1. Partial differential equations or PDEs are equations involving partial derivatives of a multivariable function. Some well known PDEs that are important in physics are the Laplace equation $f_{xx} + f_{yy} = 0$ and the wave equation $f_{xx} = c^2 f_{yy}$ for some constant c.

   (a) Show that $f(x, y) = e^x \cos y$ is a solution to the Laplace equation.

   (b) Show that $f(x, y) = e^{-(x+y)^2}$ is a solution to the wave equation.

   (c) Is $f(x, y) = x^3 - 3xy^2$ a solution to either of these PDEs?

2. Assume Clairaut's theorem holds (on some domain) for each of the following functions and compute the partial derivatives.

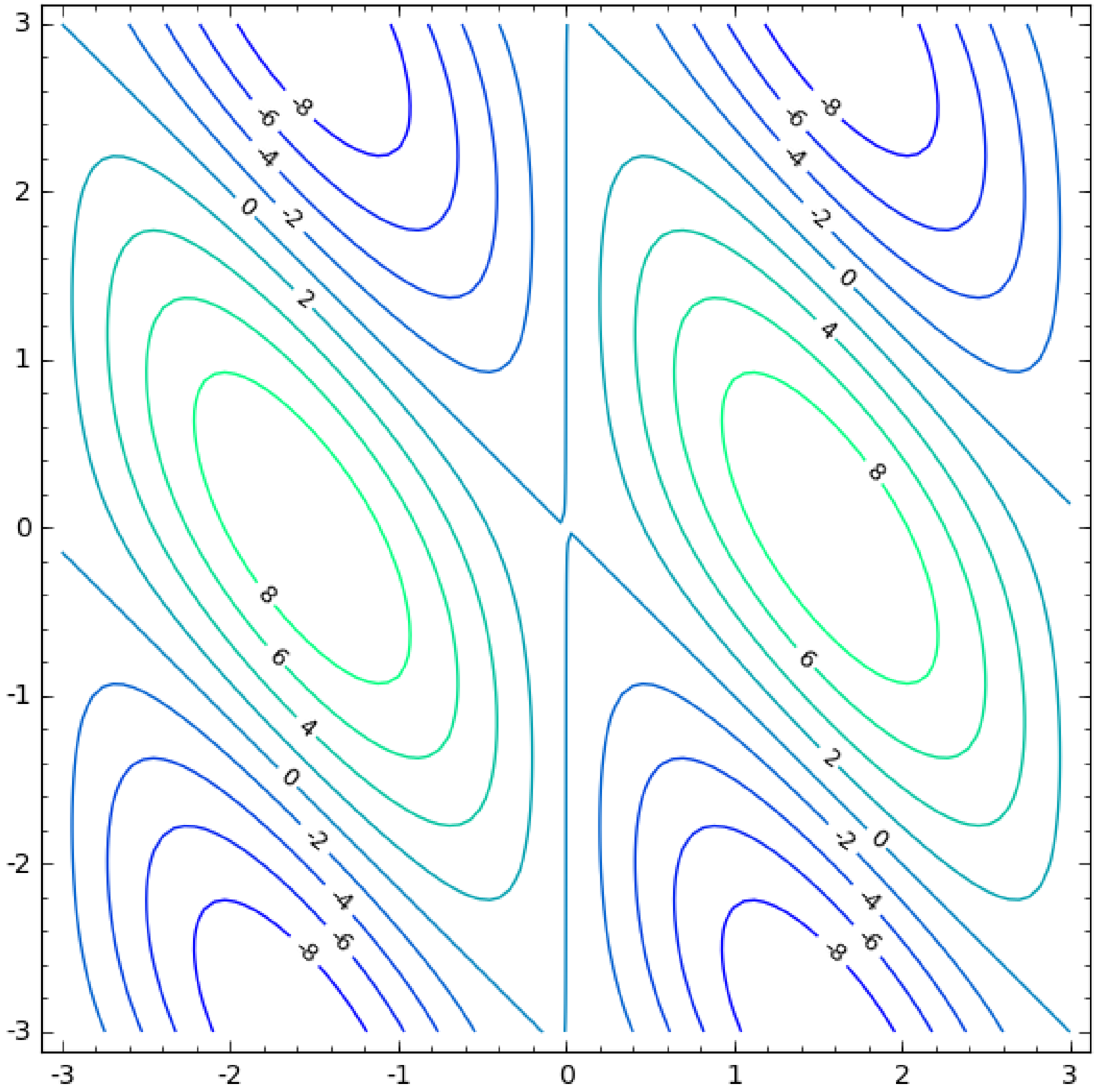
   (a) Find f_{xyxyxy} for $f(x, y) = x^2 \cos(e^y + y^2)$.

   (b) Find f_{xxxyy} for $f(x, y) = x^3y^2 - \dfrac{y}{x + \ln(x)}$.

3. Either give an example of a function $f(x, y)$ with the following partial derivatives or show that no such function can exist.

$$\frac{\partial f}{\partial x} = 2x + y\cos(xy) - y^3, \qquad \text{and}$$
$$\frac{\partial f}{\partial y} = x\cos(xy) - 3xy^2.$$

4. Consider the contour plot for $f(x, y)$ below.



Determine the sign of each of the following derivatives.

$f_x(1, 0)$ __________ $f_y(1, 0)$ __________

$f_{xx}(1, 0)$ __________ $f_{xy}(1, 0)$ __________ $f_{yy}(1, 0)$ __________

$f_x(2, 1)$ __________ $f_y(2, 1)$ __________

$f_{xx}(2, 1)$ __________ $f_{xy}(2, 1)$ __________ $f_{yy}(2, 1)$ __________