Prediction and fitted values of model m1

```
> predict(m1,type="response")
        mu1       mu2       mu3
1 0.2903467 0.1854525 0.5242007
2 0.4015290 0.3638872 0.2345838
3 0.2644265 0.6379952 0.0975783
4 0.2451446 0.1023790 0.6524764
5 0.4075272 0.2414790 0.3509939
6 0.3203514 0.5053729 0.1742757
> predict(m1,type="response")*c(45,45,60,65,44,41)
       mu1       mu2       mu3
1 13.06560  8.345364 23.589033
2 18.06881 16.374925 10.556270
3 15.86559 38.279711  5.854697
4 15.93440  6.654636 42.410967
5 17.93119 10.625075 15.443730
6 13.13441 20.720289  7.145303
```


Pearson residuals and Pearson's X^2

```
> expected <- predict(m1,type="response")*c(45,45,60,65,44,41)
> observed <- cbind(data81[,2],data81[,3],data81[,1])
> (observed-expected)/sqrt(expected)
         mu1        mu2        mu3
1 -0.2948023 -0.4657117  0.4964052
2  0.6895726 -0.3397734 -0.4789936
3 -0.4683693  0.4396739 -0.3532321
4  0.2669487  0.5215280 -0.3702135
5 -0.6922135  0.4218062  0.3960125
6  0.5147677 -0.5976090  0.3197438
> Pearsres <- (observed-expected)/sqrt(expected)
> sum(Pearsres^2)
[1] 3.926636
```

Ordinal logistic regression, parallel, see (8.13)

```
> mord1 <- vglm(cbind(data81[,1],data81[,2],data81[,3]) ~ sex+age,
cumulative(parallel=TRUE), data81)
> summary(mord1)

Call:
vglm(formula = cbind(data81[, 1], data81[, 2], data81[, 3]) ~
    sex + age, family = cumulative(parallel = TRUE), data = data81)

Coefficients:
                  Value Std. Error  t value
(Intercept):1  0.043538    0.23030  0.18905
(Intercept):2  1.654977    0.25360  6.52593
sexmale        0.576222    0.22611  2.54841
age24-40      -1.147099    0.27727 -4.13705
age41+        -2.232457    0.29042 -7.68712

Number of linear predictors:  2

Names of linear predictors: logit(P[Y<=1]), logit(P[Y<=2])

Dispersion Parameter for cumulative family:   1

Residual Deviance: 4.53207 on 7 degrees of freedom

Log-likelihood: -290.6478 on 7 degrees of freedom

Number of Iterations: 4
```



```
> expected <- predict(mord1,type="response")*c(45,45,60,65,44,41)
> expected
        mu1      mu2       mu3
1 22.989725 14.79059  7.219685
2 11.208297 16.88560 16.906100
3  6.044998 15.52576 38.429244
4 42.260664 16.43541  6.303931
5 16.330065 16.54978 11.120156
6  6.814796 13.67231 20.512892
> observed <- cbind(data81[,1],data81[,2],data81[,3])
> observed
     [,1] [,2] [,3]
[1,]   26   12    7
[2,]    9   21   15
[3,]    5   14   41
[4,]   40   17    8
[5,]   17   15   12
[6,]    8   15   18
> PearsresOrd <- (observed-expected)/sqrt(expected)
> PearsresOrd
         mu1        mu2        mu3
1  0.6278260 -0.7256102 -0.0817599
2 -0.6596107  1.0012624 -0.4635792
3 -0.4250279 -0.3872212  0.4146960
4 -0.3477506  0.1392665  0.6755195
5  0.1657824 -0.3809548  0.2638457
6  0.4540112  0.3590668 -0.5548306
> sum(PearsresOrd^2)
[1] 4.56421
```

Ordinal logistic regression, non-parallel, see (8.12)

```
> mord2 <- vglm(cbind(data81[,1],data81[,2],data81[,3]) ~ sex+age,
cumulative(parallel=FALSE), data81)
> summary(mord2)

Call:
vglm(formula = cbind(data81[, 1], data81[, 2], data81[, 3]) ~
    sex + age, family = cumulative(parallel = FALSE), data = data81)

Coefficients:
                  Value Std. Error  t value
(Intercept):1  0.064276    0.24864  0.25851
(Intercept):2  1.526134    0.30782  4.95783
sexmale:1      0.590710    0.26736  2.20945
sexmale:2      0.572248    0.26851  2.13122
age24-40:1    -1.246406    0.30572 -4.07695
age24-40:2    -0.949030    0.36284 -2.61556
age41+:1      -2.259923    0.35864 -6.30132
age41+:2      -2.102680    0.34508 -6.09329

Number of linear predictors:  2

Names of linear predictors: logit(P[Y<=1]), logit(P[Y<=2])

Dispersion Parameter for cumulative family:   1

Residual Deviance: 3.8306 on 4 degrees of freedom

Log-likelihood: -290.2970 on 4 degrees of freedom

Number of Iterations: 5
```


8.4.1 Cumulative logit model

The cumulative odds for the jth category is

$$\frac{P(z \leq C_j)}{P(z > C_j)} = \frac{\pi_1 + \pi_2 + ... + \pi_j}{\pi_{j+1} + ... + \pi_J};$$

see Figure 8.2. The cumulative logit model is

$$\log \frac{\pi_1 + ... + \pi_j}{\pi_{j+1} + ... + \pi_J} = \mathbf{x}_j^T \boldsymbol{\beta}_j. \qquad (8.12)$$

8.4.2 Proportional odds model

If the linear predictor $\mathbf{x}_j^T \boldsymbol{\beta}_j$ in (8.12) has an intercept term β_{0j} which depends on the category j, but the other explanatory variables do not depend on j, then the model is

$$\log \frac{\pi_1 + ... + \pi_j}{\pi_{j+1} + ... + \pi_J} = \beta_{0j} + \beta_1 x_1 + ... + \beta_{p-1} x_{p-1}. \qquad (8.13)$$

Table 8.4 *Results of proportional odds ordinal regression model (8.16) for the data in Table 8.1.*

Parameter	Estimate b	Standard error, s.e.(b)	Odds ratio OR (95% confidence interval)
β_{01}	-1.655	0.256	
β_{02}	-0.044	0.232	
β_1 : men	-0.576	0.226	0.56 (0.36, 0.88)
$\beta_2 : 24 - 40$	1.147	0.278	3.15 (1.83, 5.42)
$\beta_3 : > 40$	2.232	0.291	9.32 (5.28, 16.47)