Math 115A - Spring 2019
Exam 1

Full Name: ________________________________

UID: ________________________________

Instructions:

- Read each problem carefully.
- Show all work clearly and circle or box your final answer where appropriate.
- Justify your answers. A correct final answer without valid reasoning will not receive credit.
- All work including proofs should be well organized and clearly written using complete sentences.
- You may use the provided scratch paper, however this work will not be graded unless very clearly indicated there and in the exam.
- Calculators are not allowed but you may have a 3×5 inch notecard.

Page	Points	Score
1	10	
2	10	
3	10	
4	10	
Total:	40	

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1. (10 points) True or False: Prove or disprove the following statements.

   (a) The set $W = \{A + A^T \in M_{n\times n}(\mathbb{F}) \mid A \in M_{n\times n}(\mathbb{F})\}$ is a subspace of $M_{n\times n}(\mathbb{F})$.

   (b) The set $W = \{(x, y) \in \mathbb{R}^2 \mid y = x^2\}$ is a subspace of $\mathbb{R}^2$

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2. (10 points) Suppose that $T : V \to W$ and $S : U \to V$ are linear maps of finite-dimensional vector spaces with the property that $T \circ S = 0$.

   (a) Show $\operatorname{im} S \subseteq \ker T$.

   (b) Suppose S and T further satisfy that S is injective, T is surjective, and $\operatorname{im} S = \ker T$. Show that $\dim V = \dim U + \dim W$. *Hint:* You may use that since S is injective $\ker S = \{0\}$.

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3. (10 points) Let V be a vector space over a field $\mathbb{F}$ such that $\dim_{\mathbb{F}} V = 2$.

   (a) Show there exist subspaces $W_1, W_2 \subseteq V$ such that W_1 and W_2 are each one-dimensional and $V = W_1 \oplus W_2$.

   (b) Let $V = \mathbb{R}^2$ and $W_1 = \text{span}\{e_1\}$. Show that the complement W_2 is not necessarily unique. That is, give examples of two distinct subspaces W_2 and W_2' such that $V = W_1 \oplus W_2$ and $V = W_1 \oplus W_2'$.

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4. (10 points) Consider the function $T : M_{2\times 2}(\mathbb{F}) \to M_{2\times 2}(\mathbb{F})$ defined by $T(M) = HM - MH$ where H is the matrix $H = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

(a) Prove that T is a linear transformation.

(b) Find the rank and nullity of T.

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