Math 32A, Winter 2019 **WS - Week 2** **Full Name:**

1. Describe in words and sketch a picture of the region in $\mathbb{R}^3$ represented by the following inequality.

$$1 \leq \frac{x^2}{4} + y^2 \leq 4$$

2. Show the following equation represents a sphere, then find its center and radius.

$$x^2 + y^2 + z^2 - 2x - 4y + 8z + 17 = 0$$

3. Find parametric equations of the line through the point $(0, 14, -10)$, parallel to the following line.

$$\mathbf{r}(t) = \langle -1 + 2t, 6 - 3t, 3 + 9t \rangle$$

4. Find a vector equation of the line parametrizing the path of a particle that arrives at the point $(2, 0, -3)$ at $t = 0$ and $(-1, -1, 0)$ at $t = 2$.

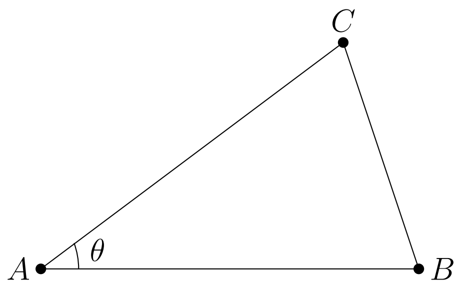
5. Show that the following lines L_1 and L_2 are **skew lines**, that is they do not intersect and they are not parallel.

$$L_1\colon \mathbf{r}(t) = \langle 1 + t, -2 + 3t, 4 - t \rangle, \qquad L_2\colon \mathbf{r}(t) = \langle 2t, 3 + t, -3 + 4t \rangle$$

6. In this problem you will use the Law of Cosines to derive the following property of the dot product.

$$\mathbf{v} \cdot \mathbf{w} = ||\mathbf{v}||\,||\mathbf{w}|| \cos\theta$$

Consider the triangle pictured below.



(a) Let $\mathbf{v}$ denote the vector $\overrightarrow{AB}$ and $\mathbf{w}$ denote the vector $\overrightarrow{AC}$, and let θ be the angle between them. Find $\mathbf{v} - \mathbf{w}$ in the picture.

(b) Use the Law of Cosines to find $||\mathbf{v} - \mathbf{w}||^2$ in terms of $||\mathbf{v}||$, $||\mathbf{w}||$, and θ.

(c) Use algebraic properties of the dot product to show the following equation holds.

$$||\mathbf{v} - \mathbf{w}||^2 = ||\mathbf{v}||^2 - 2\mathbf{v} \cdot \mathbf{w} + ||\mathbf{w}||^2$$

(d) Set the formulas from parts (b) and (c) equal to each other, then simplify to complete the proof.