Generalized Linear Model

$$f(y;\theta) = \exp\left[a(y)b(\theta) + c(\theta) + d(y)\right]$$

$$\mathrm{E}[a(Y)] = -c'(\theta)/b'(\theta).$$

$$\mathrm{var}[a(Y)] = \frac{b''(\theta)c'(\theta) - c''(\theta)b'(\theta)}{[b'(\theta)]^3}$$

Table 3.1 *Poisson, Normal and binomial distributions as members of the exponential family.*

Distribution	Natural parameter	c	d
Poisson	$\log\theta$	$-\theta$	$-\log y!$
Normal	$\frac{\mu}{\sigma^2}$	$-\frac{\mu^2}{2\sigma^2} - \frac{1}{2}\log\left(2\pi\sigma^2\right)$	$-\frac{y^2}{2\sigma^2}$
Binomial	$\log\left(\frac{\pi}{1-\pi}\right)$	$n\log\left(1-\pi\right)$	$\log\binom{n}{y}$

Table 4.3 *Data for Poisson regression example.*

y_i	2	3	6	7	8	9	10	12	15
x_i	−1	−1	0	0	0	0	1	1	1

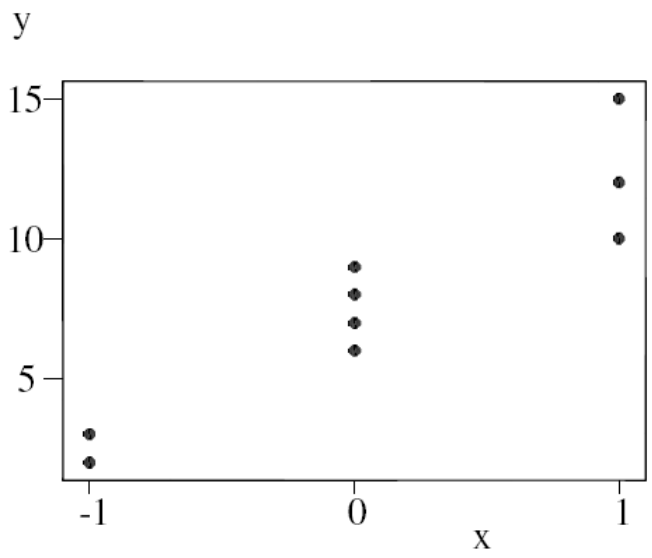

Figure 4.5 *Poisson regression example (data in Table 4.3).*

R-output from simple example

```
> y <- c(2,3,6,7,8,9,10,12,15)
> x <- c(-1,-1,0,0,0,0,1,1,1)
> ulyk <- data.frame(cbind(x,y))
> M1 <- glm(y ~x,data=ulyk,family=poisson)
> summary(M1)

Call:
glm(formula = y ~ x, family = poisson, data = ulyk)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-0.8472  -0.2601  -0.2137   0.5214   0.8788

Coefficients:
            Estimate Std. Error z value Pr(>|z|)
(Intercept)   1.8893     0.1421  13.294  < 2e-16 ***
x             0.6698     0.1787   3.748 0.000178 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for poisson family taken to be 1)

    Null deviance: 18.4206  on 8  degrees of freedom
Residual deviance:  2.9387  on 7  degrees of freedom
AIC: 41.052

Number of Fisher Scoring iterations: 4
```