Math 32B, Fall 2019 **WS - Week 2** **Full Name:**

1. Let $\mathcal{D}$ be the region in the plane bounded by $y = x^2$ and $y = 1$. Write the double integral $\iint_{\mathcal{D}} f(x, y)\, dA$ as an iterated integral in both possible orders.

2. Let $\mathcal{D}$ be the trapezoid in the plane with vertices $(0, 0)$, $(2, 0)$, $(1, 1)$, and $(0, 1)$. Write the double integral $\iint_{\mathcal{D}} f(x, y)\, dA$ as an iterated integral in both possible orders. Which one is easier?

3. Evaluate the double integral $\iint_{\mathcal{D}} \sqrt{y^3 + 1}\, dA$ where $\mathcal{D}$ is the region in the first quadrant bounded by $x = 0$, $y = 1$, and $y = \sqrt{x}$. Try the integration in both possible orders. Which one is easier?

4. Evaluate the iterated integral

$$\int_0^1 \int_{-\sqrt{1-x^2}}^0 2x \cos\left(y - \frac{y^3}{3}\right) \, dy \, dx.$$

5. Determine the projection onto the xy-plane of the region $\mathcal{W} \subseteq \mathbb{R}^3$ bounded by the planes $x = 0$, $y = 0$, $z = 0$, and $x + y + z = 1$.

6. Determine the projection onto the xy-plane of the region $\mathcal{W} \subseteq \mathbb{R}^3$ bounded by the surfaces $z = \sqrt{x^2 + y^2}$ and $x^2 + y^2 + z^2 = 1$.

7. In this problem you will integrate the function $f(x, y, z) = y$ over the region $\mathcal{W} \subseteq \mathbb{R}^3$ bounded by the surfaces $z = 8 - x^2 - y^2$ and $z = x^2 + y^2$.

(a) Sketch the region $\mathcal{W}$.

(b) Determine the projection of $\mathcal{W}$ onto the xy-plane.

(c) Notice that $\mathcal{W}$ is z-simple. This means we can write

$$\iiint_{\mathcal{W}} y\, dV = \iint_{\mathcal{D}} \int_{z_1}^{z_2} y\, dz\, dA$$

where $\mathcal{D}$ is the projection found in part (b). Use this to write the triple integral as an iterated triple integral of the form $dz\, dy\, dx$.

(d) Finally, compute the triple integral.

(e) What goes wrong if you try to compute the iterated integral instead as $dz\, dx\, dy$?