1. Recall that an affine transformation of the plane is of the form

$$G(u, v) = (au + bv + c, du + ev + f).$$

(a) Find an affine change of coordinates that takes the unit square with vertices

$$P = (0, 0), \quad Q = (1, 0), \quad R = (0, 1), \quad S = (1, 1)$$

in the uv-plane to the rectangle with vertices

$$\bar{P} = (-1, 5), \quad \bar{Q} = (3, 5), \quad \bar{R} = (-1, 8), \quad \bar{S} = (3, 8)$$

in the xy-plane.

(b) Find the Jacobian for the change of coordinates in part (a).

2. Verify that

$$f(x,y) = \begin{cases} 4xy & \text{if } 0 \le x \le 1,\ 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

is a joint density function. Suppose X and Y are random variables with joint density function f find $P\left(X \ge \frac{1}{2}\right)$.

3. A spherical shell centered at the origin has an inner radius of 5 cm and an outer radius of 6 cm. The density δ of the material increases linearly with the distance from the center. At the inner surface $\delta = 12\text{g/cm}^3$ and at the outer surface $\delta = 14\text{g/cm}^3$.

   (a) Using spherical coordinates write the density δ as a function of ρ.

   (b) Find the mass of the shell.

4. Find the Jacobian for $G : \mathbb{R}^3 \to \mathbb{R}^3$ where $G(\rho, \theta, \phi) = (x, y, z)$ is the transformation defined by spherical coordinates.

5. Let E be the ellipsoid defined by the equation

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = 1.$$

Use the change of variables $x = au$, $y = bv$, and $z = cw$ and a triple integral to find a formula for the volume of E in terms of a, b, and c.