realizations of the true occupancy and the estimated one for the same room is depicted in Figure 4. From Figure 4 it can be seen that the proposed occupancy estimator for a single-room model can give accurate results in reproducing the true occupancy. In order to quantitatively evaluate the estimation capabilities of the single-room estimators, Table III provides some performance indexes for all rooms[2]. The Mean Square Error (MSE) of the estimations is small for all rooms and the algorithm has good detection of occupied rooms (small FNs).

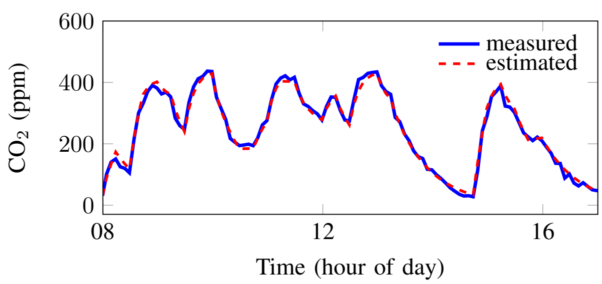

Fig. 3. Validation of model (7) against IDA-ICE for room A:225.

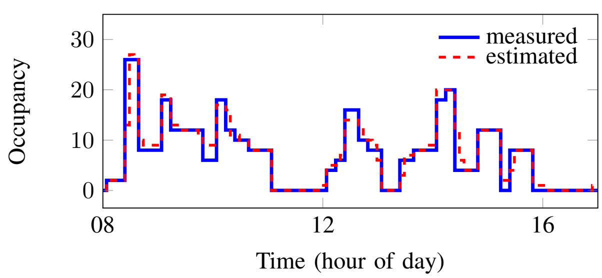

Fig. 4. Realizations of the true and estimated occupancy through the single-room estimator (17) for room A:213.

Room	MSE	Accuracy	FP	FN
A:213	0.125	0.496	0.144	0.022
A:225	0.247	0.563	0.280	0.000
A:235	0.109	0.636	0.063	0.011
B:213	0.075	0.750	0.047	0.021

TABLE III

SUMMARY OF THE PERFORMANCE INDEXES OF THE COMPLETE SINGLE-ROOM ESTIMATORS.

F. Assessing the multi-room model occupancy estimation algorithm

To evaluate the effectiveness of the proposed multi-room occupancy estimator, we collect data from IDA-ICE for all the rooms mentioned in Table I and we apply the occupancy estimator algorithm of Section IV.

[2]The performance indices are described in Appendix I.

In Figure 5, we provide the results of the occupancy estimation in one of the untrained rooms. It can be seen that the estimator is able to estimate the number of occupiers with fairly good precision, even though not as well as the single-room estimator. In order to have a better evaluation of the estimator, Table IV reports the performance indexes achieved by the estimator for all of the untrained rooms. The suggested multi-room estimator tends to have good ability on detecting occupancy levels in the rooms that are not instrumented with occupancy sensors. We noticed that there is a slight performance degradation in the estimated occupancy compared to the single-room case. This can be considered as a consequence of the assumptions made in (18), which do not hold for this simulation example (see Table I).

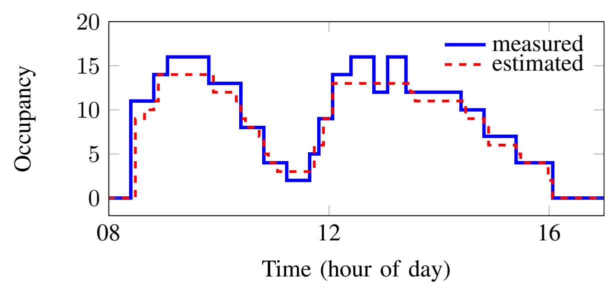

Fig. 5. Realizations of the true and estimated occupancy through the multi-room estimator for the untrained room A:213 when the model is trained on room A:225.

Trained Room	Untrained Room	MSE	Accuracy	FP	FN
A:225	A:235	0.179	0.413	0.364	0.001
A:225	A:213	0.232	0.276	0.399	0.000
A:225	B:213	0.104	0.489	0.062	0.012

TABLE IV

SUMMARY OF THE PERFORMANCE INDEXES OF THE COMPLETE ESTIMATORS .

