Math 32B - Fall 2019
Exam 2

Full Name: ______________________________

UID: ______________________________

Circle the name of your TA and the day of your discussion:

Steven Gagniere   Jason Snyder   Ryan Wilkinson

Tuesday   Thursday

Instructions:

- Read each problem carefully.
- Show all work clearly and circle or box your final answer where appropriate.
- Justify your answers. A correct final answer without valid reasoning will not receive credit.
- Simplify your answers as much as possible.
- Include units with your answer where applicable.
- Calculators are not allowed but you may have a 3×5 inch notecard.

Page	Points	Score
1	20	
2	30	
3	25	
4	25	
Total:	100	

THIS PAGE LEFT INTENTIONALLY BLANK

You may use this page for scratch work. Work found on this page will not be graded unless clearly indicated in the exam.

1. (20 points) Let $\mathbf{F}(x, y, z) = \langle 2xy^2z, 2x^2yz, x^2y^2 + 2z \rangle$ and let $\mathcal{C}$ be the line segment from $(1, 1, 3)$ to $(1, 1, -2)$.

   (a) Show that the vector field $\mathbf{F}$ is conservative using curl.

   (b) Find a function f such that $\mathbf{F} = \nabla f$.

   (c) Use part (b) to evaluate $\displaystyle\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$.

   (d) Is there a vector field $\mathbf{G}$ defined on $\mathbb{R}^3$ such that $\operatorname{curl} \mathbf{G} = \mathbf{F}$?

/20

2. (15 points) Consider the vector field

$$\mathbf{F}(x,y) = \langle F_1, F_2 \rangle = \left\langle \frac{3}{x-y}, \frac{3}{y-x} \right\rangle.$$

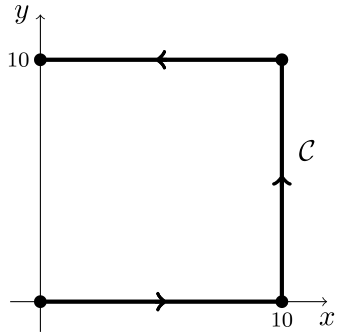
(a) Show that $\dfrac{\partial F_1}{\partial y} = \dfrac{\partial F_2}{\partial x}$.

(b) Show that $\mathbf{F}$ is defined on two distinct connected domains in the plane. On each of these domains, is $\mathbf{F}$ conservative? *Hint:* Are these domains simply connected?

3. (15 points) Find the work done by the force field $\mathbf{F}(x,y,z) = \langle x^2, y^2, z^3 \rangle$ in moving a particle along the line segment from $(0,0,0)$ to $(1,2,2)$.

/30

4. (25 points) Let $\mathcal{C}$ be the curve given by the line segments from $(0,0)$ to $(10,0)$ to $(10,10)$ to $(0,10)$ as pictured below. Evaluate $\int_{\mathcal{C}} (e^{x^2} + 2y)\,dx + (5x + 2y)\,dy$. *Hint:* Complete $\mathcal{C}$ to form a closed curve and use Green's Theorem.



/25

5. (25 points) Consider the vector field $\mathbf{F}(x, y, z) = \langle x^2, y^2, 2z \rangle$ and the surface $\mathcal{S}$ given by $z = xy$ for $0 \leq x \leq 1$ and $0 \leq y \leq 1$. Suppose $\mathcal{S}$ is oriented with upward normal. Find the flux $\displaystyle\iint_{\mathcal{S}} \mathbf{F} \cdot d\mathbf{S}$.

/25