Math 115B - Winter 2020
Midterm Exam

Full Name: ______________________________

UID: ______________________________

Instructions:

- Read each problem carefully.
- Show all work clearly and circle or box your final answer where appropriate.
- Justify your answers. A correct final answer without valid reasoning will not receive credit.
- All work including proofs should be well organized and clearly written using complete sentences.
- You may use the provided scratch paper, however this work will not be graded unless very clearly indicated there and in the exam.
- Calculators are not allowed but you may have a 3×5 inch notecard.

Page	Points	Score
1	10	
2	10	
3	10	
4	10	
Total:	40	

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You may use this page for scratch work. Work found on this page will not be graded unless clearly indicated here and in the exam.

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1. (10 points) True or False: Prove or disprove the following statements.

Let V be a finite-dimensional inner product space and let $T \colon V \to V$ be a linear operator.

(a) Suppose $\mathbb{F} = \mathbb{C}$. If T is self-adjoint then $\langle Tv, v \rangle$ is real for all $v \in V$.

(b) Suppose $\mathbb{F} = \mathbb{R}$. If T is normal then T is diagonalizable.

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2. (10 points) Let V be a finite-dimensional inner product space over the field $\mathbb{F} = \mathbb{R}$.

   (a) Let $T\colon V \to V$ be a self-adjoint linear operator whose only eigenvalues are zero and one. Show $T^m = T$ for all $m \geq 1$.

   (b) Suppose $V = W \oplus W^\perp$ for some subspace $W \subseteq V$. Let $T\colon V \to V$ be the orthogonal projection onto W along $W^\perp$. Show that T is self-adjoint.

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3. (10 points) Let T be an operator on a two-dimensional vector space V over a field $\mathbb{F}$. Recall, we say V is a T-cyclic subspace of itself if there exists a nonzero vector $v \in V$ such that the T-cyclic subspace generated by v is all of V. Prove that either V is a T-cyclic subspace of itself or T is a scalar multiple of the identity.

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4. (10 points) True or False: Prove or disprove the following statements.

   (a) If a matrix A is unitarily equivalent to a diagonal matrix then A^4 is also unitarily equivalent to a diagonal matrix.

   (b) Let V be a finite-dimensional complex vector space and let T be a linear operator whose only eigenvalue is zero. Then T is nilpotent.

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