VI. CONCLUSIONS

In this paper we have studied the problem of estimating the occupancy levels in buildings using available environmental and actuation signals. Our proposed method is centered on the CO_2 dynamics which, starting from first principles, are modeled using a nonlinear gray-box model. The parameters of this model are identified on one of the rooms using a Maximum Likelihood (ML) approach. The resulting model is utilized to construct an occupancy estimator based on regularized deconvolution; this estimator is then adapted to other rooms of the building by exploiting the knowledge of the characteristics of the rooms and their relation with the room where the model is first identified. We have built a simulated environment where we have tested the estimation scheme, showing the effectiveness of the proposed scheme.

A natural extension of the current work is the application of blind system identification techniques to the proposed scheme, so to remove the need of a training phase. More extensions may consider improving the estimations by using the knowledge of interconnection of the rooms and the locations of exits and entrances.

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APPENDIX I
PERFORMANCE INDICES

We report the performance indices that are used in Section V for evaluation of the proposed algorithm.

1) The *mean square error*

$$\mathrm{MSE}\left(\widehat{\boldsymbol{o}}\right) := \frac{\left\|\widehat{\boldsymbol{o}} - \boldsymbol{o}\right\|_2^2}{\left\|\boldsymbol{o}\right\|_2^2}, \tag{23}$$

characterizing the relative estimation errors.

2) The *accuracy*

$$\mathrm{Acc}\left(\widehat{\boldsymbol{o}}\right) := 1 - \frac{\left\|\mathbb{1}\left(\widehat{\boldsymbol{o}} - \boldsymbol{o}\right)\right\|_1}{N}, \tag{24}$$

reporting how many times the estimates are perfect by means of the ℓ_1 norm of the indicator function

$$\mathbb{1}(\boldsymbol{x}) := \begin{cases} \mathbb{1}(x(1)) \\ \vdots \\ \mathbb{1}(x(N)) \end{cases} \qquad \mathbb{1}(x(t)) := \begin{cases} 1 & \text{if } x(t) > 0 \\ 0 & \text{otherwise.} \end{cases} \tag{25}$$

3) The *false positive / false negative occupancy detection rates*

$$\mathrm{FP}(\widehat{\boldsymbol{o}}) := \widehat{\beta}(0), \qquad \mathrm{FN}(\widehat{\boldsymbol{o}}) := 1 - \widehat{\beta}(1), \tag{26}$$

describing the ability of discriminating the presence / absence of occupants in terms of false positives (when the room is estimated to be occupied while it is not) and false negatives (when the room is estimated to be empty while it is not) by means of the empirical power function

$$\widehat{\beta}(\theta) := \frac{1}{|\mathcal{N}_\theta|} \sum_{k \in \mathcal{N}_\theta} \mathbb{1}(\widehat{o}(k)), \tag{27}$$

in its turn based on the definition of the sets

$$\mathcal{N}_\theta := \{t \text{ s.t. } \mathbb{1}(o(k)) = \theta\}, \qquad \theta = \{0, 1\}, \tag{28}$$

dividing the time indexes in the sets $\mathcal{N}_0$, for the k's for which the room was not occupied, and $\mathcal{N}_1$, for the k's for which the room was occupied.

Appendix II
Notation

parameter	*description*	*unit*
$j \in \mathbb{N}_+$	room index	adim.
$t \in \mathbb{R}$	time index (continuous)	adim.
$k \in \mathbb{N}_+$	time index (discrete)	adim.
$o_j(k)$	occupancy at time k in room j	adim.
g	CO_2 generation rate per person (assumed constant and known)	$m^3_{CO_2}$ / s
$\overline{c}_j(k)$	CO_2 concentration level at time k in room j	ppm
c	CO_2 concentration level of the air injected by the ventilation system (assumed constant and known)	ppm
$c_j(k) := \overline{c}_j(k) - c$	normalized CO_2 concentration level at time k in room j	ppm
$y_j(k)$	noisy measurement of $c_j(k)$	ppm
$u_j(k) \in [0,1]$	actuation levels of the ventilation system at time k in room j	adim., %
$\dot{Q}_j^{\max}$	nominal maximum airflow of the ventilation system for room j	m^3 / s
$\dot{Q}_j^{\min}$	nominal minimum airflow of the ventilation system for room j	m^3 / s
$\dot{Q}^{\text{leak}}$	leaking air flow (e.g., from windows and doors; assumed constant for each room)	m^3 / s
v_j	volume of room j	m^3
$\mathcal{I} \subset \mathbb{N}_+$	set of rooms not instrumented with occupancy sensors	adim.

TABLE V

Summary of the most important parameters.